Lesson 14–5

Example 1 Use Sum and Difference of Angles Formulas

Find the exact value of each expression.

a. sin 150˚

Use the formula $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

$\sin 150^\circ = \sin(90^\circ + 60^\circ)$ — $\alpha = 90^\circ$, $\beta = 60^\circ$

$= \sin 90^\circ \cos 60^\circ + \cos 90^\circ \sin 60^\circ$

$= \left(1 \cdot \frac{1}{2}\right) + \left(0 \cdot \frac{\sqrt{3}}{2}\right)$ — Evaluate each expression.

$= \frac{1}{2}$ — Multiply and simplify.

b. cos (–165˚)

Use the formula $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

$\cos(-165^\circ) = \cos(60^\circ - 225^\circ)$ — $\alpha = 60^\circ$, $\beta = 225^\circ$

$= \cos 60^\circ \cos 225^\circ + \sin 60^\circ \sin 225^\circ$

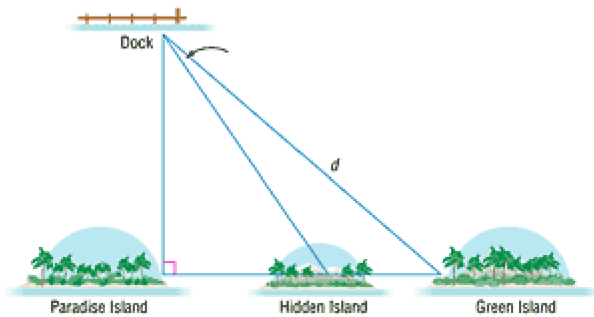
$= \left[\frac{1}{2}\cdot\left(-\frac{\sqrt{2}}{2}\right)\right] + \left[\frac{\sqrt{3}}{2}\cdot\left(-\frac{\sqrt{2}}{2}\right)\right]$ — Evaluate each expression.

$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$ — Multiply.

$= \frac{-\sqrt{2} - \sqrt{6}}{4}$ — Simplify.

Example 2 Use Sum and Difference Formulas to Solve a Problem

TRAVEL Ian plans to sail from a dock on the mainland to Green Isle by the shortest route, *d*. He knows that the distance from Paradise Island to Green Isle is 300 miles. He also knows that sin α = 0.6428 and sin β = 0.2079.



a. Find sin ($\alpha + \beta$).

To find sin ($\alpha + \beta$), you can use the sum formula for sine. However, you will need the cosine of each angle to use in the formula. Use the identity $\sin^2\theta + \cos^2\theta = 1$ to find cos α and cos β.

Find cos α.

$\sin^2 \alpha + \cos^2 \alpha = 1$	Trigonometric identity
$(0.6428)^2 + \cos^2 \alpha = 1$	$\sin \alpha = 0.6428$
$\cos^2 \alpha = 1 - (0.6428)^2$	Subtract $(0.6428)^2$ from each side.
$\cos^2 \alpha \approx 0.5868$	Simplify.
$\cos \alpha \approx 0.7660$	Take the square root of each side.

Find $\cos \beta$.

$\sin^2 \beta + \cos^2 \beta = 1$	Trigonometric identity
$(0.2079)^2 + \cos^2 \beta = 1$	$\sin \beta = 0.2079$
$\cos^2 \beta = 1 - (0.2079)^2$	Subtract $(0.2079)^2$ from each side.
$\cos^2 \beta \approx 0.9568$	Simplify.
$\cos \beta \approx 0.9782$	Take the square root of each side.

Now use the sum formula to find $\sin(\alpha + \beta)$.

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$
$$= [(0.6428) \cdot (0.9782)] + [(0.7660) \cdot (0.2079)]$$
$$\approx 0.7880$$

Therefore, $\sin(\alpha + \beta) \approx 0.7880$.

b. Find *d*.

Use the sine ratio to find d.

$\sin(\alpha + \beta) = \dfrac{300}{d}$	Sine ratio
$0.7880 = \dfrac{300}{d}$	$\sin(\alpha + \beta) \approx 0.7880$
$d = \dfrac{300}{0.7880}$	Solve for d.
≈ 380.7	

Therefore, $d \approx 380.7$ miles.

Example 3 Verify Identities

Verify that each of the following is an identity.

a. $\sin(\theta - 270^\circ) = \cos\theta$

$\sin(\theta - 270^\circ) \stackrel{?}{=} \cos\theta$	Original equation
$\sin\theta\cos 270^\circ - \cos\theta\sin 270^\circ \stackrel{?}{=} \cos\theta$	Difference of Angles Formula
$\sin\theta\cdot(0) - \cos\theta\cdot(-1) \stackrel{?}{=} \cos\theta$	Evaluate each expression.
$\cos\theta = \cos\theta$	Simplify.

b. $\cos\left(\frac{\pi}{6} + \theta\right) - \cos\left(\frac{\pi}{6} - \theta\right) = -\sin\theta$

$\cos\left(\frac{\pi}{6} + \theta\right) - \cos\left(\frac{\pi}{6} - \theta\right) \stackrel{?}{=} -\sin\theta$	Original equation
$\left(\cos\frac{\pi}{6}\cos\theta - \sin\frac{\pi}{6}\sin\theta\right) - \left(\cos\frac{\pi}{6}\cos\theta + \sin\frac{\pi}{6}\sin\theta\right) \stackrel{?}{=} -\sin\theta$	Sum of Angles and Difference of Angles Formulas
$\left(\frac{\sqrt{3}}{2}\cdot\cos\theta - \frac{1}{2}\cdot\sin\theta\right) - \left(\frac{\sqrt{3}}{2}\cdot\cos\theta + \frac{1}{2}\cdot\sin\theta\right) \stackrel{?}{=} -\sin\theta$	Evaluate each expression.
$-\sin\theta = -\sin\theta$	Simplify.