Points, Lines, and Planes

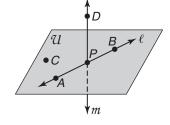
Get Ready for the Lesson

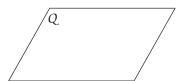
Read the introduction to Lesson 1-1 in your textbook.

- Find three pencils of different lengths and hold them upright on your desk so that the three pencil points do not lie along a single line. Can you place a flat sheet of paper or cardboard so that it touches all three pencil points?
- How many ways can you do this if you keep the pencil points in the same position?
- How will your answer change if there are four pencil points?

Read the Lesson

- 1. Complete each sentence.
 - **a.** Points that lie on the same lie are called points.
 - **b.** Points that do not lie in the same plane are called points.
 - **c.** There is exactly one through any two points.
 - **d.** There is exactly one through any three noncollinear points.
- **2.** Refer to the figure at the right. Indicate whether each statement is true or false.
 - **a.** Points *A*, *B*, and *C* are collinear.
 - **b.** The intersection of plane ABC and line m is point P.
 - **c.** Line ℓ and line m do not intersect.
 - **d.** Points A, P, and B can be used to name plane \mathcal{U} .
 - **e.** Line ℓ lies in plane ACB.
- **3.** Complete the figure at the right to show the following relationship: Lines ℓ , m, and n are coplanar and lie in plane Q. Lines ℓ and m intersect at point P. Line n intersects line m at R, but does not intersect line ℓ .





Remember What You Learned

4. Recall or look in a dictionary to find the meaning of the prefix co-. What does this prefix mean? How can it help you remember the meaning of *collinear*?

Lesson Reading Guide

Linear Measure and Precision

Get Ready for the Lesson

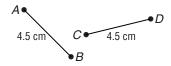
Read the introduction to Lesson 1-2 in your textbook.

- Why was it problematic for the Egyptians to define the cubit as the length of an arm from the elbow to the fingertips?
- Which unit in the customary system (used in the United States) do you think was originally based on a human body length?

Read the Lesson

- **1.** Explain the difference between a *line* and a *line segment* and why one of these can be measured, while the other cannot.
- **2.** What is the smallest length marked on a 12-inch ruler? What is the smallest length marked on a centimeter ruler?
- 3. Find the precision of each measurement.
 - **a.** 15 cm
 - **b.** 15.0 cm
- **4.** Refer to the figure at the right. Which one of the following statements is true? Explain your answer.

$$\overline{AB} = \overline{CD}$$
 $\overline{AB} \cong \overline{CD}$



- **5.** Suppose that S is a point on \overline{VW} and S is not the same point as V or W. Tell whether each of the following statements is *always*, *sometimes*, or *never* true.
 - **a.** VS = SW
 - **b.** S is between V and W.
 - \mathbf{c} , VS + VW = SW

Remember What You Learned

6. A good way to remember terms used in mathematics is to relate them to everyday words you know. Give three words that are used outside of mathematics that can help you remember that there are 100 centimeters in a meter.

Lesson Reading Guide

Distance and Midpoints

Get Ready for the Lesson

Read the introduction to Lesson 1-3 in your textbook.

- Look at the triangle in the introduction to this lesson. What is the special name for \overline{AB} in this triangle?
- Find *AB* in this figure. Write your answer both as a radical and as a decimal number rounded to the nearest tenth.

Read the Lesson

1. Match each formula or expression in the first column with one of the names in the second column.

a.
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

b.
$$\frac{a+b}{2}$$

c.
$$XY = |a - b|$$

d.
$$c^2 = a^2 + b^2$$

e.
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- v. Midpoint of a Segment on a Number Line
- **2.** Fill in the steps to calculate the distance between the points M(4, -3) and N(-2, 7).

Let
$$(x_1, y_1) = (4, -3)$$
. Then $(x_2, y_2) = (\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$.

$$d = \sqrt{(_{-} - _{-})^2 + (_{-} - _{-})^2}$$

$$MN = \sqrt{(-)^2 + (-)^2}$$

$$MN = \sqrt{(\underline{})^2 + (\underline{})^2}$$

$$MN = \sqrt{}$$

Find a decimal approximation for MN to the nearest hundredth.

Remember What You Learned

3. A good way to remember a new formula in mathematics is to relate it to one you already know. If you forget the Distance Formula, how can you use the Pythagorean Theorem to find the distance *d* between two points on a coordinate plane?

Lesson Reading Guide

Angle Measure

Get Ready for the Lesson

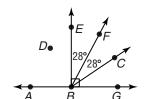
Read the introduction to Lesson 1-4 in your textbook.

- A semicircle is half a circle. How many degrees are there in a semicircle?
- How many degrees are there in a quarter circle?

Read the Lesson

- **1.** Match each description in the first column with one of the terms in the second column. Some terms in the second column may be used more than once or not at all.
 - **a.** a figure made up of two noncollinear rays with a common endpoint
 - b. angles whose degree measures are less than 90
 - c. angles that have the same measure
 - d. angles whose degree measures are between 90 and 180
 - e. a tool used to measure angles
 - f. the common endpoint of the rays that form an angle
 - g. a ray that divides an angle into two congruent angles

- 1. vertex
- 2. angle bisector
- 3. opposite rays
- 4. angle
- **5.** obtuse angles
- 6. congruent angles
- 7. right angles
- 8. acute angles
- 9. compass
- 10. protractor



- 2. Use the figure to name each of the following.
 - a. a right angle
 - **b.** an obtuse angle
 - c. an acute angle
 - **d.** a point in the interior of $\angle EBC$
 - **e.** a point in the exterior of $\angle EBA$
 - **f.** the angle bisector of $\angle EBC$
 - **g.** a point on $\angle CBE$
 - **h.** the sides of $\angle ABF$
 - i. a pair of opposite rays
 - j. the common vertex of all angles shown in the figure
 - k. a pair of congruent angles
 - 1. the angle with the greatest measure

Remember What You Learned

3. A good way to remember related geometric ideas is to compare them and see how they are alike and how they are different. Give some similarities and differences between *congruent segments* and *congruent angles*.

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1-5

Lesson Reading Guide

Angle Relationships

Get Ready for the Lesson

Read the introduction to Lesson 1-5 in your textbook.

- How many separate angles are formed if three lines intersect at a common point? (Do not use an angle whose interior includes part of another angle.)
- How many separate angles are formed if *n* lines intersect at a common point? (Do not count an angle whose interior includes part of another angle.)

Read the Lesson

- **1.** Name each of the following in the figure at the right.
 - a. two pairs of congruent angles
 - **b.** a pair of acute vertical angles
 - **c.** a pair of obtuse vertical angles
 - d. four pairs of adjacent angles
 - e. two pairs of vertical angles
 - f. four linear pairs
 - g. four pairs of supplementary angles
- **2.** Tell whether each statement is *always*, *sometimes*, or *never* true.
 - **a.** If two angles are adjacent angles, they form a linear pair.
 - **b.** If two angles form a linear pair, they are complementary.
 - c. If two angles are supplementary, they are congruent.
 - **d.** If two angles are complementary, they are adjacent.
 - **e.** When two perpendicular lines intersect, four congruent angles are formed.
 - **f.** Vertical angles are supplementary.
 - **g.** Vertical angles are complementary.
 - **h.** The two angles in a linear pair are both acute.
 - **i.** If two angles form a linear pair, one is acute and the other is obtuse.
- **3.** Complete each sentence.
 - **a.** If two angles are supplementary and x is the measure of one of the angles, then the measure of the other angle is ______.
 - **b.** If two angles are complementary and *x* is the measure of one of the angles, then the measure of the other angle is ______.

Remember What You Learned

4. Look up the nonmathematical meaning of *supplementary* in your dictionary. How can this definition help you to remember the meaning of supplementary angles?



Lesson Reading Guide

Two-Dimensional Figures

Get Ready for the Lesson

Read the introduction to Lesson 1-6 in your textbook.

Name four different shapes that can each be formed by four sticks connected to form a closed figure. Assume the sticks can be any length.

Read the Lesson

1. Tell why each figure is *not* a polygon.

a.

b.



2. Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *not regular*.

a.



b.



c.



- 3. What is another name for a regular quadrilateral?
- **4.** Match each polygon in the first column with the formula in the second column that can be used to find its perimeter. (*s* represents the length of each side of a regular polygon.)

a. regular dodecagon

i.
$$P = 8s$$

b. square

ii.
$$P = 6s$$

c. regular hexagon

iii.
$$P = a + b + c$$

d. rectangle

iv.
$$P = 12s$$

e. regular octagon

$$\mathbf{v.}\ P = 2\ell + 2w$$

f. triangle

vi.
$$P = 4s$$

Remember What You Learned

5. One way to remember the meaning of a term is to explain it to another person. How would you explain to a friend what a regular polygon is?

Three-Dimensional Figures

Get Ready for the Lesson

Read the introduction to Lesson 1-7 in your textbook.

Why do you think archeologists would want to know the details of the sizes and shapes of ancient three-dimensional structures?

Read the Lesson

a. b.

1.	Match	each	descri	iption	from	the	first	colu	ımn	with	one	of the	terms	from	the	second
	column	n. (So	me of	the te	erms i	nay	be u	sed r	nore	thar	n one	ce or r	not at a	ıll.)		

a polyhedron with two parallel congruent bases	i.	octahedro
the set of points in space that are a given distance from a	ii.	face
	iii.	icosahedro

- **c.** a regular polyhedron with eight faces
- **d.** a polyhedron that has all faces but one intersecting at one point
- e. a line segment where two faces of a polyhedron intersect
- **f.** a solid with congruent circular bases in a pair of parallel planes
- **g.** a regular polyhedron whose faces are squares
- **h.** a flat surface of a polyhedron

- iv. edge
- v. prism
- vi. dodecahedron
- vii. cylinder
- viii. sphere
- ix. cone
- x. hexahedron
- xi. pyramid
- xii. tetrahedron

2. Fi	ll in	the	missing	numbers,	words,	or	phrases	to	complete	each	sentence
--------------	-------	-----	---------	----------	--------	----	---------	----	----------	------	----------

a.	A triangular prism has vertices. It has faces: bases are congruent
	, and faces are parallelograms.
b.	A regular octahedron has vertices and faces. Each face is a(n)
c.	A hexagonal prism has vertices. It has faces: of them are the bases,
	which are congruent, and the other faces are parallelograms
d.	An octagonal pyramid has vertices and faces. The base is a(n)
	, and the other faces are
e.	There are exactly types of regular polyhedra. These are called the
	solids. A polyhedron whose faces are regular pentagons is a
	which has faces

Remember What You Learned

3. A good way to remember the characteristics of geometric solids is to think about how different solids are alike. Name a way in which pyramids and cones are alike.

Lesson Reading Guide

Inductive Reasoning and Conjecture

Get Ready for the Lesson

Read the introduction to Lesson 2-1 in your textbook.

- How could people in the ancient Orient use inductive reasoning be used to assist in farming?
- Give an example of when you might use inductive reasoning in your daily life.

Read the Lesson

- **1.** Explain in your own words the relationship between a conjecture, a counterexample, and inductive reasoning.
- 2. Make a conjecture about the next item in each sequence.

b.
$$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$$

- ${f 3.}$ State whether each conjecture is true or false. If the conjecture is false, give a counterexample.
 - a. The sum of two odd integers is even.
 - **b.** The product of an odd integer and an even integer is odd.
 - **c.** The opposite of an integer is a negative integer.
 - **d.** The perfect squares (squares of whole numbers) alternate between odd and even.

Remember What You Learned

4. Write a short sentence that can help you remember why it only takes one counterexample to prove that a conjecture is false.

Lesson 2-1

Lesson Reading Guide

Logic

Get Ready for the Lesson

Read the introduction to Lesson 2-2 in your textbook.

How can you use logic to help you answer a multiple-choice question on a standardized test if you are not sure of the correct answer?

Read the Lesson

1.	Su	pply one or two words to complete each sentence.
	a.	Two or more statements can be joined to form a statement.
		A statement that is formed by joining two statements with the word <i>or</i> is called a
		The truth or falsity of a statement is called its
•	d.	A statement that is formed by joining two statements with the word <i>and</i> is called a
•	е.	A statement that has the opposite truth value and the opposite meaning from a give statement is called the of the statement.
2.	Us	se <i>true</i> or <i>false</i> to complete each sentence.
	a.	If a statement is true, then its negation is
		If a statement is false, then its negation is
		If two statements are both true, then their conjunction is and
		their disjunction is
	d.	If two statements are both false, then their conjunction is and
		their disjunction is
		If one statement is true and another is false, then their conjunction is
		and their disjunction is
3.	Со	nsider the following statements:
-		Chicago is the capital of Illinois. q : Sacramento is the capital of California.
•	W_1	rite each statement symbolically and then find its truth value.

Remember What You Learned

a. Sacramento is not the capital of California.

4. Prefixes can often help you to remember the meaning of words or to distinguish between similar words. Use your dictionary to find the meanings of the prefixes *con* and *dis* and explain how these meanings can help you remember the difference between a *conjunction* and a *disjunction*.

b. Sacramento is the capital of California and Chicago is not the capital of Illinois.

Conditional Statements

Get Ready for the Lesson

Read the introduction to Lesson 2-3 in your textbook.

Does the second advertising statement in the introduction mean that you will not get a free phone if you sign a contract for only six months of service? Explain your answer.

Read the Lesson

- **1.** Identify the hypothesis and conclusion of each statement.
 - **a.** If you are a registered voter, then you are at least 18 years old.
 - **b.** If two integers are even, their product is even.
- 2. Complete each sentence.
 - **a.** The statement that is formed by replacing both the hypothesis and the conclusion of a conditional with their negations is the
 - **b.** The statement that is formed by exchanging the hypothesis and conclusion of a conditional is the ______.
- **3.** Consider the following statement:

You live in North America if you live in the United States.

- a. Write this conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.
- **b.** Write the inverse of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.
- c. Write the contrapositive of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.
- **d.** Write the converse of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.

Remember What You Learned

4. When working with a conditional statement and its three related conditionals, what is an easy way to remember which statements are logically equivalent to each other?

Lesson 2-3

Lesson Reading Guide

Deductive Reasoning

Get Ready for the Lesson

Read the introduction to Lesson 2-4 in your textbook.

Suppose a doctor wants to use the dose chart in your textbook to prescribe an antibiotic, but the only scale in her office gives weights in pounds. How can she use the fact that 1 kilogram is about 2.2 pounds to determine the correct dose for a patient?

Read the Lesson

If s, t, and u are three statements, match each description from the list on the left with a symbolic statement from the list on the right.

	_	_
1	negation	of t
1.	певаион	$or \iota$

2. conjunction of
$$s$$
 and u

3. converse of
$$s \to t$$

4. disjunction of
$$s$$
 and u

6. contrapositive of
$$s \to t$$

7. inverse of
$$s \to u$$

8. contrapositive of
$$s \to u$$

10. negation of
$$\sim t$$

a.
$$s \setminus l$$

b.
$$[(s \to t) \land s] \to t$$

c.
$$\sim s \rightarrow \sim u$$

d.
$$\sim u \rightarrow \sim s$$

e.
$$\sim t$$

f.
$$[(u \rightarrow t) \land (t \rightarrow s)] \rightarrow (u \rightarrow s)$$

g.
$$s \wedge u$$

h.
$$t \rightarrow s$$

j.
$$\sim t \rightarrow \sim s$$

- **11.** Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.
 - **a.** (1) Every square is a parallelogram.
 - (2) Every parallelogram is a polygon.
 - (3) Every square is a polygon.
 - **b.** (1)If two lines that lie in the same plane do not intersect, they are parallel.
 - (2) Lines ℓ and m lie in plane $\mathcal U$ and do not intersect.
 - (3) Lines ℓ and m are parallel.
 - c. (1) Perpendicular lines intersect to form four right angles.
 - (2) $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are four right angles.
 - (3) $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are formed by intersecting perpendicular lines.

Remember What You Learned

12. A good way to remember something is to explain it to someone else. Suppose that a classmate is having trouble remembering what the Law of Detachment means?



Postulates and Paragraph Proofs

Get Ready for the Lesson

Read the introduction to Lesson 2-5 in your textbook.

Postulates are often described as statements that are so basic and so clearly correct that people will be willing to accept them as true without asking for evidence or proof. Give a statement about numbers that you think most people would accept as true without evidence.

Read the Lesson

- **1.** Determine whether each of the following is a *correct* or *incorrect* statement of a geometric postulate. If the statement is incorrect, replace the underlined words to make the statement correct.
 - **a.** A plane contains at least two points that do not lie on the same line.
 - **b.** If two planes intersect, then the intersection is a line.
 - **c.** Through any four points not on the same line, there is exactly one plane.
 - d. A line contains at least one point.
 - e. If two lines are parallel, then their intersection is exactly one point.
 - f. Through any two points, there is at most one line.
- **2.** Determine whether each statement is *always*, *sometimes*, or *never* true. If the statement is not always true, explain why.
 - a. If two planes intersect, their intersection is a line.
 - **b.** The midpoint of a segment divides the segment into two congruent segments.
 - **c.** There is exactly one plane that contains three collinear points.
 - **d.** If two lines intersect, their intersection is one point.
- **3.** Use the walls, floor, and ceiling of your classroom to describe a model for each of the following geometric situations.
 - **a.** two planes that intersect in a line
 - **b.** two planes that do not intersect
 - **c.** three planes that intersect in a point

Remember What You Learned

4. A good way to remember a new mathematical term is to relate it to a word you already know. Explain how the idea of a mathematical *theorem* is related to the idea of a scientific *theory*.

PERIOD

Lesson Reading Guide

Algebraic Proof

Get Ready for the Lesson

Read the introduction to Lesson 2-6 in your textbook.

What are some of the things that lawyers might use in presenting their closing arguments to a trial jury in addition to evidence gathered prior to the trial and testimony heard during the trial?

Read the Lesson

1. Name the property illustrated by each statement.

a. If
$$a = 4.75$$
 and $4.75 = b$, then $a = b$.

b. If
$$x = y$$
, then $x + 8 = y + 8$.

c.
$$5(12 + 19) = 5 \cdot 12 + 5 \cdot 19$$

d. If x = 5, then x may be replaced with 5 in any equation or expression.

e. If
$$x = y$$
, then $8x = 8y$.

f. If
$$x = 23.45$$
, then $23.45 = x$.

g. If
$$5x = 7$$
, then $x = \frac{7}{5}$.

h. If
$$x = 12$$
, then $x - 3 = 9$.

2. Give the reason for each statement in the following two-column proof.

Given:
$$5(n-3) = 4(2n-7) - 14$$

Prove: n = 9

Statements	Reasons
1. 5(n-3)=4(2n-7)-14	1
2. $5n - 15 = 8n - 28 - 14$	2
3.5n - 15 = 8n - 42	3
4.5n - 15 + 15 = 8n - 42 + 15	4
5. $5n = 8n - 27$	5
6. $5n - 8n = 8n - 27 - 8n$	6
7. $-3n = -27$	7
$8. \frac{-3n}{-3} = \frac{-27}{-3}$	8
9. $n = 9$	9

Remember What You Learned

3. A good way to remember mathematical terms is to relate them to words you already know. Give an everyday word that is related in meaning to the mathematical term *reflexive* and explain how this word can help you to remember the Reflexive Property and to distinguish it from the Symmetric and Transitive Properties.

Proving Segment Relationships

Get Ready for the Lesson

Read the introduction to Lesson 2-7 in your textbook.

- What is the total distance that the plane will fly to get from San Diego to Dallas?
- Before leaving home, a passenger used a road atlas to determine that the distance between San Diego and Dallas is about 1350 miles. Why is the flying distance greater than that?

Read the Lesson

1. If *E* is between *Y* and *S*, which of the following statements are *always* true?

$$\mathbf{A.}\ YS + ES = YE$$

B.
$$YS - ES = YE$$

C.
$$YE > ES$$

D.
$$YE \cdot ES = YS$$

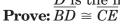
$$\mathbf{E.} \, SE \, + \, EY = SY$$

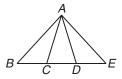
F.
$$E$$
 is the midpoint of \overline{YS} .

2. Give the reason for each statement in the following two-column proof.

Given: *C* is the midpoint of *BD*.

D is the midpoint of \overline{CE} .





Statements

Reasons

- **1.** *C* is the midpoint of *BD*.
- 2.BC = CD
- **3.** D is the midpoint of CE.
- 4. CD = DE
- 5.BC = DE
- 6.BC + CD = CD + DE
- 7.BC + CD = BD
 - CD + DE = CE
- 8.BD = CE
- 9. $\overline{BD} \cong \overline{CE}$
- 1.

Remember What You Learned

3. One way to keep the names of related postulates straight in your mind is to associate something in the name of the postulate with the content of the postulate. How can you use this idea to distinguish between the Ruler Postulate and the Segment Addition Postulate?

Proving Angle Relationships

Get Ready for the Lesson

Read the introduction to Lesson 2-8 in your textbook.

Is it possible to open a pair of scissors so that the angles formed by the two blades, a blade and a handle, and the two handles, are all congruent? If so, explain how this could happen.

Read the Lesson

1. Complete each sentence to form a statement that is always	s true.
--	---------

- **a.** If two angles form a linear pair, then they are adjacent and
- **b.** If two angles are complementary to the same angle, then they are
- **c.** If D is a point in the interior of $\angle ABC$, then $m \angle ABC = m \angle ABD + \underline{\hspace{1cm}}$
- **d.** Given \overline{RS} and a number x between and , there is exactly one ray with endpoint R, extended on either side of RS, such that the measure of the angle formed is x.
- **e.** If two angles are congruent and supplementary, then each angle is a(n)

- lines form congruent adjacent angles.
- **g.** "Every angle is congruent to itself" is a statement of the Property of angle congruence.
- **h.** If two congruent angles form a linear pair, then the measure of each angle is _____.
- i. If the noncommon sides of two adjacent angles form a right angle, then the angles are
- **2.** Determine whether each statement is always, sometimes, or never true.
 - **a.** Supplementary angles are congruent.
 - **b.** If two angles form a linear pair, they are complementary.
 - **c.** Two vertical angles are supplementary.
 - **d.** Two adjacent angles form a linear pair.
 - e. Two vertical angles form a linear pair.
 - **f.** Complementary angles are congruent.
 - **g.** Two angles that are congruent to the same angle are congruent to each other.
 - **h.** Complementary angles are adjacent angles.

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose that a classmate thinks that two angles can only be vertical angles if one angle lies above the other. How can you explain to him the meaning of vertical angles, using the word vertex in your explanation?

Parallel Lines and Transversals

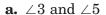
Get Ready for the Lesson

Read the introduction to Lesson 3-1 in your textbook.

- Give an example of parallel lines that can be found in your classroom.
- Give an example of parallel planes that can be found in your classroom.

Read the Lesson

- 1. Write a geometrical term that matches each definition.
 - a. two planes that do not intersect
 - **b.** lines that are not coplanar and do not intersect
 - **c.** two coplanar lines that do not intersect
 - **d.** a line that intersects two or more lines in a plane at different points
 - e. a pair of angles determined by two lines and a transversal consisting of an interior angle and an exterior angle that have different vertices and that lie on the same side of the transversal
- 2. Refer to the figure at the right. Give the special name for each angle pair.



b.
$$\angle 6$$
 and $\angle 12$

c.
$$\angle 4$$
 and $\angle 8$

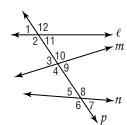
d.
$$\angle 2$$
 and $\angle 3$

e.
$$\angle 8$$
 and $\angle 12$

f.
$$\angle 5$$
 and $\angle 9$

g.
$$\angle 4$$
 and $\angle 10$

h.
$$\angle 6$$
 and $\angle 7$



Remember What You Learned

3. A good way to remember new mathematical terms is to relate them to words that you use in everyday life. Many words start with the prefix trans-, which is a Latin root meaning across. List four English words that start with trans-. How can the meaning of this prefix help you remember the meaning of transversal?

Lesson Reading Guide

Angles and Parallel Lines

Get Ready for the Lesson

Read the introduction to Lesson 3-2 in your textbook.

- Your textbook shows a painting that contains two parallel lines and a transversal. What is the name for $\angle 1$ and $\angle 2$?
- What is the relationship between these two angles?

Read the Lesson

- 1. Choose the correct word to complete each sentence.
 - **a.** If two parallel lines are cut by a transversal, then alternate exterior angles are _____ (congruent/complementary/supplementary).
 - **b.** If two parallel lines are cut by a transversal, then corresponding angles are _____ (congruent/complementary/supplementary).
 - **c.** If parallel lines are cut by a transversal, then consecutive interior angles are _____ (congruent/complementary/supplementary).
 - **d.** In a plane, if a line is perpendicular to one of two parallel lines, then it is _____ (parallel/perpendicular/skew) to the other.

Use the figure for Exercises 2 and 3.

- **2. a.** Name four pairs of vertical angles.
 - **b.** Name all angles that form a linear pair with $\angle 7$.
 - **c.** Name all angles that are congruent to $\angle 1$.
 - **d.** Name all angles that are congruent to $\angle 4$.
 - **e.** Name all angles that are supplementary to $\angle 3$.
 - **f.** Name all angles that are supplementary to $\angle 2$.
- **3.** Which conclusion(s) could you make about lines u and v if $m \angle 4 = m \angle 1$?
 - **A.** $t \parallel u$
- **B.** $t \perp u$
- **C.** $v \perp u$
- **D.** $v \perp t$
- **E.** $v \parallel t$

Remember What You Learned

4. How can you use an everyday meaning of the adjective *alternate* to help you remember the types of angle pairs for two lines and a transversal?

Slopes of Lines

Get Ready for the Lesson

Read the introduction to Lesson 3-3 in your textbook.

- If you are driving uphill on a road with a 4% grade, how many feet will the road rise for every 1000 horizontal feet traveled?
- If you are driving downhill on a road with a 7% grade, how many meters will the road fall for every 500 meters traveled?

Read the Lesson

1. Which expressions can be used to represent the slope of the line containing points (x_1, y_1) and (x_2, y_2) ? Assume that no denominator is zero.

A.
$$\frac{\Delta y}{\Delta x}$$

B.
$$\frac{\text{horizontal run}}{\text{vertical rise}}$$
 C. $\frac{y_2 - y_1}{x_2 - x_1}$ **F.** $\frac{y_1 - y_2}{x_1 - x_2}$ **G.** $\frac{x_2 - x_1}{y_2 - y_1}$

C.
$$\frac{y_2 - y_1}{x_2 - x_1}$$

D.
$$\frac{\text{change in } x}{\text{change in } y}$$

E.
$$\frac{y_2 - y_1}{x_1 - x_2}$$

F.
$$\frac{y_1 - y_2}{x_1 - x_2}$$

G.
$$\frac{x_2 - x_1}{y_2 - y_1}$$

H.
$$\frac{y_2 - x_2}{y_1 - x_1}$$

2. Match the description of a line from the first column with the description of its slope from the second column.

Type of Line

a. a horizontal line

b. a line that rises from left to right

c. a vertical line

d. a line that falls from left to right

Slope

i. a negative number

iii. undefined

iv. a positive number

3. Find the slope of each line.

a. a line parallel to a line with slope $\frac{3}{4}$

b. a line perpendicular to the *x*-axis

c. a line perpendicular to a line with slope 5

d. a line parallel to the *x*-axis

e. y-axis

Remember What You Learned

4. A good way to remember something is to explain it to someone else. Suppose your friend thinks that perpendicular lines (if neither line is vertical) have slopes that are reciprocals of each other. How could you explain to your friend that this is incorrect and give her a good way to remember the correct relationship?

NAME

Lesson Reading Guide

Equations of Lines

Get Ready for the Lesson

Read the introduction to Lesson 3-4 in your textbook.

If the rates for Julia's cell phone plan are described by the equation in your textbook, what will be the total charge (excluding taxes and fees) for a month in which Julia sends 50 text messages?

Read the Lesson

1. Identify what each formula represents.

a.
$$y - y_1 = m(x - x_1)$$

b.
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

c.
$$y = mx + b$$

2. Write the point-slope form of the equation for each line.

a. line with slope
$$-\frac{1}{2}$$
 containing $(-2, 5)$

b. line containing (-4.5, -6.5) and parallel to a line with slope 0.5

3. Which one of the following correctly describes the *y*-intercept of a line?

A. the y-coordinate of the point where the line intersects the x-axis

B. the x-coordinate of the point where the line intersects the y-axis

C. the *y*-coordinate of the point where the line crosses the *y*-axis

D. the x-coordinate of the point where the line crosses the x-axis

 \mathbf{E}_{\bullet} the ratio of the change in *y*-coordinates to the change in *x*-coordinates

4. Find the slope and *y*-intercept of each line.

a.
$$y = 2x - 7$$

b.
$$x + y = 8.5$$

c.
$$2.4x - y = 4.8$$

d.
$$y - 7 = x + 12$$

e.
$$y + 5 = -2(x + 6)$$

Remember What You Learned

5. A good way to remember something new is to relate it to something you already know. How can the slope formula help you to remember the equation for the point-slope form of a line?

Lesson Reading Guide

Proving Lines Parallel

Get Ready for the Lesson

Read the introduction to Lesson 3-5 in your textbook.

How can the workers who are striping the parking spaces in a parking lot check to see if the sides of the spaces are parallel?

Read the Lesson

1. Choose the word or phrase that best completes each sentence.

a. If two coplanar lines are cut by a transversal so that corresponding angles are congruent, then the lines are _____ (parallel/perpendicular/skew).

b. In a plane, if two lines are perpendicular to the same line, then they are _____ (perpendicular/parallel/skew).

c. For a line and a point not on the line, there exists _____ (at least one/exactly one/at most one) line through the point that is parallel to the given line.

d. If two coplanar lines are cut by a transversal so that consecutive interior angles are _____ (complementary/supplementary/congruent), then the lines are parallel.

e. If two coplanar lines are cut by a transversal so that alternate interior angles are congruent, then the lines are _____ (perpendicular/parallel/skew).

2. Which of the following conditions verify that $p \parallel q$?

A.
$$\angle 6 \cong \angle 12$$

$$\textbf{B.} \ \angle 2 \cong \angle 4$$

$$\mathbf{C}$$
. $\angle 8 \cong \angle 16$

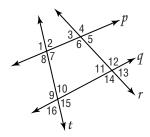
D.
$$\angle 11 \cong \angle 13$$

$$\boldsymbol{E_{\bullet}} \ \angle 6$$
 and $\angle 7$ are supplementary.

F.
$$\angle 1 \cong \angle 15$$

G.
$$\angle 7$$
 and $\angle 10$ are supplementary.

H.
$$\angle 4 \cong \angle 16$$



Remember What You Learned

3. A good way to remember something new is to draw a picture. How can a sketch help you to remember the Parallel Postulate?

Lesson Reading Guide

Perpendiculars and Distance

Get Ready for the Lesson

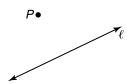
Read the introduction to Lesson 3-6 in your textbook.

Name three examples of situations in home construction where it would be important to construct parallel lines.

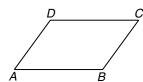
Read the Lesson

1. Fill in the blank with a word or phrase to complete each sentence.

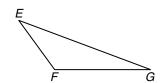
- **a.** The distance from a line to a point not on the line is the length of the segment to the line from the point.
- **b.** Two coplanar lines are parallel if they are everywhere
- **c.** In a plane, if two lines are both equidistant from a third line, then the two lines are to each other.
- **d.** The distance between two parallel lines measured along a perpendicular to the two lines is always
- **e.** To measure the distance between two parallel lines, measure the distance between one of the lines and any point on the ______.
- 2. On each figure, draw the segment that represents the distance indicated.
 - **a.** P to ℓ



b. D to \overrightarrow{AB}



c. E to \overrightarrow{FG}



d. U to \overrightarrow{RV}



Remember What You Learned

3. A good way to remember a new word is to relate it to words that use the same root. Use your dictionary to find the meaning of the Latin root *aequus*. List three words other than equal and equidistant that are derived from this root and give the meaning of each.

Lesson Reading Guide

Classifying Triangles

Get Ready for the Lesson

Read the introduction to Lesson 4-1 in your textbook.

- Why are triangles used for braces in construction rather than other shapes?
- Why do you think that isosceles triangles are used more often than scalene triangles in construction?

Read the Lesson

1. Supply the correct numbers to complete each sentence.

a. In an obtuse triangle, there are ____ acute angle(s), ___ right angle(s), and

____ obtuse angle(s).

b. In an acute triangle, there are ____ acute angle(s), ____ right angle(s), and obtuse angle(s).

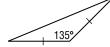
c. In a right triangle, there are ____ acute angle(s), ____ right angle(s), and obtuse angle(s).

- **2.** Determine whether each statement is *always*, *sometimes*, or *never* true.
 - a. A right triangle is scalene.
 - **b.** An obtuse triangle is isosceles.
 - c. An equilateral triangle is a right triangle.
 - **d.** An equilateral triangle is isosceles.
 - e. An acute triangle is isosceles.
 - **f.** A scalene triangle is obtuse.
- **3.** Describe each triangle by as many of the following words as apply: *acute*, *obtuse*, *right*, *scalene*, *isosceles*, or *equilateral*.

a.



b.



c.



Remember What You Learned

4. A good way to remember a new mathematical term is to relate it to a nonmathematical definition of the same word. How is the use of the word *acute*, when used to describe *acute pain*, related to the use of the word *acute* when used to describe an *acute angle* or an *acute triangle*?

Lesson Reading Guide

Angles of Triangles

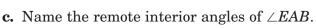
Get Ready for the Lesson

Read the introduction to Lesson 4-2 in your textbook.

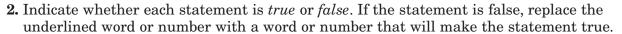
The frame of the simplest kind of kite divides the kite into four triangles. Describe these four triangles and how they are related to each other.

Read the Lesson

- 1. Refer to the figure.
 - **a.** Name the three interior angles of the triangle. (Use three letters to name each angle.)
 - **b.** Name three exterior angles of the triangle. (Use three letters to name each angle.)



- d. Find the measure of each angle without using a protractor.
 - i. $\angle DBC$
- ii. ∠ABC
- iii. ∠ACF
- **iv.** ∠*EAB*



- a. The acute angles of a right triangle are supplementary.
- **b.** The sum of the measures of the angles of any triangle is $\underline{100}$.
- ${f c.}$ A triangle can have at most one right angle or ${f \underline{acute}}$ angle.
- **d.** If two angles of one triangle are congruent to two angles of another triangle, then the third angles of the triangles are congruent.
- **e.** The measure of an exterior angle of a triangle is equal to the <u>difference</u> of the measures of the two remote interior angles.
- **f.** If the measures of two angles of a triangle are 62 and 93, then the measure of the third angle is 35.
- **g.** An <u>exterior</u> angle of a triangle forms a linear pair with an interior angle of the triangle.

Remember What You Learned

3. Many students remember mathematical ideas and facts more easily if they see them demonstrated visually rather than having them stated in words. Describe a visual way to demonstrate the Angle Sum Theorem.

NAME

Lesson Reading Guide

Congruent Triangles

Get Ready for the Lesson

Read the introduction to Lesson 4-3 in your textbook.

In the bridge shown in the photograph in your textbook, triangular beams were used to support the bridge. Why do you think these beams are used on the bridge?

Read the Lesson

1. If $\triangle RST \cong \triangle UWV$, complete each pair of congruent parts.

$$\angle R\cong$$

$$\cong \angle W$$

$$\angle T\cong$$

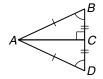
$$\overline{RT}\cong$$

$$\cong \overline{\mathit{UW}}$$

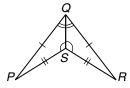
$$\cong \overline{WV}$$

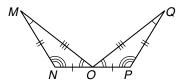
2. Identify the congruent triangles in each diagram.

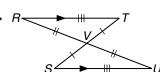
a.



b.







- **3.** Determine whether each statement says that congruence of triangles is *reflexive*, symmetric, or transitive.
 - a. If the first of two triangles is congruent to the second triangle, then the second triangle is congruent to the first.
 - **b.** If there are three triangles such that the first is congruent to the second and the second is congruent to the third, then the first triangle is congruent to the third.
 - **c.** Every triangle is congruent to itself.

Remember What You Learned

4. A good way to remember something is to explain it to someone else. Your classmate Ben is having trouble writing congruence statements for triangles because he thinks he has to match up three pairs of sides and three pairs of angles. How can you help him understand how to write correct congruence statements more easily?

Lesson 4-3

Lesson Reading Guide

Proving Congruence—SSS, SAS

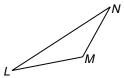
Get Ready for the Lesson

Read the introduction to Lesson 4-4 in your textbook.

Why do you think that land surveyors would use congruent right triangles rather than other congruent triangles to check a measurement?

Read the Lesson

- 1. Refer to the figure.
 - **a.** Name the sides of $\triangle LMN$ for which $\angle L$ is the included angle.



- **b.** Name the sides of $\triangle LMN$ for which $\angle N$ is the included angle.
- **c.** Name the sides of $\triangle LMN$ for which $\angle M$ is the included angle.
- **2.** Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate that you would use. If not, write *not possible*.

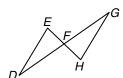


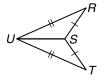


b.



c. \overline{EH} and \overline{DG} bisect each other.





Remember What You Learned

3. Find three words that explain what it means to say that two triangles are congruent and that can help you recall the meaning of the SSS Postulate.

Lesson Reading Guide

Proving Congruence—ASA, AAS

Get Ready for the Lesson

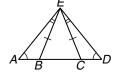
Read the introduction to Lesson 4-5 in your textbook.

Which of the triangles in the photograph in your textbook appear to be congruent?

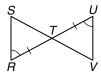
Read the Lesson

- **1.** Explain in your own words the difference between how the ASA Postulate and the AAS Theorem are used to prove that two triangles are congruent.
- **2.** Which of the following conditions are sufficient to prove that two triangles are congruent?
 - **A.** Two sides of one triangle are congruent to two sides of the other triangle.
 - **B.** The three sides of one triangles are congruent to the three sides of the other triangle.
 - C. The three angles of one triangle are congruent to the three angles of the other triangle.
 - **D.** All six corresponding parts of two triangles are congruent.
 - **E.** Two angles and the included side of one triangle are congruent to two sides and the included angle of the other triangle.
 - **F.** Two sides and a nonincluded angle of one triangle are congruent to two sides and a nonincluded angle of the other triangle.
 - **G.** Two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of the other triangle.
 - **H.** Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle.
 - **I.** Two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of the other triangle.
- **3.** Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate or theorem that you would use. If not, write *not possible*.

a.



b. T is the midpoint of \overline{RU} .



Remember What You Learned

4. A good way to remember mathematical ideas is to summarize them in a general statement. If you want to prove triangles congruent by using three pairs of corresponding parts, what is a good way to remember which combinations of parts will work?

Lesson Reading Guide

Isosceles Triangles

Get Ready for the Lesson

Read the introduction to Lesson 4-6 in your textbook.

- Why do you think that isosceles and equilateral triangles are used more often than scalene triangles in art?
- Why might isosceles right triangles be used in art?

Read the Lesson

- 1. Refer to the figure.
 - **a.** What kind of triangle is $\triangle QRS$?
 - **b.** Name the legs of $\triangle QRS$.
 - **c.** Name the base of $\triangle QRS$.
 - **d.** Name the vertex angle of $\triangle QRS$.
 - **e.** Name the base angles of $\triangle QRS$.



- **2.** Determine whether each statement is *always*, *sometimes*, or *never* true.
 - a. If a triangle has three congruent sides, then it has three congruent angles.
 - **b.** If a triangle is isosceles, then it is equilateral.
 - $\boldsymbol{c.}\,$ If a right triangle is isosceles, then it is equilateral.
 - **d.** The largest angle of an isosceles triangle is obtuse.
 - $\boldsymbol{e.}\,$ If a right triangle has a 45° angle, then it is isosceles.
 - **f.** If an isosceles triangle has three acute angles, then it is equilateral.
 - g. The vertex angle of an isosceles triangle is the largest angle of the triangle.
- ${\bf 3.}$ Give the measures of the three angles of each triangle.
 - a. an equilateral triangle
 - $\mathbf{b.}$ an isosceles right triangle
 - c. an isosceles triangle in which the measure of the vertex angle is 70
 - d. an isosceles triangle in which the measure of a base angle is 70
 - **e.** an isosceles triangle in which the measure of the vertex angle is twice the measure of one of the base angles

Remember What You Learned

4. If a theorem and its converse are both true, you can often remember them most easily by combining them into an "if-and-only-if" statement. Write such a statement for the Isosceles Triangle Theorem and its converse.

Lesson Reading Guide

Triangles and Coordinate Proof

Get Ready for the Lesson

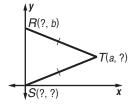
Read the introduction to Lesson 4-7 in your textbook.

- Which coordinate represents the east/west value of a location on the earth?
- Which coordinate represents the north/south value of a location on the earth?

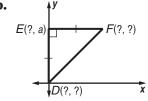
Read the Lesson

1. Find the missing coordinates of each triangle.

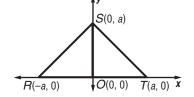
a.



b.



- 2. Refer to the figure.
 - **a.** Find the slope of \overline{SR} and the slope of \overline{ST} .
 - **b.** Find the product of the slopes of \overline{SR} and \overline{ST} . What does this tell you about \overline{SR} and \overline{ST} ?
 - **c.** What does your answer from part b tell you about $\triangle RST$?



- **d.** Find SR and ST. What does this tell you about \overline{SR} and \overline{ST} ?
- **e.** What does your answer from part d tell you about $\triangle RST$?
- **f.** Combine your answers from parts c and e to describe $\triangle RST$ as completely as possible.
- **g.** Find $m \angle SRT$ and $m \angle STR$.
- **h.** Find $m \angle OSR$ and $m \angle OST$.

Remember What You Learned

3. Many students find it easier to remember mathematical formulas if they can put them into words in a compact way. How can you use this approach to remember the slope and midpoint formulas easily?

NAME

Lesson Reading Guide

Bisectors, Medians, and Altitudes

Get Ready for the Lesson

Read the introduction to Lesson 5-1 in your textbook.

Draw any triangle and connect each vertex to the midpoint of the opposite side to form the three medians of the triangle. Is the point where the three medians intersect the midpoint of each of the medians?

Read the Lesson

- **1.** Underline the correct word or phrase to complete each sentence.
 - **a.** Three or more lines that intersect at a common point are called (parallel/perpendicular/concurrent) lines.
 - **b.** Any point on the perpendicular bisector of a segment is (parallel to/congruent to/equidistant from) the endpoints of the segment.
 - **c.** A(n) (altitude/angle bisector/median/perpendicular bisector) of a triangle is a segment drawn from a vertex of the triangle perpendicular to the line containing the opposite side.
 - **d.** The point of concurrency of the three perpendicular bisectors of a triangle is called the (orthocenter/circumcenter/centroid/incenter).
 - e. Any point in the interior of an angle that is equidistant from the sides of that angle lies on the (median/angle bisector/altitude).
 - **f.** The point of concurrency of the three angle bisectors of a triangle is called the (orthocenter/circumcenter/centroid/incenter).
- **2.** In the figure, E is the midpoint of AB, F is the midpoint of BC, and *G* is the midpoint of \overline{AC} .



- **b.** Name the medians of $\triangle ABC$.
- **c.** Name the centroid of $\triangle ABC$.
- **d.** Name the orthocenter of $\triangle ABC$.
- **e.** If AF = 12 and CE = 9, find AH and HE.

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose that a classmate is having trouble remembering whether the center of gravity of a triangle is the orthocenter, the centroid, the incenter, or the circumcenter of the triangle. Suggest a way to remember which point it is.

Lesson Reading Guide

Inequalities and Triangles

Get Ready for the Lesson

Read the introduction to Lesson 5-2 in your textbook.

- Which side of the patio is opposite the largest corner?
- Which side of the patio is opposite the smallest corner?

Read the Lesson

1. Name the property of inequality that is illustrated by each of the following.

a. If
$$x > 8$$
 and $8 > y$, then $x > y$.

b. If
$$x < y$$
, then $x - 7.5 < y - 7.5$.

c. If
$$x > y$$
, then $-3x < -3y$.

d. If x is any real number,
$$x > 0$$
, $x = 0$, or $x < 0$.

2. Use the definition of inequality to write an *equation* that shows that each inequality is true.

a.
$$20 > 12$$

b.
$$101 > 99$$

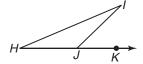
c.
$$8 > -2$$

d.
$$7 > -7$$

e.
$$-11 > -12$$

f.
$$-30 > -45$$

- **3.** In the figure, $m \angle IJK = 45$ and $m \angle H > m \angle I$.
 - **a.** Arrange the following angles in order from largest to smallest: $\angle I$, $\angle IJK$, $\angle H$, $\angle IJH$



- **b.** Arrange the sides of $\triangle HIJ$ in order from shortest to longest.
- **c.** Is $\triangle HIJ$ an acute, right, or obtuse triangle? Explain your reasoning.
- **d.** Is $\triangle HIJ$ scalene, isosceles, or equilateral? Explain your reasoning.

Remember What You Learned

4. A good way to remember a new geometric theorem is to relate it to a theorem you learned earlier. Explain how the Exterior Angle Inequality Theorem is related to the Exterior Angle Theorem, and why the Exterior Angle Inequality Theorem must be true if the Exterior Angle Theorem is true.

NAME

Lesson Reading Guide

Indirect Proof

Get Ready for the Lesson

Read the introduction to Lesson 5-3 in your textbook.

How could the author of a murder mystery use indirect reasoning to show that a particular suspect is not guilty?

Read the Lesson

1. Supply the missing words to complete the list of steps involved in writing an indirect proof.

Step 1 Assume that the conclusion is . . .

Step 2 Show that this assumption leads to a

or some other fact, such as a definition, postulate,

, or corollary.

Step 3 Point out that the assumption must be and, therefore, the

conclusion must be . .

2. State the assumption that you would make to start an indirect proof of each statement.

a. If -6x > 30, then x < -5.

b. If n is a multiple of 6, then n is a multiple of 3.

c. If *a* and *b* are both odd, then *ab* is odd.

d. If a is positive and b is negative, then ab is negative.

e. If F is between E and D, then EF + FD = ED.

f. In a plane, if two lines are perpendicular to the same line, then they are parallel.

g. Refer to the figure.



If AB = AC, then $m \angle B = m \angle C$.

h. Refer to the figure.



In $\triangle PQR$, PR + QR > QP.

Remember What You Learned

3. A good way to remember a new concept in mathematics is to relate it to something you have already learned. How is the process of indirect proof related to the relationship between a conditional statement and its contrapositive?

The Triangle Inequality

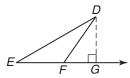
Get Ready for the Lesson

Read the introduction to Lesson 5-4 in your textbook.

If you assume that non-stop flights go directly to their destination, why will it take longer to get to Albuquerque from any city if you take two flights rather than one?

Read the Lesson

1. Refer to the figure.



Which statements are true?

A.
$$DE > EF + FD$$

$$\mathbf{B.}\ DE = EF + FD$$

$$\mathbf{C.}\ EG = EF + FG$$

D.
$$ED + DG > EG$$

E. The shortest distance from D to EG is DF.

F. The shortest distance from D to \overrightarrow{EG} is DG.

2. Complete each sentence about $\triangle XYZ$.



a. If XY = 8 and YZ = 11, then the range of values for XZ is ____ < XZ < ____.

b. If XY = 13 and XZ = 25, then YZ must be between and .

c. If $\triangle XYZ$ is isosceles with $\angle Z$ as the vertex angle, and XZ = 8.5, then the range of values for XY is $\langle XY \langle XY \rangle$.

d. If XZ = a and YZ = b, with b < a, then the range for XY is AXY < AY

Remember What You Learned

3. A good way to remember a new theorem is to state it informally in different words. How could you restate the Triangle Inequality Theorem?

Lesson Reading Guide

Inequalities Involving Two Triangles

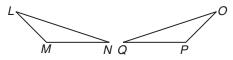
Get Ready for the Lesson

Read the introduction to Lesson 5-5 in your textbook.

Suppose the thrill ride starts from the lowest position, rises to its highest position to the left, and then falls back to its lowest position. How many times will the arm make an angle of 20° with the vertical base?

Read the Lesson

1. Refer to the figure. Write a conclusion that you can draw from the given information. Then name the theorem that justifies your conclusion.

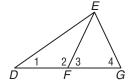


a.
$$\overline{LM} \cong \overline{OP}$$
, $\overline{MN} \cong \overline{PQ}$, and $LN > OQ$

b.
$$\overline{LM} \cong \overline{OP}$$
, $\overline{MN} \cong \overline{PQ}$, and $m \angle P < m \angle M$

c.
$$LM = 8, LN = 15, OP = 8, OQ = 15, m \angle L = 22, \text{ and } m \angle O = 21$$

2. In the figure, $\triangle EFG$ is isosceles with base \overline{FG} and F is the midpoint of \overline{DG} . Determine whether each of the following is a valid conclusion that you can draw based on the given information. (Write valid or invalid.) If the conclusion is valid, identify the definition, property, postulate, or theorem that supports it.



a.
$$\angle 3\cong \angle 4$$

b.
$$DF = GF$$

c. $\triangle DEF$ is isosceles.

d.
$$m \angle 3 > m \angle 1$$

e.
$$m \angle 2 > m \angle 4$$

f.
$$m \angle 2 > m \angle 3$$

g.
$$DE > EG$$

h.
$$DE > FG$$

Remember What You Learned

3. A good way to remember something is to think of it in concrete terms. How can you illustrate the Hinge Theorem with everyday objects?

Lesson 5-5

Angles of Polygons

Get Ready for the Lesson

Read the introduction to Lesson 6-1 in your textbook.

- How many diagonals of the scallop shell shown in your textbook can be drawn from vertex *A*?
- How many diagonals can be drawn from one vertex of an *n*-gon? Explain your reasoning.

Read the Lesson

- **1.** Write an expression that describes each of the following quantities for a regular n-gon. If the expression applies to regular polygons only, write regular. If it applies to all convex polygons, write all.
 - a. the sum of the measures of the interior angles
 - **b.** the measure of each interior angle
 - c. the sum of the measures of the exterior angles (one at each vertex)
 - d. the measure of each exterior angle
- **2.** Give the measure of an interior angle and the measure of an exterior angle of each polygon.
 - a. equilateral triangle

c. square

b. regular hexagon

- d. regular octagon
- **3.** Underline the correct word or phrase to form a true statement about regular polygons.
 - **a.** As the number of sides increases, the sum of the measures of the interior angles (increases/decreases/stays the same).
 - **b.** As the number of sides of a regular polygon increases, the measure of each interior angle (increases/decreases/stays the same).
 - **c.** As the number of sides increases, the sum of the measures of the exterior angles (increases/decreases/stays the same).
 - **d.** As the number of sides of a regular polygon increases, the measure of each exterior angle (increases/decreases/stays the same).
 - **e.** If a regular polygon has more than four sides, each interior angle will be a(n) (acute/right/obtuse) angle, and each exterior angle will be a(n) (acute/right/obtuse) angle.

Remember What You Learned

4. A good way to remember a new mathematical idea or formula is to relate it to something you already know. How can you use your knowledge of the Angle Sum Theorem (for a triangle) to help you remember the Interior Angle Sum Theorem?

Lesson 6-1

Parallelograms

Get Ready for the Lesson

Read the introduction to Lesson 6-2 in your textbook.

What is the relationship between the direction of the ruler next to the compass rose and the direction of the ruler next to the starting position?

Read the Lesson

- 1. Underline words or phrases that can complete the following sentences to make statements that are always true. (There may be more than one correct choice for some of the sentences.)
 - **a.** Opposite sides of a parallelogram are (congruent/perpendicular/parallel).
 - **b.** Consecutive angles of a parallelogram are (complementary/supplementary/congruent).
 - **c.** A diagonal of a parallelogram divides the parallelogram into two (acute/right/obtuse/congruent) triangles.
 - **d.** Opposite angles of a parallelogram are (complementary/supplementary/congruent).
 - **e.** The diagonals of a parallelogram (bisect each other/are perpendicular/are congruent).
 - **f.** If a parallelogram has one right angle, then all of its other angles are (acute/right/obtuse) angles.
- **2.** Let *ABCD* be a parallelogram with $AB \neq BC$ and with no right angles.
 - **a.** Sketch a parallelogram that matches the description above and draw diagonal BD.

In parts **b-f**, complete each sentence.

b.
$$\overline{AB} \parallel \underline{\hspace{1cm}}$$
 and $\overline{AD} \parallel \underline{\hspace{1cm}}$.

c.
$$\overline{AB} \cong \underline{\hspace{1cm}}$$
 and $\overline{BC} \cong \underline{\hspace{1cm}}$.

d.
$$\angle A \cong \underline{\hspace{1cm}}$$
 and $\angle ABC \cong \underline{\hspace{1cm}}$.

e.
$$\angle ADB \cong$$
 _____ because these two angles are ____ angles formed by the two parallel lines ____ and ___ and the transversal .

f.
$$\triangle ABD \cong$$
_____.

Remember What You Learned

3. A good way to remember new theorems in geometry is to relate them to theorems you learned earlier. Name a theorem about parallel lines that can be used to remember the theorem that says, "If a parallelogram has one right angle, it has four right angles."

Lesson Reading Guide

Tests for Parallelograms

Get Ready for the Lesson

Read the introduction to Lesson 6-3 in your textbook.

Make two observations about the angles in the roof of the covered bridge.

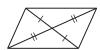
Read the Lesson

- 1. Which of the following conditions guarantee that a quadrilateral is a parallelogram?
 - A. Two sides are parallel.
 - **B.** Both pairs of opposite sides are congruent.
 - C. The diagonals are perpendicular.
 - **D.** A pair of opposite sides is both parallel and congruent.
 - E. There are two right angles.
 - **F.** The sum of the measures of the interior angles is 360.
 - G. All four sides are congruent.
 - **H.** Both pairs of opposite angles are congruent.
 - I. Two angles are acute and the other two angles are obtuse.
 - J. The diagonals bisect each other.
 - K. The diagonals are congruent.
 - L. All four angles are right angles.
- **2.** Determine whether there is enough given information to know that each figure is a parallelogram. If so, state the definition or theorem that justifies your conclusion.

a.



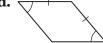
b.



c.



d.



Remember What You Learned

3. A good way to remember a large number of mathematical ideas is to think of them in groups. How can you state the conditions as one group about the *sides* of quadrilaterals that guarantee that the quadrilateral is a parallelogram?

Lesson Reading Guide

Rectangles

Get Ready for the Lesson

Read the introduction to Lesson 6-4 in your textbook.

Are the singles court and doubles court similar rectangles? Explain your answer.

Read the Lesson

- 1. Determine whether each sentence is *always*, *sometimes*, or *never* true.
 - **a.** If a quadrilateral has four congruent angles, it is a rectangle.
 - **b.** If consecutive angles of a quadrilateral are supplementary, then the quadrilateral is a rectangle.
 - **c.** The diagonals of a rectangle bisect each other.
 - **d.** If the diagonals of a quadrilateral bisect each other, the quadrilateral is a rectangle.
 - e. Consecutive angles of a rectangle are complementary.
 - **f.** Consecutive angles of a rectangle are congruent.
 - **g.** If the diagonals of a quadrilateral are congruent, the quadrilateral is a rectangle.
 - **h.** A diagonal of a rectangle bisects two of its angles.
 - **i.** A diagonal of a rectangle divides the rectangle into two congruent right triangles.
 - **j.** If the diagonals of a quadrilateral bisect each other and are congruent, the quadrilateral is a rectangle.
 - **k.** If a parallelogram has one right angle, it is a rectangle.
 - **l.** If a parallelogram has four congruent sides, it is a rectangle.
- **2.** ABCD is a rectangle with AD > AB.

Name each of the following in this figure.

- **a.** all segments that are congruent to \overline{BE}
- **b.** all angles congruent to $\angle 1$
- **c.** all angles congruent to $\angle 7$
- **d.** two pairs of congruent triangles

Remember What You Learned

3. It is easier to remember a large number of geometric relationships and theorems if you are able to combine some of them. How can you combine the two theorems about diagonals that you studied in this lesson?

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Lesson Reading Guide

Rhombi and Squares

Get Ready for the Lesson

Read the introduction to Lesson 6-5 in your textbook.

If you draw a diagonal on the surface of one of the square wheels shown in the picture in your textbook, how can you describe the two triangles that are formed?

Read the Lesson

- 1. Sketch each of the following.
 - **a.** a quadrilateral with perpendicular diagonals that is not a rhombus
- **b.** a quadrilateral with congruent diagonals that is not a rectangle

- **c.** a quadrilateral whose diagonals are perpendicular and bisect each other, but are not congruent
- **2.** List all of the following special quadrilaterals that have each listed property: parallelogram, rectangle, rhombus, square.
 - **a.** The diagonals are congruent.
 - **b.** Opposite sides are congruent.
 - **c.** The diagonals are perpendicular.
 - **d.** Consecutive angles are supplementary.
 - **e.** The quadrilateral is equilateral.
 - **f.** The quadrilateral is equiangular.
 - **g.** The diagonals are perpendicular and congruent.
 - **h.** A pair of opposite sides is both parallel and congruent.
- 3. What is the common name for a regular quadrilateral? Explain your answer.

Remember What You Learned

4. A good way to remember something is to explain it to someone else. Suppose that your classmate Luis is having trouble remembering which of the properties he has learned in this chapter apply to squares. How can you help him?

Lesson 6-5

Lesson Reading Guide

Trapezoids

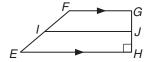
Get Ready for the Lesson

Read the introduction to Lesson 6-6 in your textbook.

How might two parallel lines be used to create a trapezoid?

Read the Lesson

1. In the figure at the right, EFGH is a trapezoid, I is the midpoint of \overline{FE} , and J is the midpoint of \overline{GH} . Identify each of the following segments or angles in the figure.



- **a.** the bases of trapezoid *EFGH*
- **b.** the two pairs of base angles of trapezoid *EFGH*
- **c.** the legs of trapezoid *EFGH*
- **d.** the median of trapezoid *EFGH*
- 2. Determine whether each statement is true or false. If the statement is false, explain why.
 - **a.** A trapezoid is a special kind of parallelogram.
 - **b.** The diagonals of a trapezoid are congruent.
 - **c.** The median of a trapezoid is parallel to the legs.
 - **d.** The length of the median of a trapezoid is the average of the length of the bases.
 - e. A trapezoid has three medians.
 - **f.** The bases of an isosceles trapezoid are congruent.
 - **g.** An isosceles trapezoid has two pairs of congruent angles.
 - **h.** The median of an isosceles trapezoid divides the trapezoid into two smaller isosceles trapezoids.

Remember What You Learned

3. A good way to remember a new geometric theorem is to relate it to one you already know. Name and state in words a theorem about triangles that is similar to the theorem in this lesson about the median of a trapezoid.

Lesson Reading Guide

Coordinate Proof and Quadrilaterals

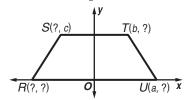
Get Ready for the Lesson

Read the introduction to Lesson 6-7 in your textbook.

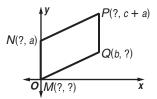
What special kinds of quadrilaterals can be placed on a coordinate system so that two sides of the quadrilateral lie along the axes?

Read the Lesson

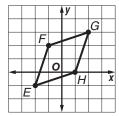
- **1.** Find the missing coordinates in each figure. Then write the coordinates of the four vertices of the quadrilateral.
 - a. isosceles trapezoid



b. parallelogram



- 2. Refer to quadrilateral *EFGH*.
 - a. Find the slope of each side.
 - **b.** Find the length of each side.



Lesson 6-7

- c. Find the slope of each diagonal.
- d. Find the length of each diagonal.
- ${f e.}$ What can you conclude about the sides of *EFGH*?
- **f.** What can you conclude about the diagonals of *EFGH*?
- **g.** Classify EFGH as a parallelogram, a rhombus, or a square. Choose the most specific term. Explain how your results from parts a-f support your conclusion.

Remember What You Learned

3. What is an easy way to remember how best to draw a diagram that will help you devise a coordinate proof?

Lesson 7-1

Lesson Reading Guide

Proportions

Get Ready for the Lesson

Read the introduction to Lesson 7-1 in your textbook.

Estimate the ratio of length to width for the background rectangles in Tiffany's Clematis Skylight.

Read the Lesson

1. Match each description in the first column with a word or phrase from the second column.

a. The ratio of two corresponding quantities

i. proportion

b. r and u in the equation $\frac{r}{s} = \frac{t}{u}$

ii. cross products

c. a comparison of two quantities

iii. means

d. ru and st in the equation $\frac{r}{s} = \frac{t}{u}$

iv. scale factor

e. an equation stating that two ratios are equal

v. extremes

f. s and t in the equation $\frac{r}{s} = \frac{t}{u}$

vi. ratio

2. If m, n, p, and q are nonzero numbers such that $\frac{m}{n} = \frac{p}{q}$, which of the following statements could be false?

$$\mathbf{A.} \ np = mq$$

$$\mathbf{B.}\,\frac{p}{n} = \frac{q}{m}$$

$$\mathbf{C}$$
. $mp = nq$

$$\mathbf{D.}\ qm=pn$$

$$\mathbf{E}_{\bullet} \, \frac{m}{n} = \frac{q}{p}$$

$$\mathbf{F.} \ \frac{q}{p} = \frac{n}{m}$$

G.
$$m:p = n:q$$

H.
$$m:n = p:q$$

- **3.** Write two proportions that match each description.
 - **a.** Means are 5 and 8; extremes are 4 and 10.
 - **b.** Means are 5 and 4; extremes are positive integers that are different from means.

Remember What You Learned

4. Sometimes it is easier to remember a mathematical idea if you put it into words without using any mathematical symbols. How can you use this approach to remember the concept of equality of cross products?

Lesson Reading Guide

Similar Polygons

Get Ready for the Lesson

Read the introduction to Lesson 7-2 in your textbook.

- Describe the figures that have similar shapes.
- What happens to the figures as your eyes move from the center to the outer edge?

Read the Lesson

- 1. Complete each sentence.
 - **a.** Two polygons that have exactly the same shape, but not necessarily the same size, are
 - b. Two polygons are congruent if they have exactly the same shape and the same
 - **c.** Two polygons are similar if their corresponding angles are _____ and their corresponding sides are _____
 - **d.** Two polygons are congruent if their corresponding angles are ______ and their corresponding sides are ______.
 - $\boldsymbol{e}_{\boldsymbol{\cdot}}$ The ratio of the lengths of corresponding sides of two similar figures is called the
 - **f.** Multiplying the coordinates of all points of a figure in the coordinate plane by a scale factor to get a similar figure is called a .
 - ${f g.}$ If two polygons are similar with a scale factor of 1, then the polygons are
- **2.** Determine whether each statement is *always*, *sometimes*, or *never* true.
 - a. Two similar triangles are congruent.
 - **b.** Two equilateral triangles are congruent.
 - **c.** An equilateral triangle is similar to a scalene triangle.
 - d. Two rectangles are similar.
 - **e.** Two isosceles right triangles are congruent.
 - **f.** Two isosceles right triangles are similar.
 - **g.** A square is similar to an equilateral triangle.
 - **h.** Two acute triangles are similar.
 - i. Two rectangles in which the length is twice the width are similar.
 - **j.** Two congruent polygons are similar.

Remember What You Learned

3. A good way to remember a new mathematical vocabulary term is to relate it to words used in everyday life. The word *scale* has many meanings in English. Give three phrases that include the word *scale* in a way that is related to proportions.

Lesson Reading Guide

Similar Triangles

Get Ready for the Lesson

Read the introduction to Lesson 7-3 in your textbook.

- What does it mean to say that triangular shapes result in rigid construction?
- What would happen if the shapes used in the construction were quadrilaterals?

Read the Lesson

- 1. State whether each condition guarantees that two triangles are *congruent* or *similar*. If the condition guarantees that the triangles are both similar and congruent, write *congruent*. If there is not enough information to guarantee that the triangles will be congruent or similar, write *neither*.
 - **a.** Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle.
 - **b.** The measures of all three pairs of corresponding sides are proportional.
 - **c.** Two angles of one triangle are congruent to two angles of the other triangle.
 - **d.** Two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of the other triangle.
 - **e.** The measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle, and the included angles are congruent.
 - **f.** The three sides of one triangle are congruent to the three sides of the other triangle.
 - ${f g.}$ The three angles of one triangle are congruent to the three angles of the other triangle.
 - **h.** One acute angle of a right triangle is congruent to one acute angle of another right triangle.
 - **i.** The measures of two sides of a triangle are proportional to the measures of two sides of another triangle.
- 2. Identify each of the following as an example of a reflexive, symmetric, or transitive property.
 - **a.** If $\triangle RST \sim \triangle UVW$, then $\triangle UVW \sim \triangle RST$.
 - **b.** If $\triangle RST \sim \triangle UVW$ and $\triangle UVW \sim \triangle OPQ$, then $\triangle RST \sim \triangle OPQ$.
 - **c.** $\triangle RST \sim \triangle RST$

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose one of your classmates is having trouble understanding the difference between SAS for congruent triangles and SAS for similar triangles. How can you explain the difference to him?

Lesson Reading Guide

Parallel Lines and Proportional Parts

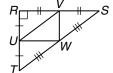
Get Ready for the Lesson

Read the introduction to Lesson 7-4 in your textbook.

Use a geometric idea to explain why the distance between Chicago Avenue and Ontario Street is shorter along Michigan Avenue than along Lake Shore Drive.

Read the Lesson

- **1.** Provide the missing words to complete the statement of each theorem. Then state the name of the theorem.
 - **a.** If a line intersects two sides of a triangle and separates the sides into corresponding segments of ______ lengths, then the line is ______ to the third side.
 - **b.** A midsegment of a triangle is ______ to one side of the triangle and its length is _____ the length of that side.
 - c. If a line is ______ to one side of a triangle and intersects the other two sides in _____ distinct points, then it separates these sides into of proportional length.
- 2. Refer to the figure at the right.
 - **a.** Name the three midsegments of $\triangle RST$.
 - **b.** If RS = 8, RU = 3, and TW = 5, find the length of each of the midsegments.



Lesson 7-4

- **c.** What is the perimeter of $\triangle RST$?
- **d.** What is the perimeter of $\triangle UVW$?
- **e.** What are the perimeters of $\triangle RUV$, $\triangle SVW$, and $\triangle TUW$?
- **f.** How are the perimeters of each of the four small triangles related to the perimeter of the large triangle?
- **g.** Would the relationship that you found in part f apply to any triangle in which the midpoints of the three sides are connected?

Remember What You Learned

3. A good way to remember a new mathematical term is to relate it to other mathematical vocabulary that you already know. What is an easy way to remember the definition of *midsegment* using other geometric terms?

Lesson Reading Guide

Parts of Similar Triangles

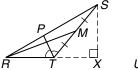
Get Ready for the Lesson

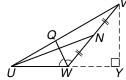
Read the introduction to Lesson 7-5 in your textbook.

- How is similarity involved in the process of making a photographic print from a negative?
- Why do photographers place their cameras on tripods?

Read the Lesson

1. In the figure, $\triangle RST \sim \triangle UVW$. Complete each proportion involving the lengths of segments in this figure by replacing the question mark. Then identify the definition or theorem from the list below that the completed proportion illustrates.





- i. Definition of congruent polygons
- ii. Definition of similar polygons
- iii. Proportional Perimeters Theorem
- iv. Angle Bisectors Theorem
- v. Similar triangles have corresponding altitudes proportional to corresponding sides.
- vi. Similar triangles have corresponding medians proportional to corresponding sides.
- vii. Similar triangles have corresponding angle bisectors proportional to corresponding sides.

a.
$$\frac{RS + ST + TR}{?} = \frac{RS}{UV}$$

b.
$$\frac{RT}{IJW} = \frac{SX}{?}$$

$$\mathbf{c.} \ \frac{RM}{UN} = \frac{?}{VW}$$

d.
$$\frac{RS}{UV} = \frac{ST}{?}$$

$$\mathbf{e.} \; \frac{RP}{PS} = \frac{?}{ST}$$

f.
$$\frac{UN}{?} = \frac{UW}{RT}$$

$$\mathbf{g.} \ \frac{TP}{WQ} = \frac{RT}{?}$$

$$\mathbf{h.} \ \frac{UW}{VW} = \frac{?}{QV}$$

Remember What You Learned

2. A good way to remember a large amount of information is to remember key words. What key words will help you remember the features of similar triangles that are proportional to the lengths of the corresponding sides?

Reading to Learn Mathematics

Geometric Mean

Get Ready for the Lesson

Read the introduction to Lesson 8-1 in your textbook.

If your eye level is halfway between the top and bottom of a painting, what additional information do you need to know to calculate the distance that creates the best view?

Read the Lesson

- 1. In the past, when you have seen the word mean in mathematics, it referred to the average or arithmetic mean of the two numbers.
 - **a.** Complete the following by writing an algebraic expression in each blank. If a and b are two positive numbers, then the geometric mean between a and b is and their arithmetic mean is .
 - **b.** Explain in words, without using any mathematical symbols, the difference between the geometric mean and the algebraic mean.
- **2.** Let r and s be two positive numbers. In which of the following equations is z equal to the geometric mean between *r* and *s*?

$$\mathbf{A.} \ \frac{s}{z} = \frac{z}{r}$$

$$\mathbf{B.} \ \frac{r}{z} = \frac{s}{z}$$

A.
$$\frac{s}{z} = \frac{z}{r}$$
 B. $\frac{r}{z} = \frac{s}{z}$ **C.** $s:z=z:r$ **D.** $\frac{r}{z} = \frac{z}{s}$ **E.** $\frac{z}{r} = \frac{z}{s}$ **F.** $\frac{z}{s} = \frac{r}{z}$

$$\mathbf{D.} \ \frac{r}{z} = \frac{z}{s}$$

E.
$$\frac{z}{r} = \frac{z}{s}$$

$$\mathbf{F.} \ \frac{z}{s} = \frac{r}{z}$$

Lesson 8-1

- **3.** Supply the missing words or phrases to complete the statement of each theorem.
 - a. The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the ______ between the measures of the two segments of the _____
 - **b.** If the altitude is drawn from the vertex of the _____ angle of a right triangle to its hypotenuse, then the measure of a ______ of the triangle is the ______ between the measure of the hypotenuse and the segment ____ adjacent to that leg.
 - **c.** If the altitude is drawn from the ______ of the right angle of a right triangle to its _____, then the two triangles formed are _____to the given triangle and to each other.

Remember What You Learned

4. A good way to remember a new mathematical concept is to relate it to something you already know. How can the meaning of *mean* in a proportion help you to remember how to find the geometric mean between two numbers?

The Pythagorean Theorem and Its Converse

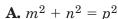
Get Ready for the Lesson

Read the introduction to Lesson 8-2 in your textbook.

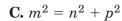
Do the two right triangles shown in the drawing appear to be similar? Explain your reasoning.

Read the Lesson

- 1. Explain in your own words the difference between how the Pythagorean Theorem is used and how the Converse of the Pythagorean Theorem is used.
- **2.** Refer to the figure. For this figure, which statements are true?



B.
$$n^2 = m^2 + p^2$$



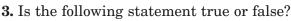
D.
$$m^2 = p^2 - n^2$$

E.
$$p^2 = n^2 - m^2$$

F.
$$n^2 - p^2 = m^2$$

$$\mathbf{G.} n = \sqrt{m^2 + p^2}$$

H.
$$p = \sqrt{m^2 - n^2}$$



A Pythagorean triple is any group of three numbers for which the sum of the squares of the smaller two numbers is equal to the square of the largest number. Explain your reasoning.

4. If x, y, and z form a Pythagorean triple and k is a positive integer, which of the following groups of numbers are also Pythagorean triples?

A. 3x, 4y, 5z

B.
$$3x$$
, $3y$, $3z$

C.
$$-3x$$
, $-3y$, $-3z$ **D.** kx , ky , kz

$$\mathbf{D}$$
. kx , ky , kz

Remember What You Learned

5. Many students who studied geometry long ago remember the Pythagorean Theorem as the equation $a^2 + b^2 = c^2$, but cannot tell you what this equation means. A formula is useless if you don't know what it means and how to use it. How could you help someone who has forgotten the Pythagorean Theorem remember the meaning of the equation $a^2 + b^2 = c^2$?

Special Right Triangles

Get Ready for the Lesson

Read the introduction to Lesson 8-3 in your textbook.

Suppose you have the four half square triangles that are each pinwheel pattern. If there are 9 squares total in each pattern, how many additional squares of material do you need to complete the pattern?

Read the Lesson

- **1.** Supply the correct number or numbers to complete each statement.
 - **a.** In a 45°-45°-90° triangle, to find the length of the hypotenuse, multiply the length of a
 - **b.** In a 30°-60°-90° triangle, to find the length of the hypotenuse, multiply the length of the shorter leg by _
 - **c.** In a 30°-60°-90° triangle, the longer leg is opposite the angle with a measure of _____.
 - d. In a 30°-60°-90° triangle, to find the length of the longer leg, multiply the length of the shorter leg by
 - **e.** In an isosceles right triangle, each leg is opposite an angle with a measure of _____.
 - **f.** In a 30°-60°-90° triangle, to find the length of the shorter leg, divide the length of the longer leg by .
 - g. In 30°-60°-90° triangle, to find the length of the longer leg, divide the length of the hypotenuse by _____ and multiply the result by ...
 - **h.** To find the length of a side of a square, divide the length of the diagonal by _
- **2.** Indicate whether each statement is *always*, *sometimes*, or *never* true.
 - a. The lengths of the three sides of an isosceles triangle satisfy the Pythagorean Theorem.
 - **b.** The lengths of the sides of a 30°-60°-90° triangle form a Pythagorean triple.
 - **c.** The lengths of all three sides of a 30°-60°-90° triangle are positive integers.

Remember What You Learned

3. Some students find it easier to remember mathematical concepts in terms of specific numbers rather than variables. How can you use specific numbers to help you remember the relationship between the lengths of the three sides in a 30°-60°-90° triangle?

DATE

Lesson Reading Guide

Trigonometry

Get Ready for the Lesson

Read the introduction to Lesson 8-4 in your textbook.

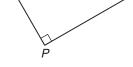
- Why is it important to determine the relative positions accurately in navigation? (Give two possible reasons.)
- What does *calibrated* mean?

Read the Lesson

1. Refer to the figure. Write a ratio using the side lengths in the figure to represent each of the following trigonometric ratios.



 $\mathbf{B.}$ cos N



C. tan *N*

D. tan *M*

E. $\sin M$

 $\mathbf{F.} \cos M$

2. Assume that you enter each of the expressions in the list on the left into your calculator. Match each of these expressions with a description from the list on the right to tell what you are finding when you enter this expression.

a. sin 20

b. cos 20

c. $\sin^{-1} 0.8$

d. $\tan^{-1} 0.8$

e. tan 20

f. $\cos^{-1} 0.8$

- **i.** the degree measure of an acute angle whose cosine is 0.8
- **ii.** the ratio of the length of the leg adjacent to the 20° angle to the length of hypotenuse in a 20°-70°-90° triangle
- iii.the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the adjacent leg is 0.8
- **iv.** the ratio of the length of the leg opposite the 20° angle to the length of the leg adjacent to it in a 20°-70°-90° triangle
- **v.** the ratio of the length of the leg opposite the 20° angle to the length of hypotenuse in a 20° - 70° - 90° triangle
- **vi.** the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the hypotenuse is 0.8

Remember What You Learned

3. How can the *co* in *cosine* help you to remember the relationship between the sines and cosines of the two acute angles of a right triangle?

Lesson 8-4

Angles of Elevation and Depression

Get Ready for the Lesson

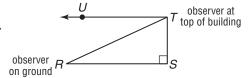
Read the introduction to Lesson 8-5 in your textbook.

What does the angle measure tell the pilot?

Read the Lesson

- 1. Refer to the figure. The two observers are looking at one another. Select the correct choice for each question.
 - **a.** What is the line of sight?

 - (i) line RS (ii) line ST (iii) line RT (iv) line TU



- **b.** What is the angle of elevation?
 - (i) $\angle RST$
- (ii) $\angle SRT$ (iii) $\angle RTS$ (iv) $\angle UTR$
- **c.** What is the angle of depression?
 - (i) $\angle RST$

- (ii) $\angle SRT$ (iii) $\angle RTS$ (iv) $\angle UTR$
- **d.** How are the angle of elevation and the angle of depression related?
 - (i) They are complementary.
 - (ii) They are congruent.
 - (iii) They are supplementary.
 - (iv) The angle of elevation is larger than the angle of depression.
- e. Which postulate or theorem that you learned in Chapter 3 supports your answer for part c?
 - (i) Corresponding Angles Postulate
 - (ii) Alternate Exterior Angles Theorem
 - (iii) Consecutive Interior Angles Theorem
 - (iv) Alternate Interior Angles Theorem
- **2.** A student says that the angle of elevation from his eye to the top of a flagpole is 135°. What is wrong with the student's statement?

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose a classmate finds it difficult to distinguish between angles of elevation and angles of depression. What are some hints you can give her to help her get it right every time?

Lesson Reading Guide

The Law of Sines

Get Ready for the Lesson

Read the introduction to Lesson 8-6 in your textbook.

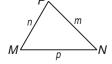
- If a triangle is a right triangle, what theorem can be used to determine the lengths of the sides?
- If a triangle is not a right triangle, can this theorem still be used to determine the lengths of the sides?

Read the Lesson

1. Refer to the figure. According to the Law of Sines, which of the following are correct statements?

A.
$$\frac{m}{\sin M} = \frac{n}{\sin N} = \frac{p}{\sin P}$$

$$\mathbf{B.} \ \frac{\sin m}{M} = \frac{\sin n}{N} = \frac{\sin p}{P}$$



$$\mathbf{C} \cdot \frac{\cos M}{m} = \frac{\cos N}{n} = \frac{\cos P}{p}$$

$$\mathbf{D}_{\bullet} \frac{\sin M}{m} + \frac{\sin N}{n} = \frac{\sin P}{p}$$

E.
$$(\sin M)^2 + (\sin N)^2 = (\sin P)^2$$

$$\mathbf{F.} \ \frac{\sin P}{p} = \frac{\sin M}{m} = \frac{\sin N}{n}$$

- **2.** State whether each of the following statements is *true* or *false*. If the statement is false, explain why.
 - a. The Law of Sines applies to all triangles.
 - **b.** The Pythagorean Theorem applies to all triangles.
 - **c.** If you are given the length of one side of a triangle and the measures of any two angles, you can use the Law of Sines to find the lengths of the other two sides.
 - **d.** If you know the measures of two angles of a triangle, you should use the Law of Sines to find the measure of the third angle.
 - **e.** A friend tells you that in triangle RST, $m \angle R = 132$, r = 24 centimeters, and s = 31 centimeters. Can you use the Law of Sines to solve the triangle? Explain.

Remember What You Learned

3. Many students remember mathematical equations and formulas better if they can state them in words. State the Law of Sines in your own words without using variables or mathematical symbols.

Lesson Reading Guide

The Law of Cosines

Get Ready for the Lesson

Read the introduction to Lesson 8-7 in your textbook.

If a triangular room and a square room have the same floor area, which room has a greater perimeter?

Read the Lesson

1. Refer to the figure. According to the Law of Cosines, which statements are correct for $\triangle DEF$?

A.
$$d^2 = e^2 + f^2 - ef \cos D$$

C.
$$d^2 = e^2 + f^2 + 2ef \cos D$$

E.
$$f^2 = d^2 + e^2 - 2de \cos F$$

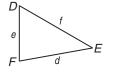
$$\mathbf{G.} \frac{\sin D}{d} = \frac{\sin E}{e} = \frac{\sin F}{f}$$

B.
$$e^2 = d^2 + f^2 - 2df \cos E$$

D.
$$f^2 = d^2 + e^2 - 2ef \cos F$$

F.
$$d^2 = e^2 + f^2$$

H.
$$d = \sqrt{e^2 + f^2 - 2ef \cos D}$$



Lesson 8-7

2. Each of the following describes three given parts of a triangle. In each case, indicate whether you would use the Law of Sines or the Law of Cosines first in solving a triangle with those given parts. (In some cases, only one of the two laws would be used in solving the triangle.)

a. SSS

b. ASA

c. AAS

d. SAS

e. SSA

- 3. Indicate whether each statement is true or false. If the statement is false, explain why.
 - a. The Law of Cosines applies to right triangles.
 - **b.** The Pythagorean Theorem applies to acute triangles.
 - **c.** The Law of Cosines is used to find the third side of a triangle when you are given the measures of two sides and the nonincluded angle.
 - **d.** The Law of Cosines can be used to solve a triangle in which the measures of the three sides are 5 centimeters, 8 centimeters, and 15 centimeters.

Remember What You Learned

4. A good way to remember a new mathematical formula is to relate it to one you already know. The Law of Cosines looks somewhat like the Pythagorean Theorem. Both formulas must be true for a right triangle. How can that be?

Lesson Reading Guide

Reflections

Get Ready for the Lesson

Read the introduction to Lesson 9-1 in your textbook.

Suppose you draw a line segment connecting a point at the peak of a mountain to its image in the lake. Where will the midpoint of this segment fall?

Read the Lesson

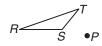
1. Draw the reflected image for each reflection described below.

a. reflection of trapezoid ABCD in the line n Label the image of ABCD as A'B'C'D'.

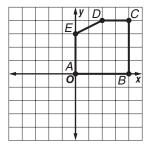
 $\begin{array}{c} B \\ D \end{array} \begin{array}{c} C \\ \end{array}$

b. reflection of $\triangle RST$ in point P Label the image of RST as R'S'T'.

Lesson 9-1



c. reflection of pentagon ABCDE in the origin Label the image of ABCDE as A'B'C'D'E'.



2. Determine the image of the given point under the indicated reflection.

- **a.** (4, 6); reflection in the *y*-axis
- **b.** (-3, 5); reflection in the *x*-axis
- **c.** (-8, -2); reflection in the line y = x
- **d.** (9, -3); reflection in the origin

3. Determine the number of lines of symmetry for each figure described below. Then determine whether the figure has point symmetry and indicate this by writing *yes* or *no*.

a. a square

b. an isosceles triangle (not equilateral)

c. a regular hexagon

- d. an isosceles trapezoid
- $\textbf{e.} \ \ a \ rectangle \ (not \ a \ square)$
- **f.** the letter E

Remember What You Learned

4. A good way to remember a new geometric term is to relate the word or its parts to geometric terms you already know. Look up the origins of the two parts of the word *isometry* in your dictionary. Explain the meaning of each part and give a term you already know that shares the origin of that part.

Lesson Reading Guide

Translations

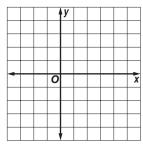
Get Ready for the Lesson

Read the introduction to Lesson 9-2 in your textbook.

How do band directors get the marching band to maintain the shape of the figure they originally formed?

Read the Lesson

- 1. Underline the correct word or phrase to form a true statement.
 - a. All reflections and translations are (opposites/isometries/equivalent).
 - **b.** The preimage and image of a figure under a reflection in a line have (the same orientation/opposite orientations).
 - **c.** The preimage and image of a figure under a translation have (the same orientation/opposite orientations).
 - **d.** The result of successive reflections over two parallel lines is a (reflection/rotation/translation).
 - e. Collinearity (is/is not) preserved by translations.
 - **f.** The translation $(x, y) \rightarrow (x + a, y + b)$ shifts every point a units (horizontally/vertically) and y units (horizontally/vertically).
- 2. Find the image of each preimage under the indicated translation.
 - **a.** (x, y); 5 units right and 3 units up
 - **b.** (x, y); 2 units left and 4 units down
 - **c.** (x, y); 1 unit left and 6 units up
 - **d.** (x, y); 7 units right
 - **e.** (4, -3); 3 units up
 - **f.** (-5, 6); 3 units right and 2 units down
 - **g.** (-7, 5); 7 units right and 5 units down
 - **h.** (-9, -2); 12 units right and 6 units down
- **3.** $\triangle RST$ has vertices R(-3,3), S(0,-2), and T(2,1). Graph $\triangle RST$ and its image $\triangle R'S'T'$ under the translation $(x,y) \rightarrow (x+3,y-2)$. List the coordinates of the vertices of the image.



Remember What You Learned

4. A good way to remember a new mathematical term is to relate it to an everyday meaning of the same word. How is the meaning of *translation* in geometry related to the idea of *translation* from one language to another?

Rotations

Get Ready for the Lesson

Read the introduction to Lesson 9-3 in your textbook.

What are two ways that each car rotates?

Read the Lesson

- **1.** List all of the following types of transformations that satisfy each description: reflection, translation, rotation.
 - **a.** The transformation is an isometry.
 - **b.** The transformation preserves the orientation of a figure.
 - c. The transformation is the composite of successive reflections over two intersecting lines.
 - **d.** The transformation is the composite of successive reflections over two parallel
 - **e.** A specific transformation is defined by a fixed point and a specified angle.
 - **f.** A specific transformation is defined by a fixed point, a fixed line, or a fixed plane.
 - **g.** A specific transformation is defined by $(x, y) \rightarrow (x + a, x + b)$, for fixed values of a and b.
 - **h.** The transformation is also called a slide.
 - **i.** The transformation is also called a flip.
 - **j.** The transformation is also called a turn.
- 2. Determine the order and magnitude of the rotational symmetry for each figure.

a.



b.







Remember What You Learned

3. What is an easy way to remember the order and magnitude of the rotational symmetry of a regular polygon?

Tessellations

Get Ready for the Lesson

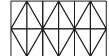
Read the introduction to Lesson 9-4 in your textbook.

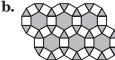
- In the pattern shown in the picture in your textbook, how many small equilateral triangles make up one regular hexagon?
- In this pattern, how many fish make up one equilateral triangle?

Read the Lesson

- 1. Underline the correct word, phrase, or number to form a true statement.
 - **a.** A tessellation is a pattern that covers a plane with the same figure or set of figures so that there are no (congruent angles/overlapping or empty spaces/right angles).
 - **b.** A tessellation that uses only one type of regular polygon is called a (uniform/regular/semi-regular) tessellation.
 - **c.** The sum of the measures of the angles at any vertex in any tessellation is (90/180/360).
 - **d.** A tessellation that contains the same arrangement of shapes and angles at every vertex is called a (uniform/regular/semi-regular) tessellation.
 - e. In a regular tessellation made up of hexagons, there are (3/4/6) hexagons meeting at each vertex, and the measure of each of the angles at any vertex is (60/90/120).
 - **f.** A uniform tessellation formed using two or more regular polygons is called a (rotational/regular/semi-regular) tessellation.
 - g. In a regular tessellation made up of triangles, there are (3/4/6) triangles meeting at each vertex, and the measure of each of the angles at any vertex is (30/60/120).
 - **h.** If a regular tessellation is made up of quadrilaterals, all of the quadrilaterals must be congruent (rectangles/parallelograms/squares/trapezoids).
- **2.** Write all of the following words that describe each tessellation: uniform, non-uniform, regular, semi-regular.

a.





Remember What You Learned

3. Often the everyday meanings of a word can help you to remember its mathematical meaning. Look up *uniform* in your dictionary. How can its everyday meanings help you to remember the meaning of a *uniform* tessellation?

Lesson Reading Guide

Dilations

Get Ready for the Lesson

Read the introduction to Lesson 9-5 in your textbook.

In addition to the example given in your textbook, give two everyday examples of scaling an object, one that makes the object larger and another that makes it smaller.

Read the Lesson

1. Each of the values of r given below represents the scale factor for a dilation. In each case, determine whether the dilation is an *enlargement*, a *reduction*, or a *congruence transformation*.

a.
$$r = 3$$

b.
$$r = 0.5$$

c.
$$r = -0.75$$

d.
$$r = -1$$

e.
$$r = \frac{2}{3}$$

f.
$$r = -\frac{3}{2}$$

g.
$$r = -1.01$$

$$h. r = 0.999$$

- **2.** Determine whether each sentence is *always*, *sometimes*, or *never* true. If the sentence is not always true, explain why.
 - **a.** A dilation requires a center point and a scale factor.
 - **b.** A dilation changes the size of a figure.
 - **c.** A dilation changes the shape of a figure.
 - **d.** The image of a figure under a dilation lies on the opposite side of the center from the preimage.
 - e. A similarity transformation is a congruence transformation.
 - **f.** The center of a dilation is its own image.
 - g. A dilation is an isometry.
 - $\boldsymbol{h}\boldsymbol{.}$ The scale factor for a dilation is a positive number.
 - i. Dilations produce similar figures.

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose that your classmate Lydia is having trouble understanding the relationship between *similarity* transformations and congruence transformations. How can you explain this to her?

Vectors

Get Ready for the Lesson

Read the introduction to Lesson 9-6 in your textbook.

Why do pilots often head their planes in a slightly different direction from their destination?

Read the Lesson

- 1. Supply the missing words or phrases to complete the following sentences.
 - is a directed segment representing a quantity that has both magnitude and direction.
 - **b.** The length of a vector is called its
 - **c.** Two vectors are parallel if and only if they have the same or ___ direction.
 - if it is drawn with initial point at the origin. **d.** A vector is in
 - **e.** Two vectors are equal if and only if they have the same and the
 - **f.** The sum of two vectors is called the ______.
 - **g.** A vector is written in if it is expressed as an ordered pair.
 - **h.** The process of multiplying a vector by a constant is called
- **2.** Write each vector described below in component form.
 - **a.** a vector in standard position with endpoint (a, b)
 - **b.** a vector with initial point (a, b) and endpoint (c, d)
 - **c.** a vector in standard position with endpoint (-3, 5)
 - **d.** a vector with initial point (2, -3) and endpoint (6, -8)
 - **e.** $\overrightarrow{\mathbf{a}} + \overrightarrow{\mathbf{b}}$ if $\overrightarrow{\mathbf{a}} = \langle -3, 5 \rangle$ and $\overrightarrow{\mathbf{b}} = \langle 6, -4 \rangle$
 - **f.** $5\mathbf{\hat{u}}$ if $\mathbf{\hat{u}} = \langle 8, -6 \rangle$
 - **g.** $-\frac{1}{2}\mathbf{\vec{v}}$ if $\mathbf{\vec{v}} = \langle -15, 24 \rangle$
 - **h.** $0.5\mathbf{\hat{u}} + 1.5\mathbf{\hat{v}}$ if $\mathbf{\hat{u}} = \langle 10, -10 \rangle$ and $\mathbf{\hat{v}} = \langle -8, 6 \rangle$

Remember What You Learned

3. A good way to remember a new mathematical term is to relate it to a term you already know. You learned about scale factors when you studied similarity and dilations. How is the idea of a scalar related to scale factors?

Circles and Circumference

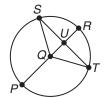
Get Ready for the Lesson

Read the introduction to Lesson 10-1 in your textbook.

How could you measure the approximate distance around the circular carousel using everyday measuring devices?

Read the Lesson

- 1. Refer to the figure.
 - a. Name the circle.
 - **b.** Name four radii of the circle.
 - c. Name a diameter of the circle.
 - **d.** Name two chords of the circle.



- 2. Match each description from the first column with the best term from the second column. (Some terms in the second column may be used more than once or not at all.)
 - a. a segment other than the diameter endpoints on a circle

 - **b.** the set of all points in a plane that are the same distance from a given point
 - **c.** the distance between the center of a circle and any point on the circle
 - **d.** a chord that passes through the center of a circle
 - **e.** a segment whose endpoints are the center and any point on a circle
 - **f.** a chord made up of two collinear radii
 - **g.** the distance around a circle
- **3.** Which equations correctly express a relationship in a circle?

A.
$$d = 2r$$

B.
$$C = \pi r$$

C.
$$C = 2d$$

D.
$$d = \frac{C}{\pi}$$

i. radius

iii. chord

iv. circle

ii. diameter

v. circumference

E.
$$r=\frac{d}{\pi}$$

F.
$$C = r^2$$

G.
$$C=2\pi r$$

H.
$$d = \frac{1}{2}r$$

Remember What You Learned

4. A good way to remember a new geometric term is to relate the word or its parts to geometric terms you already know. Look up the origins of the two parts of the word diameter in your dictionary. Explain the meaning of each part and give a term you already know that shares the origin of that part.

Lesson Reading Guide

Measuring Angles and Arcs

Get Ready for the Lesson

Read the introduction to Lesson 10-2 in your textbook.

- What is the measure of the angle formed by the hour hand and the minute hand of the clock at 5:00?
- What is the measure of the angle formed by the hour hand and the minute hand at 10:30? (Hint: How has each hand moved since 10:00?)

Read the Lesson

- **1.** Refer to $\bigcirc P$. \overline{AC} is a diameter. Indicate whether each statement is *true* or *false*.
 - **a.** \widehat{DAB} is a major arc.
 - **b.** \widehat{ADC} is a semicircle.
 - **c.** $\widehat{AD} \cong \widehat{CD}$
 - **d.** \widehat{DA} and \widehat{AB} are adjacent arcs.
 - **e.** $\angle BPC$ is an acute central angle.
 - **f.** $\angle DPA$ and $\angle BPA$ are supplementary central angles.
- 2. Refer to the figure in Exercise 1. Give each of the following arc measures.
 - **a.** $m\widehat{AB}$

b. $m\widehat{CD}$

c. $m\widehat{BC}$

d. \widehat{mADC}

e. $m\widehat{DAB}$

f. \widehat{mDCB}

 \mathbf{g} . mDAC

- **h.** \overrightarrow{mBDA}
- **3.** Underline the correct word or number to form a true statement.
 - **a.** The arc measure of a semicircle is (90/180/360).
 - **b.** Arcs of a circle that have exactly one point in common are (congruent/opposite/adjacent) arcs.
 - **c.** The measure of a major arc is greater than (0/90/180) and less than (90/180/360).
 - **d.** Suppose a set of central angles of a circle have interiors that do not overlap. If the angles and their interiors contain all points of the circle, then the sum of the measures of the central angles is (90/270/360).
 - **e.** The measure of an arc formed by two adjacent arcs is the (sum/difference/product) of the measures of the two arcs.
 - **f.** The measure of a minor arc is greater than (0/90/180) and less than (90/180/360).

Remember What You Learned

4. A good way to remember something is to explain it to someone else. Suppose your classmate Luis does not like to work with proportions. What is a way that he can find the length of a minor arc of a circle without solving a proportion?

Lesson Reading Guide

Arcs and Chords

Get Ready for the Lesson

NAME

Read the introduction to Lesson 10-3 in your textbook.

What do you observe about any two of the grooves in the waffle iron shown in the picture in your textbook?

Read the Lesson

1. Supply the missing words or phrases to form true statements.

a. In a circle, if a radius is ______ to a chord, then it bisects the chord and its

b. In a circle or in _____ circles, two _____ are congruent if and only if their corresponding chords are congruent.

c. In a circle or in _____ circles, two chords are congruent if they are from the center.

d. A polygon is inscribed in a circle if all of its ______ lie on the circle.

e. All of the sides of an inscribed polygon are ______ of the circle.

2. If $\bigcirc P$ has a diameter 40 centimeters long, and AC = FD = 24 centimeters, find each measure.



 $\mathbf{b.} AG$

c. *PE*

d. *PH*

e. *HE*

f. *FG*

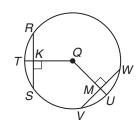


a. $m\widehat{RT}$

b. $m\widehat{ST}$

c. $m\widehat{VW}$

d. $m\widehat{VU}$



4. Find the measure of each arc of a circle that is circumscribed about the polygon.

a. an equilateral triangle

b. a regular pentagon

c. a regular hexagon

d. a regular decagon

e. a regular dodecagon

f. a regular *n*-gon

Remember What You Learned

5. Some students have trouble distinguishing between *inscribed* and *circumscribed* figures. What is an easy way to remember which is which?

Lesson Reading Guide

Inscribed Angles

Get Ready for the Lesson

Read the introduction to Lesson 10-4 in your textbook.

- Why do you think regular hexagons are used rather than squares for the "hole" in a socket?
- Why do you think regular hexagons are used rather than regular polygons with more sides?

Read the Lesson

- **1.** Underline the correct word or phrase to form a true statement.
 - **a.** An angle whose vertex is on a circle and whose sides contain chords of the circle is called a(n) (central/inscribed/circumscribed) angle.
 - **b.** Every inscribed angle that intercepts a semicircle is a(n) (acute/right/obtuse) angle.
 - **c.** The opposite angles of an inscribed quadrilateral are (congruent/complementary/supplementary).
 - **d.** An inscribed angle that intercepts a major arc is a(n) (acute/right/obtuse) angle.
 - **e.** Two inscribed angles of a circle that intercept the same arc are (congruent/complementary/supplementary).
 - **f.** If a triangle is inscribed in a circle and one of the sides of the triangle is a diameter of the circle, the diameter is (the longest side of an acute triangle/a leg of an isosceles triangle/the hypotenuse of a right triangle).
- **2.** Refer to the figure. Find each measure.

a.
$$m \angle ABC$$

b.
$$m\widehat{CD}$$

c.
$$m\widehat{AD}$$

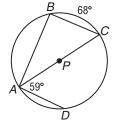
d.
$$m \angle BAC$$

e.
$$m \angle BCA$$

f.
$$\widehat{mAB}$$

g.
$$\widehat{mBCD}$$

h.
$$\widehat{mBDA}$$



Remember What You Learned

3. A good way to remember a geometric relationship is to visualize it. Describe how you could make a sketch that would help you remember the relationship between the measure of an inscribed angle and the measure of its intercepted arc.

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10-5

Lesson Reading Guide

Tangents

Get Ready for the Lesson

Read the introduction to Lesson 10-5 in your textbook.

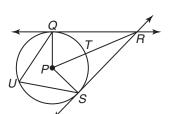
How is the hammer throw event related to the mathematical concept of a tangent line?

Read the Lesson

- 1. Refer to the figure. Name each of the following.
 - **a.** two lines that are tangent to $\bigcirc P$
 - **b.** two points of tangency
 - c. two chords of the circle
 - d. three radii of the circle
 - e. two right angles
 - **f.** two congruent right triangles
 - **g.** the hypotenuse or hypotenuses in the two congruent right triangles
 - **h.** two congruent central angles
 - i. two congruent minor arcs
 - j. an inscribed angle
- **2.** Explain the difference between an *inscribed polygon* and a *circumscribed polygon*. Use the words *vertex* and *tangent* in your explanation.

Remember What You Learned

3. A good way to remember a mathematical term is to relate it to a word or expression that is used in a nonmathematical way. Sometimes a word or expression used in English is derived from a mathematical term. What does it mean to "go off on a tangent," and how is this meaning related to the geometric idea of a *tangent* line?



Lesson Reading Guide

Secants, Tangents, and Angle Measures

Get Ready for the Lesson

Read the introduction to Lesson 10-6 in your textbook.

- How would you describe $\angle C$ in the figure in your textbook?
- When you see a rainbow, where is the sun in relation to the circle of which the rainbow is an arc?

Read the Lesson

- **1.** Underline the correct word to form a true statement.
 - **a.** A line can intersect a circle in at most (one/two/three) points.
 - **b.** A line that intersects a circle in exactly two points is called a (tangent/secant/radius).
 - **c.** A line that intersects a circle in exactly one point is called a (tangent/secant/radius).
 - **d.** Every secant of a circle contains a (radius/tangent/chord).
- **2.** Determine whether each statement is *always*, *sometimes*, or *never* true.
 - **a.** A secant of a circle passes through the center of the circle.
 - **b.** A tangent to a circle passes through the center of the circle.
 - **c.** A secant-secant angle is a central angle of the circle.
 - **d.** A vertex of a secant-tangent angle is a point on the circle.
 - **e.** A secant-tangent angle passes through the center of the circle.
 - **f.** The vertex of a tangent-tangent angle is a point on the circle.
 - **g.** If one side of a secant-tangent angle passes through the center of the circle, the angle is a right angle.
 - **h.** The measure of a secant-secant angle is one-half the positive difference of the measures of its intercepted arcs.
 - i. The sum of the measures of the arcs intercepted by a tangent-tangent angle is 360.
 - **j.** The two arcs intercepted by a tangent-tangent angle are congruent.

Remember What You Learned

3. Some students have trouble remembering the difference between a *secant* and a *tangent*. What is an easy way to remember which is which?

Special Segments in a Circle

Get Ready for the Lesson

Read the introduction to Lesson 10-7 in your textbook.

- What kinds of angles of the circle are formed at the points of the star?
- What is the sum of the measures of the five angles of the star?

Read the Lesson

- **1.** Refer to $\bigcirc O$. Name each of the following.
 - a. a diameter
 - **b.** a chord that is not a diameter
 - **c.** two chords that intersect in the interior of the circle
 - **d.** an exterior point
 - e. two secant segments that intersect in the exterior of the circle
 - f. a tangent segment
 - g. a right angle
 - h. an external secant segment
 - i. a secant-tangent angle with vertex on the circle
 - **j.** an inscribed angle
- **2.** Supply the missing length to complete each equation.

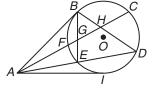
a.
$$BH \cdot HD = FH \cdot$$
_____ **b.** $AC \cdot AF = AD \cdot$ _____

$$\mathbf{c.} \ AD \cdot AE = AB \cdot \underline{\hspace{1cm}}$$

$$\mathbf{d.}\,AB = \underline{\hspace{1cm}}$$

e.
$$AF \cdot AC = (\underline{\hspace{1cm}})^2$$

e.
$$AF \cdot AC = ($$
 $)^2$ **f.** $EG \cdot$ $= FG \cdot GC$



Remember What You Learned

3. Some students find it easier to remember geometric theorems if they restate them in their own words. Restate Theorem 10.16 in a way that you find easier to remember.

Lesson Reading Guide

Equations of Circles

Get Ready for the Lesson

Read the introduction to Lesson 10-8 in your textbook.

In a series of concentric circles, what is the same about all the circles, and what is different?

Read the Lesson

1. Identify the center and radius of each circle.

a.
$$(x-2)^2 + (y-3)^2 = 16$$

c.
$$(x-2)^{2} + (y-3)^{2}$$

e.
$$x^2 + (y - 10)^2 = 144$$

b.
$$(x + 1)^2 + (y + 5)^2 = 9$$

d.
$$(x-8)^2 + (y+1)^2 = 36$$

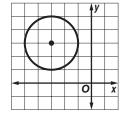
f.
$$(x + 3)^2 + y^2 = 5$$

2. Write an equation for each circle.

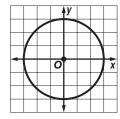
- **a.** center at origin, r = 8
- **b.** center at (3, 9), r = 1
- **c.** center at (-5, -6), r = 10
- **d.** center at (0, -7), r = 7
- **e.** center at (12, 0), d = 12
- **f.** center at (-4, 8), d = 22
- **g.** center at (4.5, -3.5), r = 1.5
- **h.** center at $(0, 0), r = \sqrt{13}$

3. Write an equation for each circle.

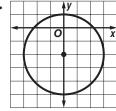
a.



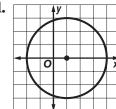
c.



b.



d.



Remember What You Learned

4. A good way to remember a new mathematical formula or equation is to relate it to one you already know. How can you use the Distance Formula to help you remember the standard equation of a circle?

Areas of Parallelograms

Get Ready for the Lesson

Read the introduction to Lesson 11-1 in your textbook.

How many 22-yard squares could fit in an acre?

Read the Lesson

1. Which expression gives the area of the parallelogram? (Hint: There can be more than one correct response.)

A. *ab*

 $\mathbf{B}. cb$

C. ed

D. *af*

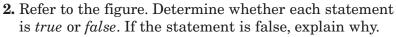
E. ce

F. cd

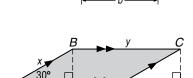
 $\mathbf{G}. df$

H. *bf*

I. cf



a. \overline{AB} is an altitude of the parallelogram.



Lesson 11-1

b. \overline{CD} is a base of parallelogram ABCD.

c. The perimeter of ABCD is (2x + 2y) units².

 $\mathbf{d.} \; BE = CF$

e.
$$BE = \frac{\sqrt{3}}{2}x$$

f. The area of ABCD is 2xy units².

Remember What You Learned

3. A good way to remember a new formula in geometry is to relate it to a formula you already know. How can you use the formula for the area of a rectangle to help you remember the formula for the area of a parallelogram?

Areas of Triangles, Trapezoids, and Rhombi

Get Ready for the Lesson

Read the introduction to Lesson 11-2 in your textbook.

Classify the polygons in the panels of the beach umbrella.

Read the Lesson

1. Match each area formula from the first column with the corresponding polygon in the second column.

a.
$$A = \ell w$$

b.
$$A = \frac{1}{2}d_1d_2$$

c.
$$A = s^2$$

$${\bf d.}~A = \frac{1}{2}h(b_1 + b_2)$$

e.
$$A = \frac{1}{2}bh$$

$$\mathbf{f.} \ \ A = bh$$

- **2.** Determine whether each statement is *always*, *sometimes*, or *never* true. In each case, explain your reasoning.
 - **a.** The area of a square is half the product of its diagonals.
 - **b.** The area of a triangle is half the product of two of its sides.
 - ${f c.}$ You can find the area of a rectangle by multiplying base times height.
 - d. You can find the area of a rectangle by multiplying the lengths of any two of its sides.
 - e. The area of a trapezoid is the product of its height and the sum of the bases.
 - **f.** The square of the length of a side of a square is equal to half the product of its diagonals.

Remember What You Learned

3. A good way to remember a new geometric formula is to state it in words. Write a short sentence that tells how to find the area of a trapezoid in a way that is easy to remember.

Areas of Regular Polygons and Circles

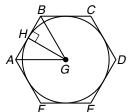
Get Ready for the Lesson

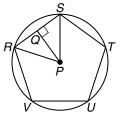
Read the introduction to Lesson 11-3 in your textbook.

Describe what type of triangles you could form by drawing the radii from the center of the octagon.

Read the Lesson

- **1.** *ABCDEF* and *RSTUV* are regular polygons. Name each of the following in one of the figures.
 - a. a circumscribed polygon
 - **b.** an inscribed polygon
 - c. an apothem of a regular hexagon
 - d. an isosceles triangle
 - **e.** a 30°-60°-90° triangle
 - **f.** a central angle with a measure of 72





- 2. Refer to the figures in Exercise 1. Match each item in the first column with an expression in the second column.
 - a. perimeter of ABCDEF

i. $\pi(PS)^2$

b. circumference of circle G

ii. $2\pi(PR)$

c. perimeter of RSTUV

iii. $\frac{5}{9}(RS)(PQ)$

d. area of circle G

iv. 3(AB)(HG)

e. area of RSTUV

 $\mathbf{v.} \ 6(CD)$

f. area of *ABCDEF*

vi. $\pi(GH)^2$

g. area of circle P

h. circumference of circle *P*

- **vii.** 5(*UV*) viii. $2\pi(GH)$
- 3. Explain in your own words how to find the area of a circle if you know the circumference.

Remember What You Learned

4. A good way to remember something is to explain it to someone else. Suppose your classmate Joelle is having trouble remembering which formula is for circumference and which is for area. How can you help her?

Areas of Composite Figures

Get Ready for the Lesson

Read the introduction to Lesson 11-4 in your textbook.

How do you think the areas of the figures outlined in the picture of the sail are related?

Read the Lesson

1. Use dashed segments to show how each figure can be subdivided into figures for which you have learned area formulas. Name the smaller figures that you have formed as specifically as possible and indicate whether any of them are congruent to each other.

a.





2. In the figure, *B* is the midpoint of *ABC*. Complete the following steps to derive a formula for the area of the shaded region in terms of the radius r of the circle.



The area of circle P is $\qquad \qquad$.

 $m \angle ABC =$ because

 $\widehat{mAB} = \widehat{mBC}$ because

 $\overline{AB} \cong \overline{BC}$ because

Therefore, $\triangle ABC$ is a(n) triangle.

AC =_____, so AB =____ and BC =____.

The area of $\triangle ABC$ is $\frac{1}{2} \cdot \underline{} = \underline{}$.

Therefore, the area of the shaded region is given by

Remember What You Learned

3. Rolando is having trouble remembering when to subtract an area when finding the area of a composite figure. How can you help him remember?

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Lesson Reading Guide

Geometric Probability and Areas of Sectors

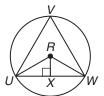
Get Ready for the Lesson

Read the introduction to Lesson 11-5 in your textbook.

To find the probability of winning at darts, would you use geometric probability to compare areas or lengths?

Read the Lesson

- 1. Explain the difference between a sector of a circle and a segment of a circle
- **2.** Suppose you are playing a game of darts with a target like the one shown at the right. If your dart lands inside equilateral $\triangle UVW$, you get a point. Assume that every dart will land on the target. The radius of the circle is 1. Complete the following steps to figure out the probability of getting a point. f simala D i



The area of circle R is			
$\triangle URW$ is a(n) triangle because \overline{RU} and \overline{RW} are			
of the same			
$\angle URW$ is a(n) angle of the circle, and $m\angle URW =$			
$m\angle RUX = \underline{\qquad}$ and $m\angle RWX = \underline{\qquad}$.			
The angle measures in $\triangle RUX$ are,, and			
\overline{RU} is a of the circle, so $RU=$			
\overline{RX} is the leg of $\triangle RUX$ opposite the angle, so $RX =$			
Also, \overline{UX} is the leg of $\triangle RUX$ opposite the angle, so $UX =$			
$UW =$, so the area of $\triangle URW$ is $\frac{1}{2} \cdot$ =			
Then the area of $\triangle IWW = 3$.			

Remember What You Learned

3. Many students find it difficult to remember a large number of geometric formulas. How can you use the formula for the area of a circle to find the area of a sector of a circle without having to learn a new formula?

of ______ to ____, which is approximately (to the nearest thousandth).

Therefore, the probability that the dart will fall inside the triangle is the ratio

Lesson 11-5

Lesson Reading Guide

Representations of Three-Dimensional Figures

Get Ready For the Lesson

Read the introduction to Lesson 12-1 in your textbook.

Artists use three-point perspective to draw three-dimensional objects with a high degree of realism. Why do three-point perspective drawings look more realistic than isometric drawings?

Read the Lesson

Complete the following table.

Word	Definition
1. corner view	
2. perspective view	
3. cross section	
4. reflection symmetry	

- **5.** A three-point perspective drawing has three ______ points. Each of these points is aligned with the _____, width, and length of the figure.
- **6.** A cross section of a solid occurs when a _____ intersects a solid figure.

Remember What You Learned

7. Look up the word isometry in a dictionary. Compare its definition with the definition of corner view and perspective view. Why are corner views considered isometric views but three-dimensional point perspective views not considered isometric views?

Lesson Reading Guide

Surface Areas of Prisms

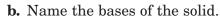
Get Ready for the Lesson

Read the introduction to Lesson 12-2 in your textbook.

How could the architects figure out lateral area of the building?

Read the Lesson

- 1. Determine whether each sentence is always, sometimes, or never true.
 - **a.** A base of a prism is a face of the prism.
 - **b.** A face of a prism is a base of the prism.
 - c. The lateral faces of a prism are rectangles.
 - **d.** If a base of a prism has n vertices, then the prism has n faces.
 - **e.** If a base of a prism has n vertices, then the prism has n lateral edges.
 - f. In a right prism, the lateral edges are also altitudes.
 - g. The bases of a prism are congruent regular polygons.
 - **h.** Any two lateral edges of a prism are perpendicular to each other.
 - i. In a rectangular prism, any pair of opposite faces can be called the bases.
 - j. All of the lateral faces of a prism are congruent to each other.
- **2.** Explain the difference between the *lateral area* of a prism and the *surface area* of a prism. Your explanation should apply to both right and oblique prisms. Do not use any formulas in your explanation.
- **3.** Refer to the figure.
 - a. Name this solid with as specific a name as possible.



- c. Name the lateral faces.
- d. Name the edges.
- e. Name an altitude of the solid.
- **f.** If a represents the area of one of the bases, P represents the perimeter of one of the bases, and x = AF, write an expression for the surface area of the solid that involves a, P, and x.

Remember What You Learned

4. A good way to remember a new mathematical term is to relate it to an everyday use of the same word. How can the way the word *lateral* is used in sports help you remember the meaning of the *lateral area* of a solid?

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Lesson Reading Guide

Surface Areas of Cylinders

Get Ready for the Lesson

Read the introduction to Lesson 12-3 in your textbook.

If the surface area of the half-pipe includes an added flat section in the middle, then how does the surface area of the half-pipe compare to the full-pipe?

Read the Lesson

- 1. Underline the correct word or phrase to form a true statement.
 - **a.** The bases of a cylinder are (rectangles/regular polygons/circles).
 - **b.** The (axis/radius/diameter) of a cylinder is the segment whose endpoints are the centers of the bases.
 - **c.** The net of a cylinder is composed of two congruent (rectangles/circles) and one (rectangle/semicircle).
 - **d.** In a right cylinder, the axis of the cylinder is also a(n) (base/lateral edge/altitude).
 - e. A cylinder that is not a right cylinder is called an (acute/obtuse/oblique) cylinder.
- **2.** Match each description from the first column with an expression from the second column that represents its value.
 - **a.** the lateral area of a right cylinder in which the radius of each base is x cm and the length of the axis is y cm
 - **b.** the surface area of a right prism with square bases in which the length of a side of a base is *x* cm and the length of a lateral edge is *y* cm
 - **c.** the surface area of a right cylinder in which the radius of a base is *x* cm and the height is *y* cm
 - **d.** the surface area of regular hexahedron (cube) in which the length of each edge is x cm
 - **e.** the lateral area of a triangular prism in which the bases are equilateral triangles with side length x cm and the height is y cm
 - **f.** the surface area of a right cylinder in which the diameter of the base is *x* cm and the length of the axis is *y* cm

i.
$$(2x^2 + 4xy)$$
 cm²

ii.
$$(2\pi xy + 2\pi x^2)$$
 cm²

iii.
$$3xy$$
 cm²

iv.
$$6x^2 \text{ cm}^2$$

$$\mathbf{v.} \ 2\pi xy \ \mathrm{cm}^2$$

$$\mathbf{vi.} \left(\frac{\pi x^2}{2} + \pi xy \right) \mathbf{cm}^2$$

Remember What You Learned

3. Often the best way to remember a mathematical formula is to think about where the different parts of the formula come from. How can you use this approach to remember the formula for the surface area of a cylinder?

Surface Areas of Pyramids

Get Ready for the Lesson

Read the introduction to Lesson 12-4 in your textbook.

Why do you think that the architect for the new entrance to the Louvre decided to use a pyramid rather than a rectangular prism?

Read the Lesson

- **1.** In the figure, *ABCDE* has congruent sides and congruent angles.
 - **a.** Describe this pyramid with as specific a name as possible.



- **b.** Use the figure to name the base of this pyramid.
- **c.** Describe the base of the pyramid.
- **d.** Name the vertex of the pyramid.
- **e.** Name the lateral faces of the pyramid.
- **f.** Describe the lateral faces.
- **g.** Name the lateral edges of the pyramid.
- **h.** Name the altitude of the pyramid.
- **i.** Write an expression for the height of the pyramid.
- **j.** Write an expression for the slant height of the pyramid.
- **2.** In a regular square pyramid, let s represent the side length of the base, h represent the height, α represent the apothem, and ℓ represent the slant height. Also, let L represent the lateral area and let T represent the surface area. Which of the following relationships are correct?

$$\mathbf{A.} \ s = 2a$$

B.
$$a^2 + \ell^2 = h^2$$

C.
$$L = 4\ell s$$

D.
$$h = \sqrt{\ell^2 - a^2}$$

E.
$$\left(\frac{s}{2}\right)^2 + h^2 = \ell^2$$
 F. $T = s^2 + 2\ell s$

$$\mathbf{F.} \ T = s^2 + 2\ell s$$

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose that one of your classmates is having trouble remembering the difference between the height and the *slant height* of a regular pyramid. How can you explain this concept?

Surface Areas of Cones

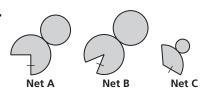
Get Ready for the Lesson

Read the introduction to Lesson 12-5 in your textbook.

If you wanted to build a tepee of a certain size, how would it help you to know the formula for the lateral area of a cone?

Read the Lesson

1.



- **a.** Which net will give the cone with the greatest lateral area?
- **b.** Which net will give the tallest cone?
- **2.** Refer to the figure at the right. Suppose you have removed the circular base of the cone and cut from *V* to *A* so that you can unroll the lateral surface onto a flat table.
 - **a.** How can you be sure that the flattened-out piece is a sector of a circle?



- **b.** How do you know that the flattened-out piece is not a full circle?
- **3.** Suppose you have a right cone with radius r, diameter d, height h, and slant height ℓ . Which of the following relationships involving these lengths are correct?

A.
$$r = 2d$$

$$\mathbf{B.}\;r+h=\ell$$

$$\mathbf{C} \cdot r^2 + h^2 = \ell^2$$

D.
$$r^2 + \ell^2 = h^2$$

$$\mathbf{E}_{\bullet} r = \sqrt{\ell^2 - h^2}$$

F.
$$h = \pm \sqrt{\ell^2 - r^2}$$

Remember What You Learned

4. One way to remember a new formula is to relate it to a formula you already know. Explain how the formulas for the lateral areas of a pyramid and a cone are similar.

Lesson Reading Guide

Surface Areas of Spheres

Get Ready for the Lesson

Read the introduction to Lesson 12-6 in your textbook.

How would knowing the formula for the surface area of a sphere help make the world's largest soccer ball?

Read the Lesson

- **1.** In the figure, P is the center of the sphere. Name each of the following in the figure.
 - a. three radii of the sphere
 - **b.** a diameter of the sphere
 - c. two chords of the sphere
 - d. a great circle of the sphere
 - e. a tangent to the sphere
 - **f.** the point of tangency
- 2. Determine whether each sentence is *sometimes*, *always*, or *never* true.
 - **a.** If a sphere and a plane intersect in more than one point, their intersection will be a great circle.
 - **b.** A great circle has the same center as the sphere.
 - ${f c.}$ The endpoints of a radius of a sphere are two points on the sphere.
 - d. A chord of a sphere is a diameter of the sphere.
 - e. A radius of a great circle is also a radius of the sphere.
- **3.** Match each surface area formula with the name of the appropriate solid.

$$\mathbf{a.} \ T = \pi r \ell + \pi r^2$$

$$\mathbf{b.} \ T = Ph + 2B$$

$$\mathbf{c.} \ T = 4\pi r^2$$

d.
$$T = \frac{1}{2}P\ell + B$$

e.
$$T = 2\pi rh + 2\pi r^2$$

f.
$$T = 3\pi r^2$$

Remember What You Learned

4. Many students have trouble remembering all of the formulas they have learned in this chapter. What is an easy way to remember the formula for the surface area of a sphere?

Lesson Reading Guide

Volumes of Prisms and Cylinders

Get Ready for the Lesson

Read the introduction to Lesson 13-1 in your textbook.

In the cartoon, why was Shoe confused when the teacher said the class was going to discuss volumes?

Read the Lesson

- **1.** In each case, write a formula for the volume *V* of the solid in terms of the given variables.
 - **a.** a rectangular box with length a, width b, and height c
 - **b.** a rectangular box with square bases with side length x, and with height y
 - **c.** a cube with edges of length *e*
 - **d.** a triangular prism whose bases are isosceles right triangles with legs of length x, and whose height is y
 - **e.** a prism whose bases are regular polygons with perimeter P and apothem a, and whose height is h
 - **f.** a cylinder whose bases each have radius *r*, and whose height is three times the radius of the bases
 - **g.** a regular octagonal prism in which each base has sides of length s and apothem a, and whose height is t
 - **h.** a cylinder with height h whose bases each have diameter d
 - i. an oblique cylinder whose bases have radius a and whose height is b
 - **j.** a regular hexagonal prism whose bases have side length s, and whose height is h

Remember What You Learned

2. A good way to remember a mathematical concept is to explain it to someone else. Suppose that your younger sister, who is in eighth grade, is having trouble understanding why square units are used to measure area, but cubic units are needed to measure volume. How can you explain this to her in a way that will make it easy for her to understand and remember the correct units to use?

Lesson 13-1

Lesson Reading Guide

Volumes of Pyramids and Cones

Get Ready for the Lesson

Read the introduction to Lesson 13-2 in your textbook.

In addition to reflecting more light, why do you think the architect of the Transamerica Pyramid may have designed the building as a square pyramid rather than a rectangular prism?

Read the Lesson

- **1.** In each case, two solids are described. Determine whether the first solid or the second solid has the greater volume, or if the two solids have the same volume. (Answer by writing *first*, *second*, or *same*.)
 - **a.** First solid: A rectangular prism with length x, width y, and height z Second solid: A rectangular prism with length 2x, width y, height z
 - **b.** First solid: a rectangular prism that has a square base with side length *x* and that has height *y*

Second solid: a square pyramid whose base has side length *x* and that has height *y*

- **c.** First solid: a right cone whose base has radius *x* and that has height *y* Second solid: an oblique cone whose base has radius *x* and that has height *y*
- **d.** First solid: a cone whose base has radius *x*, and whose height is *y* Second solid: a cylinder whose bases have radius *x*, and whose height is *y*
- **e.** First solid: a cone whose base has radius *x* and whose height is *y* Second solid: a square pyramid whose base has side length *x* and whose height is *y*
- **2.** Supply the missing numbers to form true statements.

a.	the length, width, and height of a rectangular box are all doubled, its volume wil
	e multiplied by

- **b.** If the radius of a cylinder is tripled and the height is unchanged, the volume will be multiplied by ____.
- **c.** In a square pyramid, if the side length of the base is multiplied by 1.5 and the height is doubled, the volume will be multiplied by ____.
- **d.** In a cone, if the radius of the base is tripled and the height is doubled, the volume will be multiplied by _____.
- **e.** In a cube, if the edge length is multiplied by 5, the volume will be multiplied by _____.

Remember What You Learned

3. Many students find it easier to remember mathematical formulas if they can put them in words. Use words to describe in one sentence how to find the volume of any pyramid or cone.

NAME

Lesson Reading Guide

Volumes of Spheres

Get Ready for the Lesson

Read the introduction to Lesson 13-3 in your textbook.

How would you estimate the radius of Earth based on Eratosthenes' estimate of its circumference?

Read the Lesson

1. Name all solids from the following list for which each volume formula can be used: prism, pyramid, cone, cylinder, sphere, hemisphere.

a.
$$V = Bh$$

b.
$$V = \frac{4}{3}\pi r^3$$

c.
$$V = \frac{1}{3}Bh$$

$$\mathbf{d.}\ V=\pi r^2h$$

e.
$$V = \frac{1}{3}\pi r^2 h$$

f.
$$V = \frac{2}{3}\pi r^3$$

2. Let r represent the radius and d represent the diameter of a sphere. Determine whether each formula below can be used to find the volume of a sphere, a hemisphere, or neither.

a.
$$V = \frac{2\pi r^3}{3}$$

b.
$$V = \frac{1}{6}\pi d^3$$

c.
$$V = \frac{1}{3}\pi r^3$$

d.
$$V = \frac{3}{4}\pi r^3$$

e.
$$V = \frac{\pi d^3}{12}$$

f.
$$V = \frac{4}{3}\pi r^2 h$$

3. Compare the volumes of these three solids. Then complete the sentence below.







Of the three solids shown above, the has the largest volume and the has the smallest volume.

Remember What You Learned

4. A good way to remember something is to explain it to someone else. Suppose that your classmate Loretta knows that the expressions $\frac{4}{3}\pi r^3$ and $4\pi r^2$ are used in finding measurements related to spheres, but can't remember which one is used to find the surface area of a sphere and which one is used to find the volume. How can you help her to remember which is which?

Lesson Reading Guide

Congruent and Similar Solids

Get Ready for the Lesson

Read the introduction to Lesson 13-4 in your textbook.

If you want to make a miniature with a scale factor of 1:64, how can you use the actual object to find the measurements you should use to construct the miniature?

Read the Lesson

- 1. Determine whether each statement is always, sometimes, or never true.
 - a. Two cubes are similar.
 - **b.** Two cones are similar.
 - **c.** Two cylinders in which the height is twice the diameter are similar.
 - **d.** Two cylinders with the same volume are congruent.
 - e. A prism with a square base and a square pyramid are similar.
 - **f.** Two rectangular prisms with equal surface areas are similar.
 - g. Nonsimilar solids have different volumes.
 - **h.** Two hemispheres with the same radius are congruent.
- **2.** Supply the missing ratios.

a.	If the ratio of the diameters of two spheres is 3:1, then the ratio of their surface areas
	is, and the ratio of their volumes is
b.	If the ratio of the radii of two hemispheres is 2:5, then the ratio of their surface areas
	is, and the ratio of their volumes is
c.	If two cones are similar and the ratio of their heights is $\frac{4}{3}$, then the ratio of their
	volumes is, and the ratio of their surface areas is
d.	If two cylinders are similar and the ratio of their surface areas is 100:49, then the
	ratio of the radii of their bases is, and the ratio of their volumes is

Remember What You Learned

3. A good way to remember a new mathematical concept is to relate it to something you already know. How can what you know about the units used to measure lengths, areas, and volumes help you to remember the theorem about the ratios of surface areas and volumes of similar solids?

Lesson Reading Guide

Coordinates in Space

Get Ready for the Lesson

Read the introduction to Lesson 13-5 in your textbook.

Why would a mesh be created first?

Read the Lesson

1. Refer to the figure. Match each point from the first column with its coordinates from the second column.



i. (3, 0, 0)

b. *B*

ii. (3, 0, -4)

c. *O*

iii. (3, -2, 0)

 $\mathbf{d}.\ J$

iv. (3, -2, -4)

 \mathbf{e} . H

 $\mathbf{v}. (0, 0, 0)$

f. K

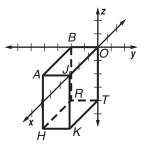
vi. (0, -2, 0)

g. T

vii. (0, -2, -4)

h. *R*

viii. (0, 0, -4)



2. Which of the following expressions give the distance between the points at (4, -1, -5) and (-3, 2, -9)?

A.
$$\sqrt{7^2 + (-3)^2 + 4^2}$$

B.
$$\sqrt{1^2+1^2+(-14)^2}$$

C.
$$\sqrt{2^2+2^2+4^2}$$

D.
$$\left(\frac{1}{2}, \frac{1}{2}, -7\right)$$

E.
$$\sqrt{(-3-4)^2+(-1-2)^2+(-9+5)^2}$$

F.
$$\sqrt{24}$$

G.
$$\sqrt{(-3+4)^2+[2+(-1)]^2+[-9+(-5)]^2}$$

H.
$$\sqrt{74}$$

Remember What You Learned

3. A good way to remember new mathematical formulas is to relate them to ones you already know. How can you use your knowledge of the Distance and Midpoint Formulas in two dimensions to remember the formulas in three dimensions?