Lesson 14–7

Example 1 Solve Equations for a Given Interval

Find all solutions of each equation for the given interval.

a. $4(1 - \cos^2 \theta) = 1;\ 0^\circ \leq \theta \leq 360^\circ$

$4(1 - \cos^2 \theta) = 1$	Original equation
$4(\sin^2 \theta) = 1$	$1 - \cos^2 \theta = \sin^2 \theta$
$\sin^2 \theta = \frac{1}{4}$	Divide each side by 4.
$\sin \theta = \pm\frac{1}{2}$	Take the square root of each side.
$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$	

The solutions are 30˚, 150˚, 210˚, and 330˚.

b. $\sin \theta \cos \theta = -\sin \theta;\ 0 \leq \theta < \frac{3\pi}{2}$

$\sin \theta \cos \theta = -\sin \theta$	Original equation
$\sin \theta \cos \theta + \sin \theta = 0$	Solve for 0.
$\sin \theta (\cos \theta + 1) = 0$	Factor.

Use the Zero Product Property.

$\sin \theta = 0$ or $\cos \theta + 1 = 0$

$\theta = 0$ or π

$\cos \theta = -1$

$\theta = \pi$

The solutions are 0 and π.

Example 2 **Solve Trigonometric Equations.**

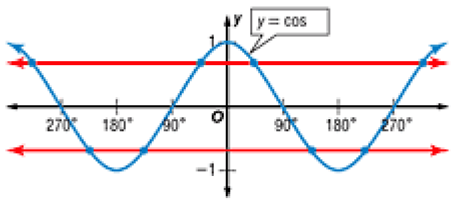
a. Solve $2\cos^2\theta = 1$ for all values of θ if θ is measured in radians.

$2\cos^2\theta = 1$ — Original equation

$\cos^2\theta = \frac{1}{2}$ — Divide each side by 2.

$\cos\theta = \pm\sqrt{\frac{1}{2}}$ or $\pm\frac{\sqrt{2}}{2}$ — Take the square root of each side.

Look at the graph of $y = \cos\theta$ to find solutions of $\cos\theta = \pm\frac{\sqrt{2}}{2}$



The solutions are $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, and $\frac{7\pi}{4}$, and so on, and $-\frac{\pi}{4}$, $-\frac{3\pi}{4}$, $-\frac{5\pi}{4}$, and $-\frac{7\pi}{4}$, and so on. The solutions in the interval 0 to 2π are $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, and $\frac{7\pi}{4}$. The period of the cosine function is 2π radians, but $\frac{\pi}{4} + \pi = \frac{5\pi}{4}$ and $\frac{3\pi}{4} + \pi = \frac{7\pi}{4}$. So the solutions can be written as $\frac{\pi}{4} + k\pi$ and $\frac{3\pi}{4} + k\pi$.

b. Solve $2 \tan \theta \cos \theta = \tan \theta$ if θ is measured in degrees.

$2 \tan \theta \cos \theta = \tan \theta$	Original equation
$2 \tan \theta \cos \theta - \tan \theta = 0$	Solve for 0.
$\tan \theta (2 \cos \theta - 1) = 0$	Factor.

Solve for θ in the interval 0° to 360°.

$\tan \theta = 0$ or $2 \cos \theta - 1 = 0$

$\theta = 0^\circ$ or 180°

$\cos \theta = \frac{1}{2}$

$\theta = 60^\circ$ or 300°

The solutions are $0^\circ + k \cdot 180^\circ$, $60^\circ + k \cdot 360^\circ$, and $300^\circ + k \cdot 360^\circ$.

Example 3 Solve Trigonometric Equations Using Identities

Solve $\cos \theta \tan \theta - 2 \cos^2 \theta = -1$.

$\cos \theta \tan \theta - 2 \cos^2 \theta = -1$	Original equation.
$\cos \theta \left(\frac{\sin \theta}{\cos \theta}\right) - 2 \cos^2 \theta = -1$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
$\sin \theta - 2 \cos^2 \theta = -1$	Multiply.
$\sin \theta - 2(1 - \sin^2 \theta) = -1$	$\cos^2 + \sin^2 \theta = 1$
$\sin \theta - 2 + 2 \sin^2 \theta = -1$	Distributive Property
$2 \sin^2 \theta + \sin \theta - 1 = 0$	Rearrange terms and solve for 0.
$(2 \sin \theta - 1)(\sin \theta + 1) = 0$	Factor.

$2 \sin \theta - 1 = 0$ or $\sin \theta + 1 = 0$

$\sin \theta = \frac{1}{2}$ $\quad$ $\sin \theta = -1$

$\theta = 30^\circ$ and 150° $\quad$ $\theta = 270^\circ$

CHECK

$\theta = 30^\circ$

$$\cos \theta \tan \theta - 2 \cos^2 \theta = -1$$

$$\cos 30^\circ \tan 30^\circ - 2 \cos^2 30^\circ = -1$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{3} - 2 \cdot \left(\frac{\sqrt{3}}{2}\right)^2 \stackrel{?}{=} -1$$

$$-1 = -1 \checkmark$$

$\theta = 150^\circ$

$$\cos \theta \tan \theta - 2 \cos^2 \theta = -1$$

$$\cos 150^\circ \tan 150^\circ - 2 \cos^2 150^\circ = -1$$

$$-\frac{\sqrt{3}}{2} \cdot \left(-\frac{\sqrt{3}}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)^2 \stackrel{?}{=} -1$$

$$-1 = -1 \checkmark$$

$$\cos \theta \tan \theta - 2 \cos^2 \theta = -1$$

$$\cos 270^\circ \tan 270^\circ - 2 \cos^2 270^\circ = -1$$

The expression $\tan 270^\circ$ is undefined.
Thus, 270° is not a solution.

The solutions are $30^\circ + k \cdot 360^\circ$ and $150^\circ + k \cdot 360^\circ$.

Example 4 Determine Whether a Solution Exists

Solve $2\cos^2\theta = 3\sin\theta$.

$2\cos^2\theta = 3\sin\theta$	Original equation
$2(1-\sin^2\theta) = 3\sin\theta$	$\cos^2 + \sin^2\theta = 1$
$2 - 2\sin^2\theta = 3\sin\theta$	Distributive Property
$2 - 2\sin^2\theta - 3\sin\theta = 0$	Solve for 0.
$2\sin^2\theta + 3\sin\theta - 2 = 0$	Rearrange terms and multiply by –1.
$(2\sin\theta - 1)(\sin\theta + 2) = 0$	Factor.

$2\sin\theta - 1 = 0$ or $\sin\theta + 2 = 0$

$2\sin\theta = 1$ $\quad$ $\sin\theta = -2$

$\sin\theta = \frac{1}{2}$ $\quad$ Not possible since $\sin\theta$ cannot be less than –1.

$\theta = 30^\circ$ or 150°

Thus, the solutions are $30^\circ + k \cdot 360^\circ$ and $150^\circ + k \cdot 360^\circ$.

Example 5 Use a Trigonometric Equation

WEATHER The monthly normal high temperatures for Cairns, Australia, can be approximately modeled by the sine function $y = 2.5\sin\left(\frac{\pi}{6}x - 5.236\right) + 29.5$, where y is the temperature and x is the integer representing the month. The months are represented by the integers 1, 2, …, 12 and the temperature is in degrees Celsius.

a. In approximately what month is the temperature 27°C?

$y = 2.5\sin\left(\frac{\pi}{6}x - 5.236\right) + 29.5$	Original equation
$27 = 2.5\sin\left(\frac{\pi}{6}x - 5.236\right) + 29.5$	$y = 27$; The temperature is 27°C.
$-2.5 = 2.5\sin\left(\frac{\pi}{6}x - 5.236\right)$	Subtract 29.5 from each side.
$-1 = \sin\left(\frac{\pi}{6}x - 5.236\right)$	Divide each side by 2.5.
$-1.57 \approx \frac{\pi}{6}x - 5.236$	$\mathrm{Sin}^{-1}\,{-1} \approx -1.57$ radians
$3.666 \approx \frac{\pi}{6}x$	Add 5.236 to each side.
$7.0 \approx x$	Divide each side by $\frac{\pi}{6}$.

Since this is just an estimate and the months are given in integers, the temperature of 27°C occurs in month 7 which is July.

b. How do the temperatures in Cairns, Australia, compare to temperatures in the U.S?

If you graph the function $y = 2.5\ \sin\left(\frac{\pi}{6}x - 5.236\right) + 29.5$ on a calculator, you can see that the temperatures in Cairns are high in our winter months and lower in our summer months. That is because Australia is in the Southern Hemisphere.