Lesson 14-6

# Example 1 Double-Angle Formulas

Find the exact value of each expression if  $\cos \theta = -\frac{2}{5}$  and  $\theta$  is between 180° and 270°.

### a. $\sin 2\theta$

Use the identity  $\sin 2\theta = 2 \sin \theta \cos \theta$ . First, find the value of  $\cos \theta$ .

$$\sin^{2} \theta = 1 - \cos^{2} \theta \qquad \cos^{2} \theta + \sin^{2} \theta = 1$$
  

$$\sin^{2} \theta = 1 - (-\frac{2}{5})^{2} \qquad \cos \theta = -\frac{2}{5}$$
  

$$\sin^{2} \theta = \frac{21}{25} \qquad \text{Subtract.}$$
  

$$\sin \theta = \pm \frac{\sqrt{21}}{5} \qquad \text{Take the square root of each side.}$$

Since  $\theta$  is in the third quadrant, sin is negative. Thus, sin  $\theta = -\frac{\sqrt{21}}{5}$ .

Now find  $\sin 2\theta$ .  $\sin 2\theta = 2 \sin \theta$  co

$$\sin 2\theta = 2 \sin \theta \cos \theta \qquad \text{Double-Angle Formula} \\ \sin 2\theta = 2(-\frac{\sqrt{21}}{5})(-\frac{2}{5}) \qquad \sin \theta = -\frac{\sqrt{21}}{5}, \cos \theta = -\frac{2}{5} \\ = \frac{4\sqrt{21}}{25} \qquad \text{Multiply.}$$

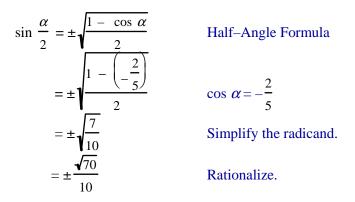
The value of 
$$\sin 2\theta$$
 is  $\frac{4\sqrt{21}}{25}$ 

## b. $\cos 2\theta$

Use the identity 
$$\cos 2\theta = 1 - 2 \sin^2 \theta$$
.  
 $\cos 2\theta = 1 - 2 \sin^2 \theta$  Double-Angle Formula  
 $= 1 - 2(-\frac{\sqrt{21}}{5})^2$   $\sin \theta = -\frac{\sqrt{21}}{5}$   
 $= -\frac{17}{25}$  Simplify.

## Example 2 Half–Angle Formulas

Find  $\sin \frac{\alpha}{2}$  if  $\cos \alpha = -\frac{2}{5}$  and  $\alpha$  is in the second quadrant.



Since  $\alpha$  is between 90° and 180°,  $\frac{\alpha}{2}$  is between 45° and 90°. Thus,  $\sin \frac{\alpha}{2}$  is positive and equals  $\frac{\sqrt{70}}{10}$ .

# **Example 3** Evaluate Using Half–Angle Formulas

Find the exact value of each expression by using the half–angle formulas. a. cos 112.5\*

$$\cos 112.5 = \cos \frac{225}{2}$$

$$= -\sqrt{\frac{1+\cos 225^{\circ}}{2}}$$

$$= -\sqrt{\frac{1+\cos 225^{\circ}}{2}}$$

$$= -\sqrt{\frac{1+\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= -\sqrt{\frac{1+\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= -\sqrt{\frac{2-\sqrt{2}}{2}}$$

$$= -\sqrt{\frac{2-\sqrt{2}}{4}}$$
Simplify the radicand.
$$= -\frac{\sqrt{2-\sqrt{2}}}{2}$$
Simplify the denominator.

b. 
$$\sin \frac{5\pi}{8}$$
  
 $\sin \frac{5\pi}{8} = \sin \frac{5\pi}{2}$   
 $= \sqrt{\frac{1 - \cos \alpha}{2}}$   
 $= \sqrt{\frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2}}$   
 $= \sqrt{\frac{2 + \sqrt{2}}{2}}$   
 $= \frac{\sqrt{2 + \sqrt{2}}}{2}$   
 $\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}; \frac{5\pi}{8} \text{ is in the second quadrant where sine is positive}}$   
 $\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$   
Simplify the radicand.  
 $= \frac{\sqrt{2 + \sqrt{2}}}{2}$   
Simplify the denominator.

Example 4 Verify Identities Verify that  $\left(\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}\right) = \frac{\sin x}{2}$  is an identity.  $\left(\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}\right) \stackrel{?}{=} \frac{\sin x}{2}$ Original equation  $\frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2} \stackrel{?}{=} \frac{\sin x}{2}$ Rewrite.  $\frac{\sin 2\left(\frac{x}{2}\right)}{\frac{2}{2}} \stackrel{?}{=} \frac{\sin x}{\frac{2}{2}}$ Simplify.  $\frac{\sin x}{2} = \frac{\sin x}{2}$  $\frac{x}{2}$ .