

1-1 Enrichment

Fano Plane

Euclid presented two axioms.

- (1) Two points determine a line.
- (2) If a line ℓ is a line in a plane and point P is a point not on ℓ , then there is a unique line through P that is parallel to ℓ .

In Euclidean geometry, there are an infinite number of lines and planes.

In the late 1800's Gino Fano first presented the idea of geometry where there is a finite number of lines and points in a plane called a projective plane.

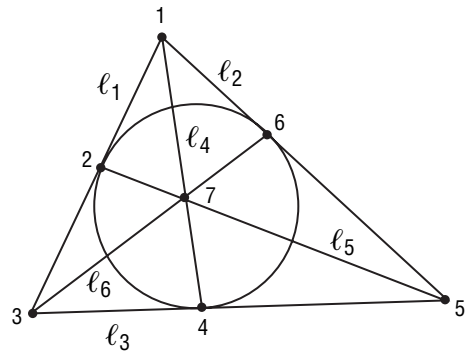
A projective plane consists of a set of points P' and lines L' that are formed by the points in P' . It is defined by the following axioms.

- (1) Two distinct points in P' are only on one line.
- (2) Any two lines from L' intersect in a unique point (there are no parallel lines).
- (3) There is at least one set of four points on P' , of which no three of the points are collinear.

Refer to the Fano Plane shown at the right.

Line 1 is represented by ℓ_1 .

1. What line contains the points 1 and 2? 2 and 7? 5 and 6? Are there any violations of the first axiom?
2. Which point is the intersection of line 1 and line 2? line 4 and line 2? line 3 and line 6? Are there any violations of the second axiom?
3. Why is it that regardless of which four points you choose, one point will not be on the same line as the other three?
4. Is the Fano Plane a projective plane?

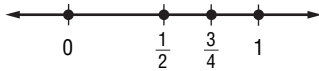


1-2 Enrichment***Around the World***

Given two fractions on a number line, it is possible to quickly find another fraction that is between the first two. Add the numerators of the original fractions to create a new numerator and then add the denominators of the original fractions to create a new denominator. This new fraction will be between the other two.

Example Find a number that is between $\frac{1}{2}$ and $\frac{3}{4}$.

Step 1: Graph on a number line.



Step 2: Add the numerators and denominators of the original fractions.

$$\frac{1+3}{2+4} = \frac{4}{6} \text{ or } \frac{2}{3}$$

So, $\frac{2}{3}$ is between $\frac{1}{2}$ and $\frac{3}{4}$.

Exercises

- Use the distance on the number line to show $\frac{2}{3}$ is between $\frac{1}{2}$ and $\frac{3}{4}$.
 - Find the sum of the distance from $\frac{1}{2}$ to $\frac{2}{3}$ and the distance from $\frac{2}{3}$ to $\frac{3}{4}$.
 - Find the sum of the distances.
 - Compare this sum to the distance from $\frac{1}{2}$ to $\frac{3}{4}$.

Find a fraction that is between each pair of numbers.

- $\frac{1}{3}$ and $\frac{3}{5}$
- $\frac{4}{7}$ and $\frac{2}{9}$
- $\frac{1}{9}$ and $\frac{5}{9}$
- 3 and 4

- How can you use decimals to show $\frac{2}{3}$ is between $\frac{1}{2}$ and $\frac{3}{4}$?

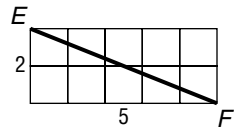
1-3 Enrichment

Lengths on a Grid

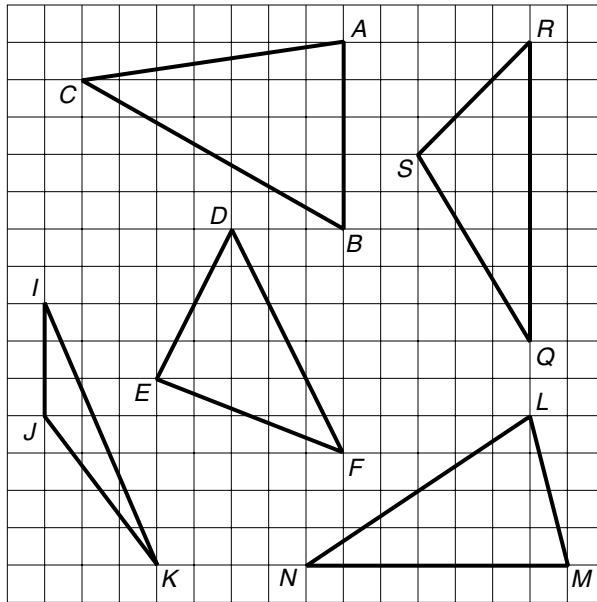
Evenly-spaced horizontal and vertical lines form a grid.

You can easily find segment lengths on a grid if the endpoints are grid-line intersections. For horizontal or vertical segments, simply count squares. For diagonal segments, use the Pythagorean Theorem (proven in Chapter 7). This theorem states that in any right triangle, if the length of the longest side (the side opposite the right angle) is c and the two shorter sides have lengths a and b , then $c^2 = a^2 + b^2$.

Example Find the measure of \overline{EF} on the grid at the right. Locate a right triangle with \overline{EF} as its longest side.



$$EF = \sqrt{2^2 + 5^2} = \sqrt{29} \approx 5.4 \text{ units}$$



Find each measure to the nearest tenth of a unit.

- | | | | |
|--------------------|--------------------|--------------------|--------------------|
| 1. \overline{IJ} | 2. \overline{MN} | 3. \overline{RS} | 4. \overline{QS} |
| 5. \overline{IK} | 6. \overline{JK} | 7. \overline{LM} | 8. \overline{LN} |

Use the grid above. Find the perimeter of each triangle to the nearest tenth of a unit.

- | | | | |
|--------------------|---------------------|---------------------|---------------------|
| 9. $\triangle ABC$ | 10. $\triangle QRS$ | 11. $\triangle DEF$ | 12. $\triangle LMN$ |
|--------------------|---------------------|---------------------|---------------------|

13. Of all the segments shown on the grid, which is longest? What is its length?
14. On the grid, 1 unit = 0.5 cm. How can the answers above be used to find the measures in centimeters?
15. Use your answer from exercise 8 to calculate the length of segment \overline{LN} in centimeters. Check by measuring with a centimeter ruler.
16. Use a centimeter ruler to find the perimeter of triangle IJK to the nearest tenth of a centimeter.

1-4 Enrichment**Angle Relationships**

Angles are measured in degrees ($^{\circ}$). Each degree of an angle is divided into 60 minutes ($'$), and each minute of an angle is divided into 60 seconds ($''$).

$$60' = 1^{\circ}$$

$$60'' = 1'$$

$$67\frac{1}{2}^{\circ} = 67^{\circ}30'$$

$$70.4^{\circ} = 70^{\circ}24'$$

$$90^{\circ} = 89^{\circ}60'$$

Two angles are complementary if the sum of their measures is 90° . Find the complement of each of the following angles.

1. $35^{\circ}15'$

2. $27^{\circ}16'$

3. $15^{\circ}54'$

4. $29^{\circ}18'22''$

5. $34^{\circ}29'45''$

6. $87^{\circ}2'3''$

Two angles are supplementary if the sum of their measures is 180° . Find the supplement of each of the following angles.

7. $120^{\circ}18'$

8. $84^{\circ}12'$

9. $110^{\circ}2'$

10. $45^{\circ}16'24''$

11. $39^{\circ}21'54''$

12. $129^{\circ}18'36''$

13. $98^{\circ}52'59''$

14. $9^{\circ}2'32''$

15. $1^{\circ}2'3''$

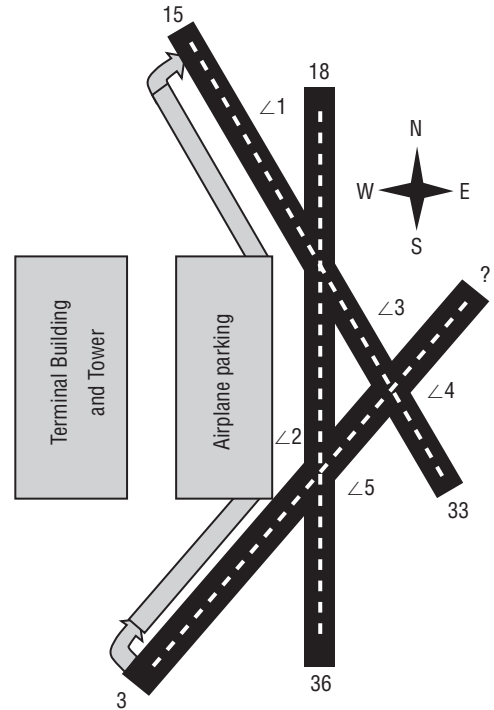
1-5

Enrichment

Runway Angles

Airport runways are numbered by dividing their bearing, the angle measured clockwise from due north, by 10. Because there are 360° in a circle, runways are numbered from 1 to 36. For instance, a runway that runs east to west would be number 27 on the east end of the runway and 9 on the west end.

1. What is the number of the unlabeled runway in the diagram?
2. Find the measure of $\angle 1$.
3. Find the measure of $\angle 2$.
4. Find the measure of $\angle 3$.
5. Find the measure of $\angle 4$.
6. Name a pair of vertical angles.
7. Name a pair of supplementary angles.
8. Explain why the difference between the opposite ends of the runway is always 18.
9. Airports are designed with more than one runway so that pilots landing planes will not have to deal with the difficult situation of landing in a strong crosswind. Design a runway system with the fewest number of runways so that a pilot will never have a crosswind angle of more than 30 degrees. Number each of the runways in your plan.



1-6 Enrichment

Perimeter and Area of Irregular Shapes

Two formulas that are used frequently in mathematics are perimeter and area of a rectangle.

Perimeter: $P = 2\ell + 2w$

Area: $A = \ell w$, where ℓ is the length and w is the width

However, many figures are combinations of two or more rectangles creating **irregular shapes**. To find the area of an irregular shape, it helps to separate the shape into rectangles, calculate the formula for each rectangle, then find the sum of the areas.

Example Find the area of the figure at the right.

Separate the figure into two rectangles.

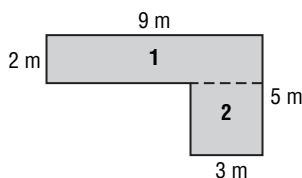
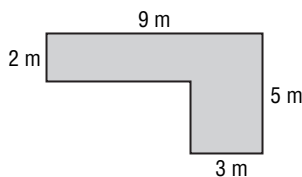
$$A = \ell w$$

$$A_1 = 9 \cdot 2 \quad A_2 = 3 \cdot 3$$

$$= 18 \quad = 9$$

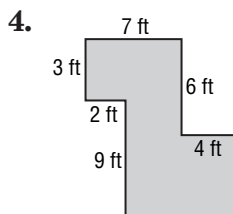
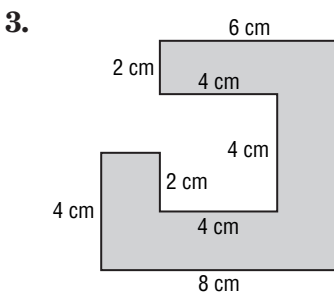
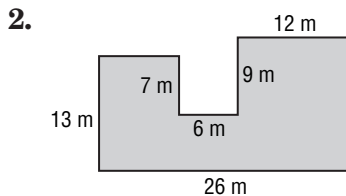
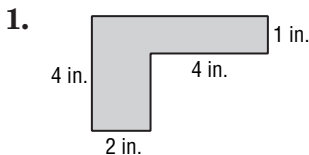
$$18 + 9 = 27$$

The area of the irregular shape is 27 m^2 .



Exercises

Find the area of each irregular shape.



For Exercises 5–8, find the perimeter of the figures in Exercises 1–4.

5. _____ 6. _____ 7. _____ 8. _____

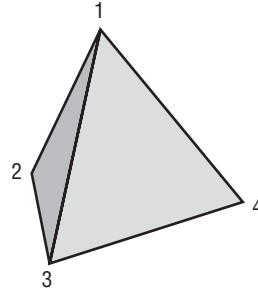
9. Describe the steps you used to find the perimeter in Exercise 1.

1-7

Enrichment

Polyhedrons

As you know, any three noncollinear points determine a unique plane. Consider what happens with four noncoplanar points.



1. How many unique planes can be determined by four noncoplanar points?
2. Drawing all of these planes forms a closed polyhedron. Classify this polyhedron by name.

Now consider a set of n points such that no more than three points in this set are coplanar.

3. If every set of three points determines a unique plane and these planes are drawn, the result will be a closed polyhedron. What will be the shape of each of the faces of such a polyhedron?

4. Complete the table to show the number of faces of the resulting polyhedron for each number of noncoplanar points (column 2).
5. Extend your pattern to find each of the following.
 - a. the number of triangular faces formed using a set of 20 noncoplanar points

| Points | Faces (column 2) | Edges (column 3) |
|--------|---------------------|---------------------|
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |

- b. the number of points necessary to form a polyhedron of 20 faces

6. Generalize your pattern to write the expression representing the number of triangular faces formed using a set of n noncoplanar points.

7. Determine whether Euler's formula is true for each of the polyhedrons formed in this activity. If so, fill in the number of edges for each polyhedron in the table (column 3).

2-1 Enrichment**Counterexamples**

When you make a conclusion after examining several specific cases, you have used **inductive reasoning**. However, you must be cautious when using this form of reasoning. By finding only one **counterexample**, you disprove the conclusion.

Example Is the statement $\frac{1}{x} \leq 1$ true when you replace x with 1, 2, and 3? Is the statement true for all reals? If possible, find a counterexample.

$\frac{1}{1} = 1$, $\frac{1}{2} < 1$, and $\frac{1}{3} < 1$. But when $x = \frac{1}{2}$, then $\frac{1}{x} = 2$. This counterexample shows that the statement is not always true.

Exercises

- The coldest day of the year in Chicago occurred in January for five straight years. Is it safe to conclude that the coldest day in Chicago is always in January?
- Suppose John misses the school bus four Tuesdays in a row. Can you safely conclude that John misses the school bus every Tuesday?
- Is the equation $\sqrt{k^2} = k$ true when you replace k with 1, 2, and 3? Is the equation true for all integers? If possible, find a counterexample.
- Is the statement $2x = x + x$ true when you replace x with $\frac{1}{2}$, 4, and 0.7? Is the statement true for all real numbers? If possible, find a counterexample.
- Suppose you draw four points A , B , C , and D and then draw \overline{AB} , \overline{BC} , \overline{CD} , and \overline{DA} . Does this procedure give a quadrilateral always or only sometimes? Explain your answers with figures.
- Suppose you draw a circle, mark three points on it, and connect them. Will the angles of the triangle be acute? Explain your answers with figures.

2-2 Enrichment

Sudoku

Sudoku is a math puzzle that requires logic to solve. A Sudoku puzzle is typically a 9×9 grid with each square subdivided into nine 3×3 squares. The puzzle starts with some of the numbers given and the goal is to fill in the rest using the following rules.

- Each row and each column has every number, 1 through 9, with none repeated.
- Each 3×3 grid must contain every number, 1 through 9, with none repeated.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | | | | 1 | | 2 | | |
| 1 | 7 | | 3 | | 5 | 9 | | 8 |
| 2 | | 8 | 4 | 7 | 9 | | 3 | |
| 9 | | | 1 | | 6 | | | 7 |
| | 8 | 6 | 9 | | 7 | 1 | | |
| | | 3 | | | | | 5 | 9 |
| | | 2 | 7 | | | 3 | 1 | |
| 3 | 5 | | | | 1 | | | 6 |
| 8 | | | 5 | | 3 | 7 | | |

Exercises

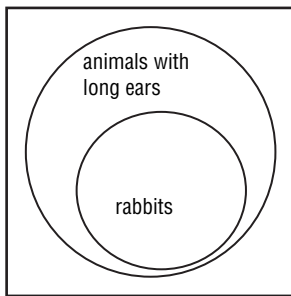
1. What is a good starting point? Why?
2. Explain how you can use the second rule to have all the numbers to solve the larger puzzle.
3. Complete the puzzle.

2-3 Enrichment

Venn Diagrams

A type of drawing called a **Venn diagram** can be useful in explaining conditional statements. A Venn diagram uses circles to represent sets of objects.

Consider the statement “All rabbits have long ears.” To make a Venn diagram for this statement, a large circle is drawn to represent all animals with long ears. Then a smaller circle is drawn inside the first to represent all rabbits. The Venn diagram shows that every rabbit is included in the group of long-eared animals.



The set of rabbits is called a **subset** of the set of long-eared animals.

The Venn diagram can also explain how to write the statement, “All rabbits have long ears,” in if-then form. Every rabbit is in the group of long-eared animals, so if an animal is a rabbit, then it has long ears.

For each statement, draw a Venn diagram. Then write the sentence in if-then form.

1. Every dog has long hair.
2. All rational numbers are real.

3. People who live in Iowa like corn.
4. Staff members are allowed in the faculty lounge.

2-4 Enrichment

Valid and Faulty Arguments

Consider the statements at the right. (1) Boots is a cat.
 What conclusions can you make? (2) Boots is purring.
 (3) A cat purrs if it is happy.

From statements 1 and 3, it is correct to conclude that Boots purrs if it is happy. However, it is faulty to conclude from only statements 2 and 3 that Boots is happy. The if-then form of statement 3 is *If a cat is happy, then it purrs*.

Advertisers often use faulty logic in subtle ways to help sell their products. By studying the arguments, you can decide whether the argument is valid or faulty.

Decide if each argument is valid or faulty.

1. (1) If you buy Tuff Cote luggage, it will survive airline travel.
 (2) Justin buys Tuff Cote luggage.
 Conclusion: Justin's luggage will survive airline travel.
2. (1) If you buy Tuff Cote luggage, it will survive airline travel.
 (2) Justin's luggage survived airline travel.
 Conclusion: Justin has Tuff Cote luggage.
3. (1) If you use Clear Line long distance service, you will have clear reception.
 (2) Anna has clear long distance reception.
 Conclusion: Anna uses Clear Line long distance service.
4. (1) If you read the book *Beautiful Braids*, you will be able to make beautiful braids easily.
 (2) Nancy read the book *Beautiful Braids*.
 Conclusion: Nancy can make beautiful braids easily.
5. (1) If you buy a word processor, you will be able to write letters faster.
 (2) Tania bought a word processor.
 Conclusion: Tania will be able to write letters faster.
6. (1) Great swimmers wear AquaLine swimwear.
 (2) Gina wears AquaLine swimwear.
 Conclusion: Gina is a great swimmer.
7. Write an example of faulty logic that you have seen in an advertisement.

2-5 Enrichment***Even and Odd***

It is commonly known that to determine if a number is even, you check to see if the last number is divisible by 2. However, this is not the definition of an even number. The definition of an even number states that a number is even if it can be written as $2k$ for some integer k .

The following proof uses this definition to show that the sum of two even numbers is even.

Suppose m and n are even. By the definition, they can be written as $m = 2\ell$ and $n = 2j$ for some integers ℓ and j . We need to show that $m + n$ can be written as $2k$ for some integer k to prove that the sum is even. Now, the sum $m + n$ can be written as $2\ell + 2j$ or $2(\ell + j)$ using the distributive property. Since ℓ and j are both integers, the sum $\ell + j$ is equal to some integer k . So, $m + n$ can be written as $2k$ for some integer k . Therefore, the sum $m + n$ is even.

The definition of an odd number states that a number is odd if it can be written as $2k + 1$ for some integer k .

Use the definitions of even and odd numbers to write paragraph proof for each statement.

1. The sum of two odd numbers is even.

2. The product of two odd numbers is odd.

3. The product of two even numbers is even.

2-6 Enrichment

Symmetric, Reflexive, and Transitive Properties

Equality has three important properties.

Reflexive $a = a$

Symmetric If $a = b$, then $b = a$.

Transitive If $a = b$ and $b = c$, then $a = c$.

Other relations have some of the same properties. Consider the relation “is next to” for objects labeled X , Y , and Z . Which of the properties listed above are true for this relation?

X is next to X . *False*

If X is next to Y , then Y is next to X . *True*

If X is next to Y and Y is next to Z , then X is next to Z . *False*

Only the symmetric property is true for the relation “is next to.”

For each relation, state which properties (*symmetric, reflexive, transitive*) are true.

1. is the same size as

2. is a family descendant of

3. is in the same room as

4. is the identical twin of

5. is warmer than

6. is on the same line as

7. is a sister of

8. is the same weight as

9. Find two other examples of relations, and tell which properties are true for each relation.

2-7 Enrichment**Midpoint Counterpoint**

The midpoint M of \overline{AB} when A is (x_1, y_1) and B is (x_2, y_2) is found by using the formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

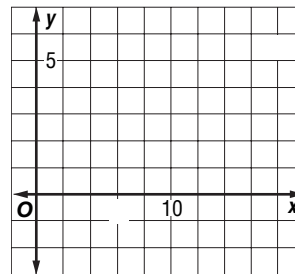
Suppose point P is a point on \overline{AB} located $\frac{1}{4}$ of the distance from A to B . Juan says the coordinates of P can be found by using the formula

$$P = \left(\frac{x_1 + x_2}{4}, \frac{y_1 + y_2}{4} \right).$$

1. Is Juan's formula for P valid? Explain your answer.
2. Use midpoints to find a formula for the coordinates of P . Write your formula in terms of $x_1, y_1, x_2,$ and y_2 .

For Exercises 3–5, use the coordinate plane at the right.

3. Graph $A(2, -2)$ and $B(14, 4)$.
4. Graph point P between A and B so that AP is $\frac{1}{4}(AB)$. What are its coordinates?
5. Graph point C so that B is between A and C and BC is $\frac{1}{4}(AB)$. What are the coordinates of point C ?



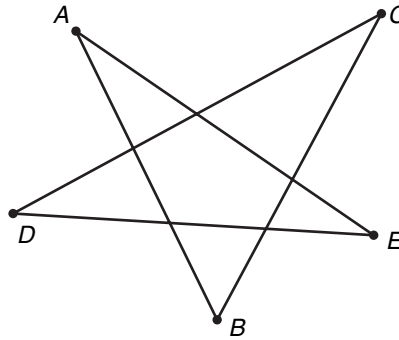
2-8

Enrichment

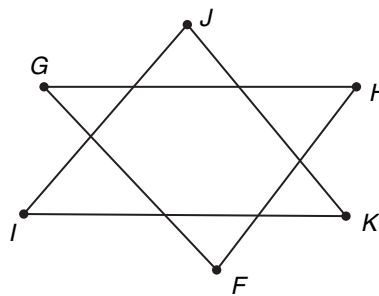
Stars

There are many different types of stars. Stars can have 5 points, 6 points, 7 points, or more. The sum of the angles of the star changes depending on the number of points.

- Find the sum of the measures of the angles in the 5-pointed star.



- Find the sum of the measures of the angles in the 6-pointed star.

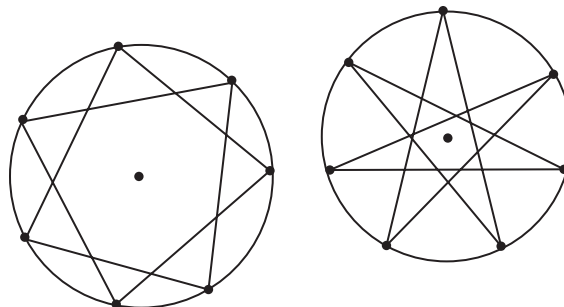


- Complete the table for the sum of the measures of the angles in a star with the number of points given.

| | | | | |
|------------------|---|---|---|---|
| Number of points | 5 | 6 | 7 | 8 |
| Sum of angles | | | | |

- Make a conjecture about the formula for the sum of the measures of the angles for a star with n points. Using this formula, what will be the sum of the angles in a star with 12 points?

- Use the figures at the right to determine if this formula will always work? Explain.



3-1 Enrichment***Spherical Geometry***

On a map, longitude and latitude appear to be lines. However, longitude and latitude exist on a sphere rather than on a flat surface. In order to accurately apply geometry to longitude and latitude, we must consider spherical geometry.

The first four axioms in spherical geometry are the same as those in the Euclidean Geometry you have studied. However, in spherical geometry, the meanings of lines and angles are a different.

1. A straight line can be drawn between any two points.
However, a straight line in spherical geometry is a **great circle**. A great circle is a circle that goes around the sphere and contains the diameter of the sphere.
2. A finite line segment can be extended infinitely in both directions.
A line of infinite length in spherical geometry will just go around itself an infinite number of times.
3. A circle can be drawn with any center or radius.
So, in spherical geometry, a great circle is both a line and a circle.
4. Right angles can be found on the sphere.
Latitude and longitude meet at right angles on a sphere.

The fifth axiom of Euclidean Geometry states that given any straight line and a point not on it, there exists one and only one straight line that passes through that point and never intersects the first line. The fifth axiom is also known as the Parallel Postulate.

Exercises

1. Get a ball. Wrap two rubber bands around the ball to represent two lines (great circles) on the sphere. How many points of intersection are there?
2. Try to draw two lines (great circles) or wrap two rubber bands around a ball that do not intersect. Is it possible?
3. Make a conjecture about the number of points of intersection of any two lines (great circles) in spherical geometry.
4. Does the fifth axiom, or Parallel Postulate, hold for spherical geometry? Explain.

3-2

Enrichment

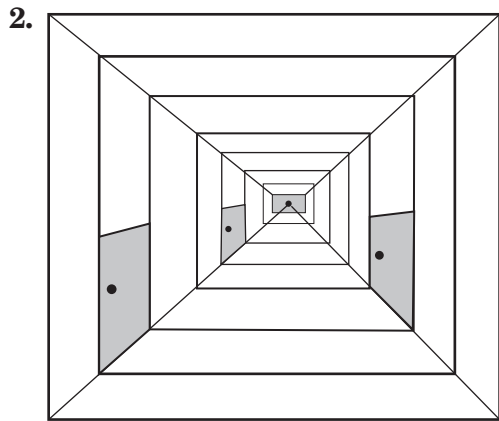
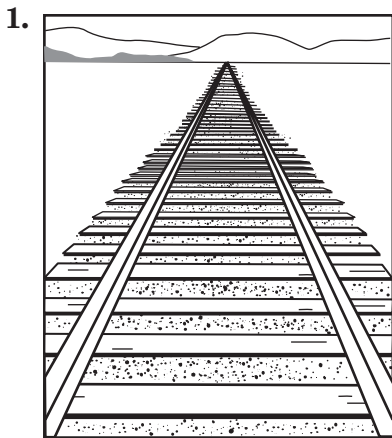
Vanishing Point

If you look down a road that does not have any curves or bends in it, the sides of the road that are parallel appear to meet at a single point. This is called the vanishing point and has been used in artwork since the 1400's.

The picture below shows a straight road going into the distance. The parallel lines of the left and right sides of the road have been traced to show the vanishing point.



In the following pictures, draw lines to find the vanishing point or points.

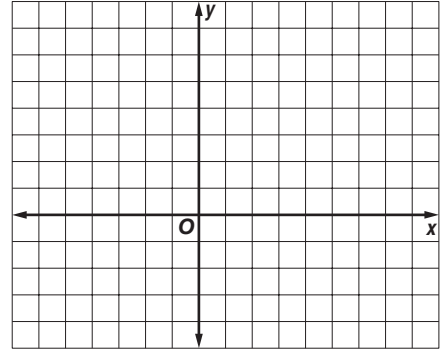


3-3 Enrichment

Slopes and Polygons

In coordinate geometry, the slopes of two lines determine if the lines are parallel or perpendicular. This knowledge can be useful when working with polygons.

- The coordinates of the vertices of a triangle are $A(-6, 4)$, $B(8, 6)$, and $C(4, -4)$. Graph $\triangle ABC$.
- J , K , and L are midpoints of \overline{AB} , \overline{BC} , and \overline{AC} , respectively. Find the coordinates of J , K , and L . Draw $\triangle JKL$.
- Which segments appear to be parallel?
- Show that the segments named in Exercise 3 are parallel by finding the slopes of all six segments.



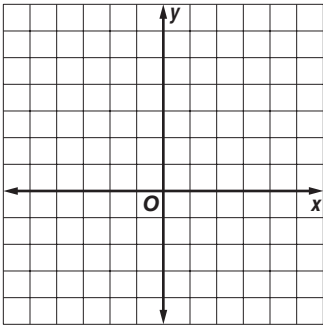
The coordinates of the vertices of right $\triangle PQR$ are given. Find the slope of each side of the triangle. Then name the hypotenuse.

5. $P(5, 1)$ $Q(1, -1)$ $R(-2, 5)$
 slope of \overline{PQ} =
 slope of \overline{QR} =
 slope of \overline{PR} =
 hypotenuse:

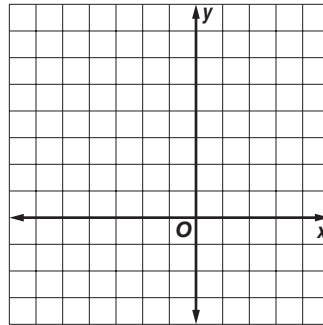
6. $P(-2, -3)$ $Q(5, 1)$ $R(2, 3)$
 slope of \overline{PQ} =
 slope of \overline{QR} =
 slope of \overline{PR} =
 hypotenuse:

The coordinates of quadrilateral $PQRS$ are given. Graph quadrilateral $PQRS$ and find the slopes of the diagonals. State whether the diagonals are perpendicular.

7. $P(-2, 6)$, $Q(4, 0)$, $R(1, -4)$, $S(-5, 2)$



8. $P(0, 6)$, $Q(3, 0)$, $R(-4, -2)$, $S(-5, 4)$



3-4 Enrichment

Polygons on a Coordinate Grid

When equations are graphed on a coordinate grid, their lines can intersect in a way that the segments determined by their intersection points form the sides of a polygon.

1. The following equations when graphed will contain the sides of a polygon. Without graphing the lines, make a prediction about what kind of figure the lines will create.

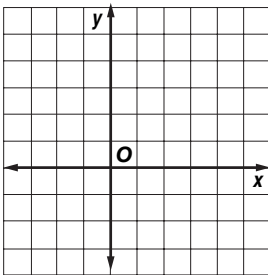
$$y = \frac{1}{2}x + 3$$

$$y = \frac{1}{2}x - 2$$

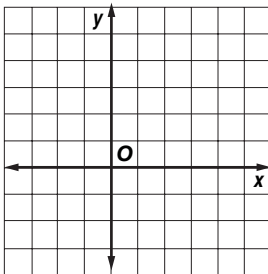
$$y = 2x + 1$$

$$y = 2x - 3$$

2. Graph the lines from Exercise 1 to determine whether your prediction was correct.



3. Find the equations of the lines that form the sides to the polygon shown below. What type of polygon is it? Explain your reasoning.



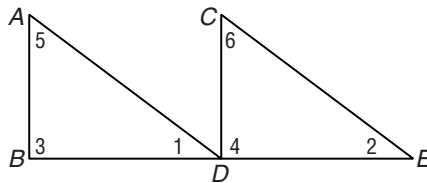
3-5

Enrichment

Scrambled-Up Proof

The reasons necessary to complete the following proof are scrambled up below. To complete the proof, number the reasons to match the corresponding statements.

- Given:** $\overline{CD} \perp \overline{BE}$
 $\overline{AB} \perp \overline{BE}$
 $\overline{AD} \cong \overline{CE}$
 $\overline{BD} \cong \overline{DE}$



Prove: $\overline{AD} \parallel \overline{CE}$

Proof:

| Statements | Reasons |
|---|---|
| 1. $\overline{CD} \perp \overline{BE}$ | Definition of Right Triangle |
| 2. $\overline{AB} \perp \overline{BE}$ | Given |
| 3. $\angle 3$ and $\angle 4$ are right angles. | Given |
| 4. $\triangle ABD$ and $\triangle CDE$ are right triangles. | Definition of Perpendicular Lines |
| 5. $\overline{AD} \cong \overline{CE}$ | Given |
| 6. $\overline{BD} \cong \overline{DE}$ | CPCTC |
| 7. $\triangle ABD \cong \triangle CDE$ | In a plane, if two lines are cut by a transversal so that a pair of corresponding angles is congruent, then the lines are parallel. (Theorem 7-5) |
| 8. $\angle 1 \cong \angle 2$ | Given |
| 9. $\overline{AD} \parallel \overline{CE}$ | HL |

3-6

Enrichment

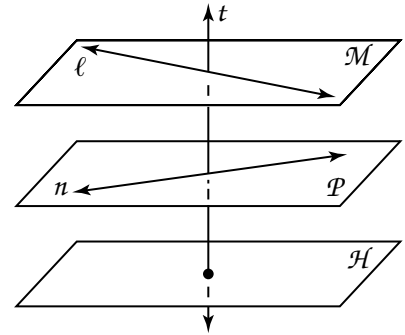
Parallelism in Space

In space geometry, the concept of parallelism must be extended to include two planes and a line and a plane.

Definition: Two planes are parallel if and only if they do not intersect.

Definition: A line and a plane are parallel if and only if they do not intersect.

Thus, in space, two lines can be intersecting, parallel, or skew while two planes or a line and a plane can only be intersecting or parallel. In the figure at the right, $t \perp \mathcal{M}$, $t \perp \mathcal{P}$, $\mathcal{P} \parallel \mathcal{H}$, and ℓ and n are skew.



The following five statements are theorems about parallel planes.

Theorem: Two planes perpendicular to the same line are parallel.

Theorem: Two planes parallel to the same plane are parallel.

Theorem: A line perpendicular to one of two parallel planes is perpendicular to the other.

Theorem: A plane perpendicular to one of two parallel planes is perpendicular to the other.

Theorem: If two parallel planes each intersect a third plane, then the two lines of intersection are parallel.

Use the figure given above for Exercises 1–10. State *yes* or *no* to tell whether the statement is true.

- | | | | | |
|--|-----------------------------|--|---------------------------------|---------------------------|
| 1. $\mathcal{M} \parallel \mathcal{P}$ | 2. $\ell \parallel n$ | 3. $\mathcal{M} \parallel \mathcal{H}$ | 4. $\ell \parallel \mathcal{P}$ | 5. $\ell \perp t$ |
| 6. $n \parallel \mathcal{H}$ | 7. $\ell \perp \mathcal{P}$ | 8. $t \parallel \mathcal{H}$ | 9. $\mathcal{M} \perp t$ | 10. $t \perp \mathcal{H}$ |

Make a small sketch to show that each statement is false.

- | | |
|---|--|
| 11. If two lines are parallel to the same plane, then the lines are parallel. | 12. If two planes are parallel, then any line in one plane is parallel to any line in the other plane. |
| 13. If two lines are parallel, then any plane containing one of the lines is parallel to any plane containing the other line. | 14. If two lines are parallel, then any plane containing one of the lines is parallel to the other line. |

4-1 Enrichment

Reading Mathematics

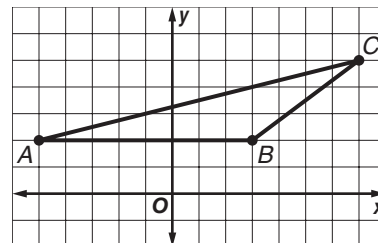
When you read geometry, you may need to draw a diagram to make the text easier to understand.

Example Consider three points, A , B , and C on a coordinate grid. The y -coordinates of A and B are the same. The x -coordinate of B is greater than the x -coordinate of A . Both coordinates of C are greater than the corresponding coordinates of B . Is triangle ABC acute, right, or obtuse?

To answer this question, first draw a sample triangle that fits the description.

Side \overline{AB} must be a horizontal segment because the y -coordinates are the same. Point C must be located to the right and up from point B .

From the diagram you can see that triangle ABC must be obtuse.



Exercises

Draw a simple triangle on grid paper to help you.

- Consider three points, R , S , and T on a coordinate grid. The x -coordinates of R and S are the same. The y -coordinate of T is between the y -coordinates of R and S . The x -coordinate of T is less than the x -coordinate of R . Is angle R of triangle RST acute, right, or obtuse?
- Consider three noncollinear points, J , K , and L on a coordinate grid. The y -coordinates of J and K are the same. The x -coordinates of K and L are the same. Is triangle JKL acute, right, or obtuse?
- Consider three noncollinear points, D , E , and F on a coordinate grid. The x -coordinates of D and E are opposites. The y -coordinates of D and E are the same. The x -coordinate of F is 0. What kind of triangle must $\triangle DEF$ be: scalene, isosceles, or equilateral?
- Consider three points, G , H , and I on a coordinate grid. Points G and H are on the positive y -axis, and the y -coordinate of G is twice the y -coordinate of H . Point I is on the positive x -axis, and the x -coordinate of I is greater than the y -coordinate of G . Is triangle GHI scalene, isosceles, or equilateral?

4-2 Enrichment

Finding Angle Measures in Triangles

You can use algebra to solve problems involving triangles.

Example In triangle ABC , $m\angle A$ is twice $m\angle B$, and $m\angle C$ is 8 more than $m\angle B$. What is the measure of each angle?

Write and solve an equation. Let $x = m\angle B$.

$$m\angle A + m\angle B + m\angle C = 180$$

$$2x + x + (x + 8) = 180$$

$$4x + 8 = 180$$

$$4x = 172$$

$$x = 43$$

So, $m\angle A = 2(43)$ or 86 , $m\angle B = 43$, and $m\angle C = 43 + 8$ or 51 .

Solve each problem.

- In triangle DEF , $m\angle E$ is three times $m\angle D$, and $m\angle F$ is 9 less than $m\angle E$. What is the measure of each angle?
- In triangle RST , $m\angle T$ is 5 more than $m\angle R$, and $m\angle S$ is 10 less than $m\angle T$. What is the measure of each angle?
- In triangle JKL , $m\angle K$ is four times $m\angle J$, and $m\angle L$ is five times $m\angle J$. What is the measure of each angle?
- In triangle XYZ , $m\angle Z$ is 2 more than twice $m\angle X$, and $m\angle Y$ is 7 less than twice $m\angle X$. What is the measure of each angle?
- In triangle GHI , $m\angle H$ is 20 more than $m\angle G$, and $m\angle I$ is 8 more than $m\angle G$. What is the measure of each angle?
- In triangle MNO , $m\angle M$ is equal to $m\angle N$, and $m\angle O$ is 5 more than three times $m\angle N$. What is the measure of each angle?
- In triangle STU , $m\angle U$ is half $m\angle T$, and $m\angle S$ is 30 more than $m\angle T$. What is the measure of each angle?
- In triangle PQR , $m\angle P$ is equal to $m\angle Q$, and $m\angle R$ is 24 less than $m\angle P$. What is the measure of each angle?
- Write your own problems about measures of triangles.

4-3

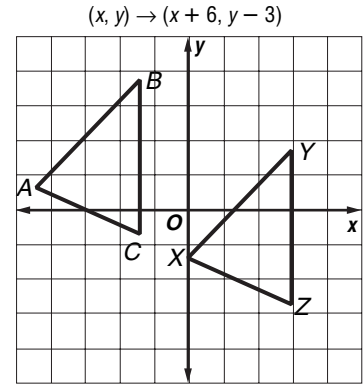
Enrichment

Transformations in The Coordinate Plane

The following statement tells one way to relate points to corresponding translated points in the coordinate plane.

$$(x, y) \rightarrow (x + 6, y - 3)$$

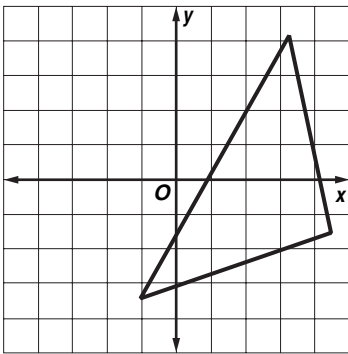
This can be read, "The point with coordinates (x, y) is mapped to the point with coordinates $(x + 6, y - 3)$." With this transformation, for example, $(3, 5)$ is mapped or moved to $(3 + 6, 5 - 3)$ or $(9, 2)$. The figure shows how the triangle ABC is mapped to triangle XYZ .



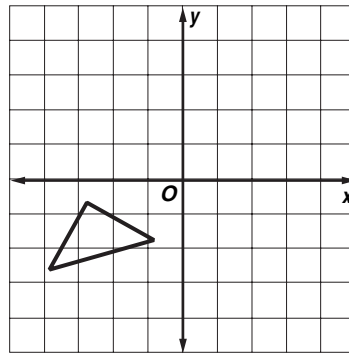
1. Does the transformation above appear to be a congruence transformation? Explain your answer.

Draw the transformation image for each figure. Then tell whether the transformation is or is not a congruence transformation.

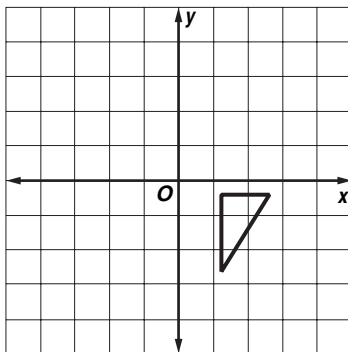
2. $(x, y) \rightarrow (x - 4, y)$



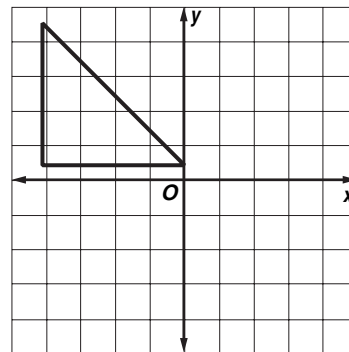
3. $(x, y) \rightarrow (x + 8, y + 7)$



4. $(x, y) \rightarrow (-x, -y)$



5. $(x, y) \rightarrow \left(-\frac{1}{2}x, y\right)$

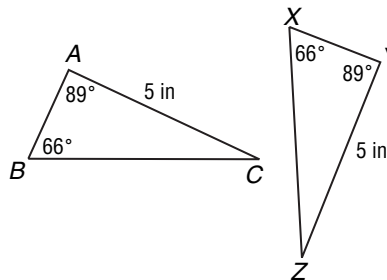


4-4 Enrichment

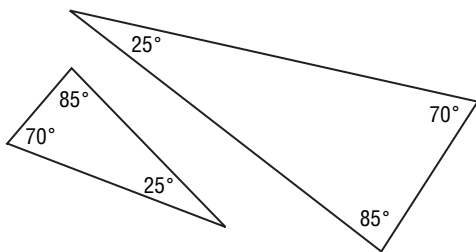
What About AAS to Prove Triangle Congruence?

You have learned about the SSS and ASA postulates to prove that two triangles are congruent. Do you think that triangles can be proven congruent by AAS?

AAS represents the conjecture that two triangles are congruent if they have two pairs of corresponding angles congruent and corresponding non-included sides congruent. For example, in the triangles at the right, $\angle A \cong \angle Y$, $\angle B \cong \angle X$, and $\overline{AC} \cong \overline{YZ}$.



1. Refer to the drawing above. What is the measure of the missing angle in the first triangle? What is the measure of the missing angle in the second triangle?
2. With the information from Exercise 1, what postulate can you now use to prove that the two triangles are congruent?
3. Can you draw two triangles that have two consecutive congruent angles and the following side congruent that are not congruent? Explain
4. The triangles below demonstrate AAA.



- a. Are the triangles above congruent? Explain.
- b. Can you draw two triangles that have all three angles congruent that are not congruent? Can AAA be used to prove that two triangles are congruent?
- c. What characteristics do these two triangles drawn by AAA seem to possess?

4-5 Enrichment***Congruent Triangles in the Coordinate Plane***

If you know the coordinates of the vertices of two triangles in the coordinate plane, you can often decide whether the two triangles are congruent. There may be more than one way to do this.

1. Consider $\triangle ABD$ and $\triangle CDB$ whose vertices have coordinates $A(0, 0)$, $B(2, 5)$, $C(9, 5)$, and $D(7, 0)$. Briefly describe how you can use what you know about congruent triangles and the coordinate plane to show that $\triangle ABD \cong \triangle CDB$. You may wish to make a sketch to help get you started.

2. Consider $\triangle PQR$ and $\triangle KLM$ whose vertices are the following points.

 $P(1, 2)$ $Q(3, 6)$ $R(6, 5)$ $K(-2, 1)$ $L(-6, 3)$ $M(-5, 6)$

Briefly describe how you can show that $\triangle PQR \cong \triangle KLM$.

3. If you know the coordinates of all the vertices of two triangles, is it *always* possible to tell whether the triangles are congruent? Explain.

4-6

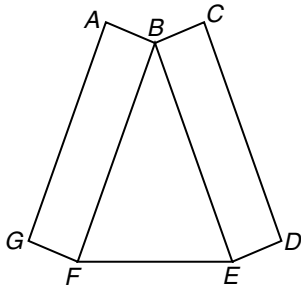
Enrichment

Triangle Challenges

Some problems include diagrams. If you are not sure how to solve the problem, begin by using the given information. Find the measures of as many angles as you can, writing each measure on the diagram. This may give you more clues to the solution.

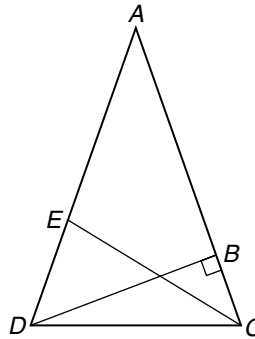
- 1. Given:** $BE = BF$, $\angle BFG \cong \angle BEF \cong \angle BED$, $m\angle BFE = 82$ and $ABFG$ and $BCDE$ each have opposite sides parallel and congruent.

Find $m\angle ABC$.



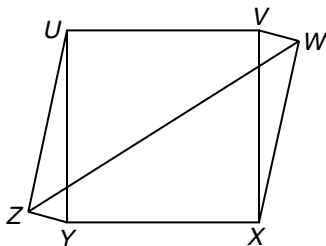
- 2. Given:** $AC = AD$, and $\overline{AB} \perp \overline{BD}$, $m\angle DAC = 44$ and \overline{CE} bisects $\angle ACD$.

Find $m\angle DEC$.



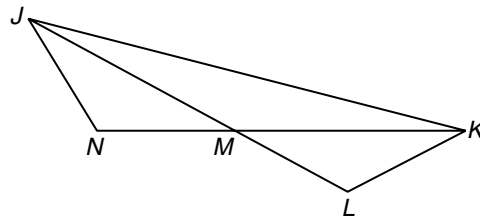
- 3. Given:** $m\angle UZY = 90$, $m\angle ZWX = 45$, $\triangle YZU \cong \triangle VWX$, $UVXY$ is a square (all sides congruent, all angles right angles).

Find $m\angle WZY$.



- 4. Given:** $m\angle N = 120$, $\overline{JN} \cong \overline{MN}$, $\triangle JNM \cong \triangle KLM$.

Find $m\angle JKM$.

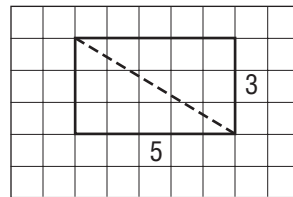
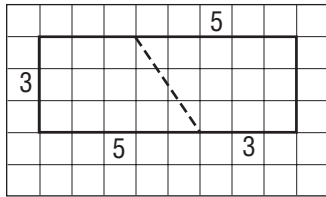


4-7

Enrichment

Rectangle Paradox

Use grid paper to draw the two rectangles below. Cut them into pieces along the dashed lines. Then rearrange the pieces to form a new rectangle.



1. Make a sketch of the new rectangle. Label the whole number lengths

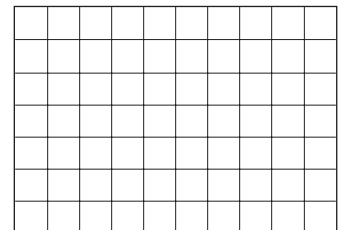
2. What is the combined area of the two original rectangles?

3. What is the area of the new rectangle?

4. Make a conjecture as to why there is a discrepancy between the answers to Exercises 1 and 2.

5. Use a straight edge to precisely draw the four shapes that make up the larger rectangle. What does your drawing show?

6. How could you use slope to show that your drawing is accurate?



Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

Lesson 4-7

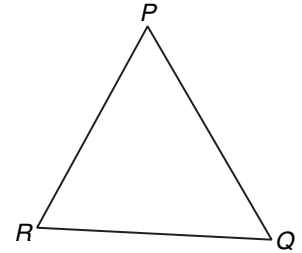
5-1

Enrichment

Inscribed and Circumscribed Circles

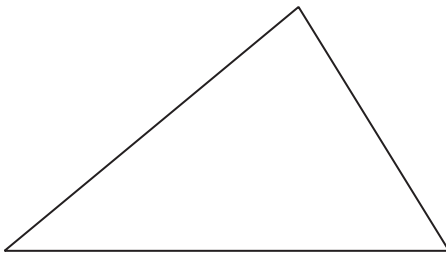
The three angle bisectors of a triangle intersect in a single point called the **incenter**. This point is the center of a circle that just touches the three sides of the triangle. Except for the three points where the circle touches the sides, the circle is inside the triangle. The circle is said to be inscribed in the triangle.

1. With a compass and a straightedge, construct the inscribed circle for $\triangle PQR$ by following the steps below.
 - Step 1** Construct the bisectors of $\angle R$ and $\angle Q$. Label the point where the bisectors meet A .
 - Step 2** Construct a perpendicular segment from A to \overline{RQ} . Use the letter B to label the point where the perpendicular segment intersects \overline{RQ} .
 - Step 3** Use a compass to draw the circle with center at A and radius \overline{AB} .

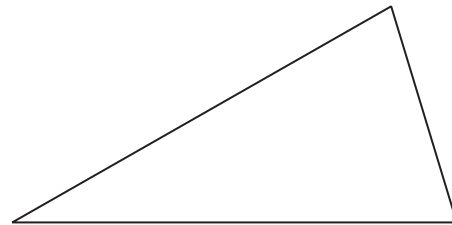


Construct the inscribed circle in each triangle.

2.

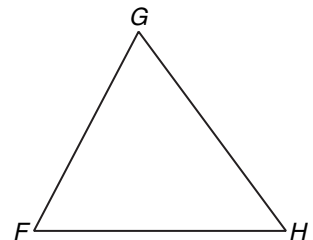


3.



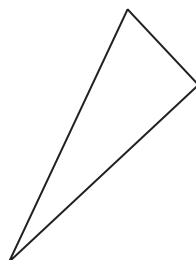
The three perpendicular bisectors of the sides of a triangle also meet in a single point. This point is the center of the circumscribed circle, which passes through each vertex of the triangle. Except for the three points where the circle touches the triangle, the circle is outside the triangle.

4. Follow the steps below to construct the circumscribed circle for $\triangle FGH$.
 - Step 1** Construct the perpendicular bisectors of \overline{FG} and \overline{FH} . Use the letter A to label the point where the perpendicular bisectors meet.
 - Step 2** Draw the circle that has center A and radius \overline{AF} .

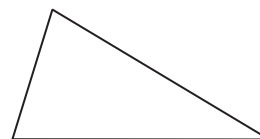


Construct the circumscribed circle for each triangle.

5.



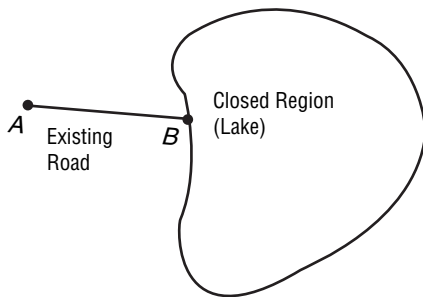
6.



5-2 Enrichment

Construction Problem

The diagram below shows segment AB adjacent to a closed region. The problem requires that you construct another segment XY to the right of the closed region such that points A , B , X , and Y are collinear. You are not allowed to touch or cross the closed region with your compass or straightedge.



Follow these instructions to construct a segment XY so that it is collinear with segment AB .

- Construct the perpendicular bisector of \overline{AB} . Label the midpoint as point C , and the line as m .
- Mark two points P and Q on line m that lie well above the closed region. Construct the perpendicular bisector n of \overline{PQ} . Label the intersection of lines m and n as point D .
- Mark points R and S on line n that lie well to the right of the closed region. Construct the perpendicular bisector k of \overline{RS} . Label the intersection of lines n and k as point E .
- Mark point X on line k so that X is below line n and so that \overline{EX} is congruent to \overline{DC} .
- Mark points T and V on line k and on opposite sides of X , so that \overline{XT} and \overline{XV} are congruent. Construct the perpendicular bisector l of \overline{TV} . Call the point where the line l hits the boundary of the closed region point Y . \overline{XY} corresponds to the new road.

5-3 Enrichment**More Counterexamples**

Some statements in mathematics can be proven false by **counterexamples**. Consider the following statement.

For any numbers a and b , $a - b = b - a$.

You can prove that this statement is false in general if you can find one example for which the statement is false.

Let $a = 7$ and $b = 3$. Substitute these values in the equation above.

$$7 - 3 \stackrel{?}{=} 3 - 7$$

$$4 \neq -4$$

In general, for any numbers a and b , the statement $a - b = b - a$ is false. You can make the equivalent verbal statement: subtraction is *not* a commutative operation.

In each of the following exercises a , b , and c are any numbers. Prove that the statement is false by counterexample.

1. $a - (b - c) \stackrel{?}{=} (a - b) - c$

2. $a \div (b \div c) \stackrel{?}{=} (a \div b) \div c$

3. $a \div b \stackrel{?}{=} b \div a$

4. $a \div (b + c) \stackrel{?}{=} (a \div b) + (a \div c)$

5. $a + (bc) \stackrel{?}{=} (a + b)(a + c)$

6. $a^2 + a^2 \stackrel{?}{=} a^4$

7. Write the verbal equivalents for Exercises 1, 2, and 3.

8. For the Distributive Property $a(b + c) = ab + ac$ it is said that multiplication distributes over addition. Exercises 4 and 5 prove that some operations do not distribute. Write a statement for each exercise that indicates this.

5-4 Enrichment

Constructing Triangles

The measurements of the sides of a triangle are given. If a triangle having sides with these measurements is not possible, then write *impossible*. If a triangle is possible, draw it and measure each angle with a protractor.

1. $AR = 5$ cm $m\angle A =$

$RT = 3$ cm $m\angle R =$

$AT = 6$ cm $m\angle T =$

2. $PI = 8$ cm $m\angle P =$

$IN = 3$ cm $m\angle I =$

$PN = 2$ cm $m\angle N =$

3. $ON = 10$ cm $m\angle O =$

$NE = 5.3$ cm $m\angle N =$

$OE = 4.6$ cm $m\angle E =$

4. $TW = 6$ cm $m\angle T =$

$WO = 7$ cm $m\angle W =$

$TO = 2$ cm $m\angle O =$

5. $BA = 3.1$ cm $m\angle B =$

$AT = 8$ cm $m\angle A =$

$BT = 5$ cm $m\angle T =$

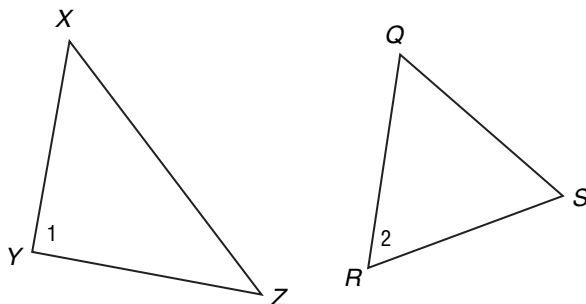
6. $AR = 4$ cm $m\angle A =$

$RM = 5$ cm $m\angle R =$

$AM = 3$ cm $m\angle M =$

5-5 Enrichment***Hinge Theorem***

The Hinge Theorem that you studied in this section states that if two sides of a triangle are congruent to two sides of another triangle and the included angle in one triangle has a greater measure than the included angle in the other, then the third side of the first triangle is longer than the third side of the second triangle. In this activity, you will investigate whether the converse, inverse and contrapositive of the Hinge Theorem are also true.



Hypothesis: $XY = QR$, $YZ = RS$, $m\angle 1 > m\angle 2$

Conclusion: $XZ > QS$

1. What is the converse of the Hinge Theorem?
2. Can you find any counterexamples to prove that the converse is false?
3. What is the inverse of the Hinge Theorem?
4. Can you find any counterexamples to prove that the inverse is false?
5. What is the contrapositive of the Hinge Theorem?
6. Can you find any counterexamples to prove that the contrapositive is false?

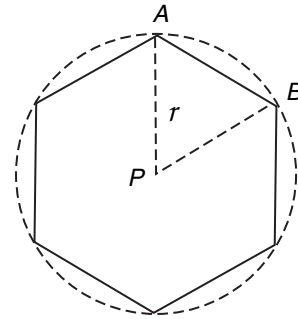
6-1

Enrichment

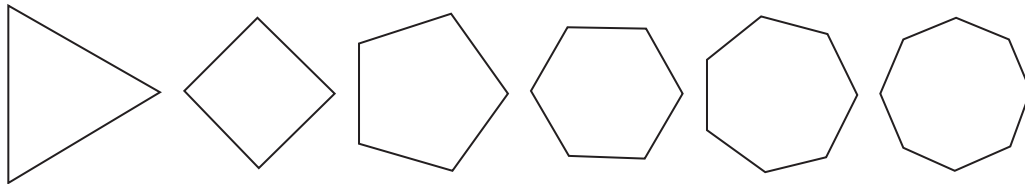
Central Angles of Regular Polygons

You have learned about the interior and exterior angles of a polygon. Regular polygons also have **central angles**. A central angle is measured from the center of the polygon.

The center of a polygon is the point equidistant from all of the vertices of the polygon, just as the center of a circle is the point equidistant from all of the points on the circle. The central angle is the angle drawn with the vertex at the center of the circle and the sides of angle drawn through consecutive vertices of the polygon. One of the central angles that can be drawn in this regular hexagon is $\angle APB$. You may remember from making circle graphs that there are 360° around the center of a circle.



1. By using logic or by drawing sketches, find the measure of the central angle of each regular polygon.



2. Make a conjecture about how the measure of a central angle of a regular polygon relates to the measures of the interior angles and exterior angles of a regular polygon.

3. **CHALLENGE** In obtuse $\triangle ABC$, \overline{BC} is the longest side. \overline{AC} is also a side of a 21-sided regular polygon. \overline{AB} is also a side of a 28-sided regular polygon. The 21-sided regular polygon and the 28-sided regular polygon have the same center point P . Find n if \overline{BC} is a side of a n -sided regular polygon that has center point P .

(Hint: Sketch a circle with center P and place points A , B , and C on the circle.)

6-2

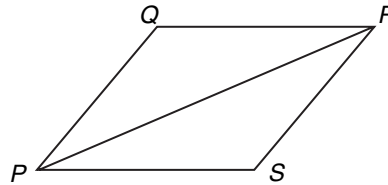
Enrichment

Diagonals of Parallelograms

In some drawings the diagonal of a parallelogram appears to be the angle bisector of both opposite angles. When might that be true?

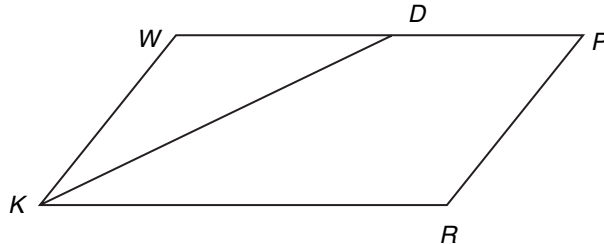
1. **Given:** Parallelogram $PQRS$ with diagonal \overline{PR} .
 \overline{PR} is an angle bisector of $\angle QPS$ and $\angle QRS$.

What type of parallelogram is $PQRS$? Justify your answer.



2. **Given:** Parallelogram $WPRK$ with angle bisector \overline{KD} , $DP = 5$, and $WD = 7$.

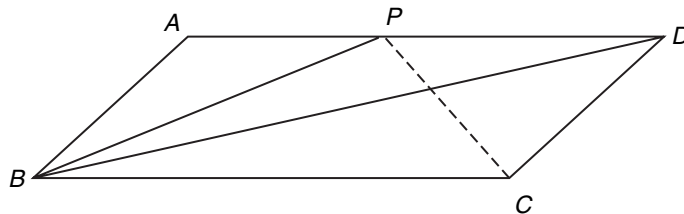
Find WK and KR .



3. Refer to Exercise 2. Write a statement about parallelogram $WPRK$ and angle bisector \overline{KD} .

4. **Given:** Parallelogram $ABCD$ with diagonal \overline{BD} and angle bisector \overline{BP} .
 $PD = 5$, $BP = 6$, and $CP = 6$.

Find AB and BC .



6-3 Enrichment***Tests for Parallelograms***

By definition, a quadrilateral is a parallelogram *if and only if* both pairs of opposite sides are parallel. What conditions other than both pairs of opposite sides parallel will guarantee that a quadrilateral is a parallelogram? In this activity, several possibilities will be investigated by drawing quadrilaterals to satisfy certain conditions. Remember that any test that seems to work is not guaranteed to work unless it can be formally proven.

Complete.

1. Draw a quadrilateral with one pair of opposite sides congruent.
Must it be a parallelogram?
2. Draw a quadrilateral with both pairs of opposite sides congruent.
Must it be a parallelogram?
3. Draw a quadrilateral with one pair of opposite sides parallel and the other pair of opposite sides congruent. Must it be a parallelogram?
4. Draw a quadrilateral with one pair of opposite sides both parallel and congruent. Must it be a parallelogram?
5. Draw a quadrilateral with one pair of opposite angles congruent.
Must it be a parallelogram?
6. Draw a quadrilateral with both pairs of opposite angles congruent.
Must it be a parallelogram?
7. Draw a quadrilateral with one pair of opposite sides parallel and one pair of opposite angles congruent. Must it be a parallelogram?

6-4

Enrichment

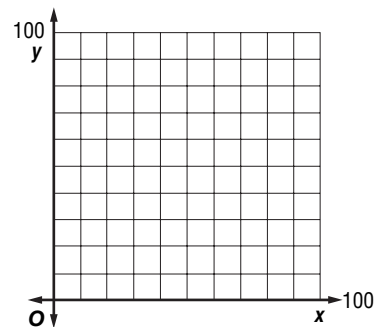
Constant Perimeter

Douglas wants to fence a rectangular region of his back yard for his dog. He bought 200 feet of fence.

- Complete the table to show the dimensions of five different rectangular pens that would use the entire 200 feet of fence. Then find the area of each rectangular pen.
- Do all five of the rectangular pens have the same area? If not, which one has the larger area?

| Perimeter | Length | Width | Area |
|-----------|--------|-------|------|
| 200 | 80 | | |
| 200 | 70 | | |
| 200 | 60 | | |
| 200 | 50 | | |
| 200 | 45 | | |

- Write a rule for finding the dimensions of a rectangle with the largest possible area for a given perimeter.
- Let x represent the length of a rectangle and y the width. Write the formula for all rectangles with a perimeter of 200. Then graph this relationship on the coordinate plane at the right.

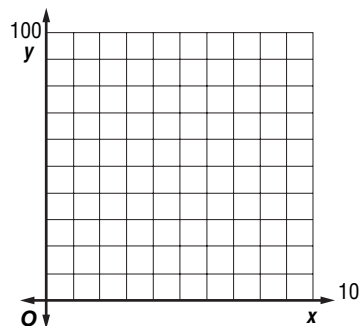


Julio read that a dog the size of his new pet, Bennie, should have at least 100 square feet in his pen. Before going to the store to buy fence, Douglas made a table to determine the dimensions for Bennie's rectangular pen.

- Complete the table to find five possible dimensions of a rectangular fenced area of 100 square feet.
- Douglas wants to save money by purchasing the least number feet of fencing to enclose the 100 square feet e. What will be the dimensions of the completed pen?

| Area | Length | Width | How much fence to buy |
|------|--------|-------|-----------------------|
| 100 | | | |
| 100 | | | |
| 100 | | | |
| 100 | | | |
| 100 | | | |

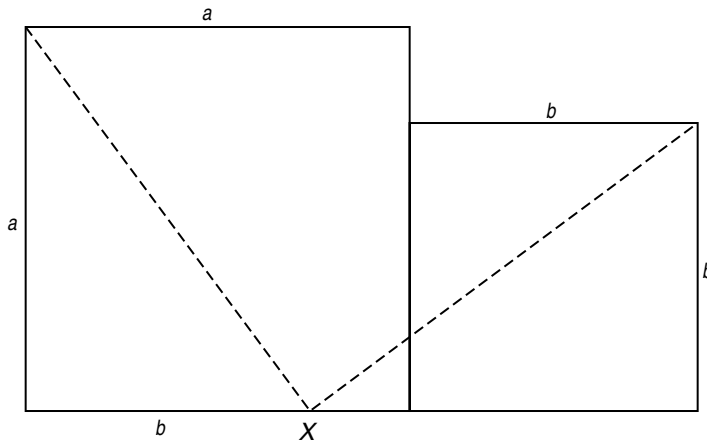
- Write a rule for finding the dimensions of a rectangle with the least possible perimeter for a given area.
- For length x and width y , write a formula for the area of a rectangle with an area of 100 square feet. Then graph the formula.



6-5 Enrichment

Creating Pythagorean Puzzles

By drawing two squares and cutting them in a certain way, you can make a puzzle that demonstrates the Pythagorean Theorem. A sample puzzle is shown. You can create your own puzzle by following the instructions below.



- Carefully construct a square and label the length of a side as a . Then construct a smaller square to the right of it and label the length of a side as b , as shown in the figure above. The bases should be adjacent and collinear.
- Mark a point X that is b units from the left edge of the larger square. Then draw the segments from the upper left corner of the larger square to point X , and from point X to the upper right corner of the smaller square.
- Cut out and rearrange your five pieces to form a larger square. Draw a diagram to show your answer.
- Verify that the length of each side is equal to $\sqrt{a^2 + b^2}$.

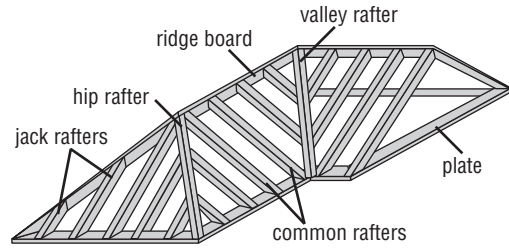
6-6

Enrichment

Quadrilaterals in Construction

Quadrilaterals are often used in construction work.

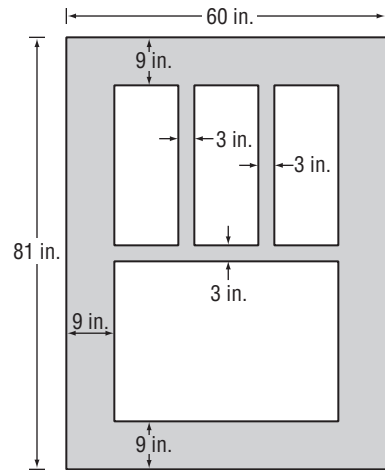
1. The diagram at the right represents a roof frame and shows many quadrilaterals. Find the following shapes in the diagram and shade in their edges.



Roof Frame

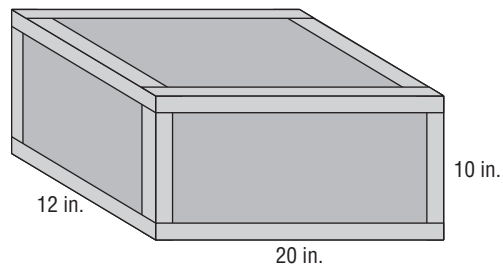
- a. isosceles triangle
- b. scalene triangle
- c. rectangle
- d. rhombus
- e. trapezoid (not isosceles)
- f. isosceles trapezoid

2. The figure at the right represents a window. The wooden part between the panes of glass is 3 inches wide. The frame around the outer edge is 9 inches wide. The outside measurements of the frame are 60 inches by 81 inches. The height of the top and bottom panes is the same. The top three panes are the same size.



- a. How wide is the bottom pane of glass?
- b. How wide is each top pane of glass?
- c. How high is each pane of glass?

3. Each edge of this box has been reinforced with a piece of tape. The box is 10 inches high, 20 inches wide, and 12 inches deep. What is the length of the tape that has been used?



Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

6-7 Enrichment

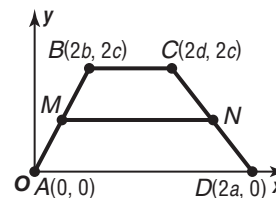
Coordinate Proofs

An important part of planning a coordinate proof is correctly placing and labeling the geometric figure on the coordinate plane.

Example Draw a diagram you would use in a coordinate proof of the following theorem.

The median of a trapezoid is parallel to the bases of the trapezoid.

In the diagram, note that one vertex, A , is placed at the origin. Also, the coordinates of B , C , and D use $2a$, $2b$, and $2c$ in order to avoid the use of fractions when finding the coordinates of the midpoints, M and N .



When doing coordinate proofs, the following strategies may be helpful.

1. If you are asked to prove that segments are parallel or perpendicular, use slopes.
2. If you are asked to prove that segments are congruent or have related measures, use the distance formula.
3. If you are asked to prove that a segment is bisected, use the midpoint formula.

For each of the following theorems, a diagram has been provided to be used in a formal proof. Name the missing coordinates in the diagram. Then, using the *Given* and the *Prove* statements, prove the theorem.

1. The median of a trapezoid is parallel to the bases of the trapezoid. (Use the diagram given in the example above.)

Coordinates of M and N :

Given: Trapezoid $ABCD$ has median \overline{MN} .

Prove: $\overline{BC} \parallel \overline{MN}$ and $\overline{AD} \parallel \overline{MN}$

Proof:

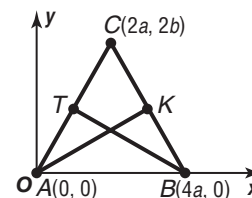
2. The medians to the legs of an isosceles triangle are congruent.

Coordinates of T and K :

Given: $\triangle ABC$ is isosceles with medians \overline{TB} and \overline{KA} .

Prove: $\overline{TB} \cong \overline{KA}$

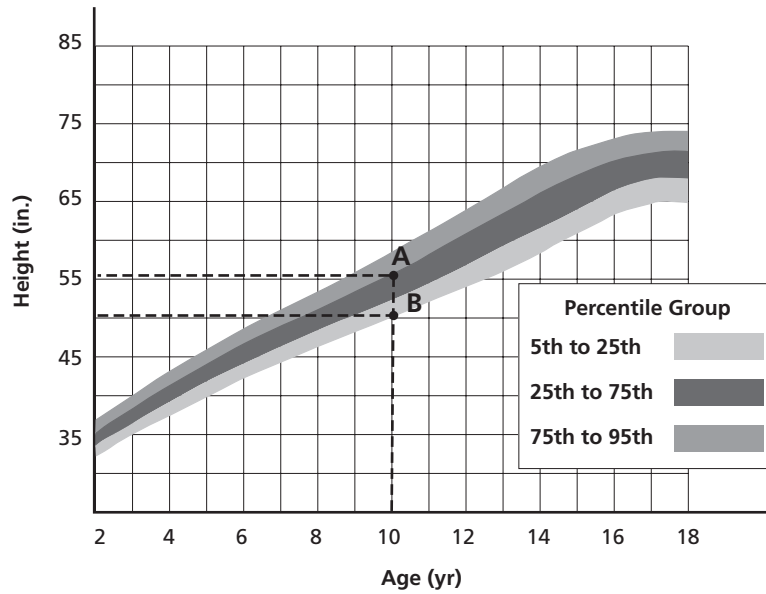
Proof:



7-1 Enrichment

Growth Charts

It is said that when a child has reached the age of 2 years, he is roughly half of his adult height. The growth chart below shows the growth according to percentiles for boys.



- Use the chart to determine the approximate height for a boy at age 2 if he is in the 75th to 95th percentile.
- Using the rule that the height at age 2 is approximately half of his adult height, set up a proportion to solve for the adult height of the boy in exercise 1. Solve your proportion.
- Use the chart to approximate the height at age 18 for a boy if he is in the 75th to 95th percentile. How does this answer compare to the answer to problem 1?
- Repeat this process for a boy who is in the 5th to 25th percentile.
- Is using the rule that a boy is half of his adult height at age 2 years a good approximation? Explain.

7-2

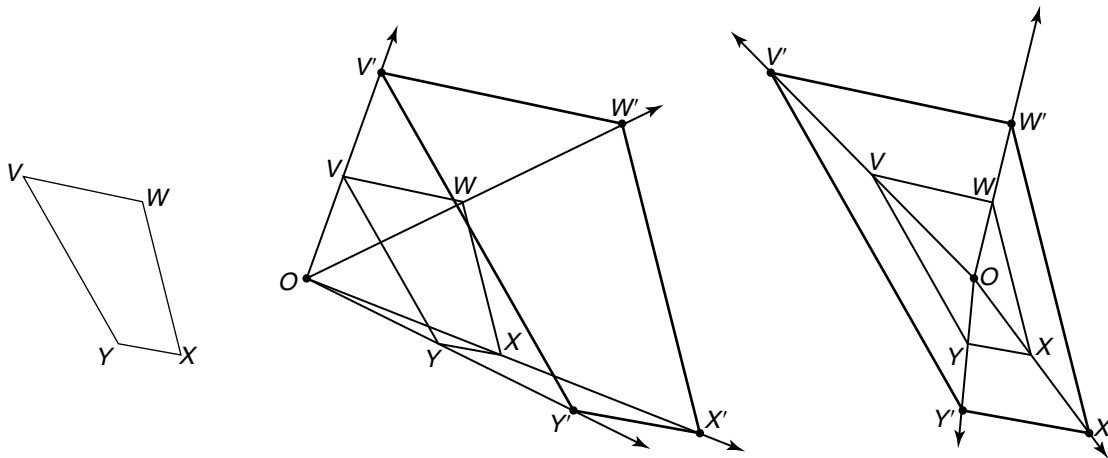
Enrichment

Constructing Similar Polygons

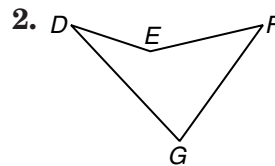
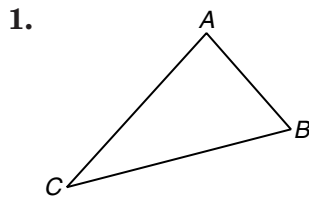
Here are four steps for constructing a polygon that is similar to and with sides twice as long as those of an existing polygon.

- Step 1** Choose any point either inside or outside the polygon and label it O .
- Step 2** Draw rays from O through each vertex of the polygon.
- Step 3** For vertex V , set the compass to length OV . Then locate a new point V' on ray OV such that $VV' = OV$. Thus, $OV' = 2(OV)$.
- Step 4** Repeat Step 3 for each vertex. Connect points V' , W' , X' and Y' to form the new polygon.

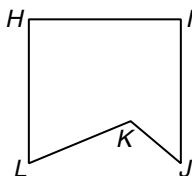
Two constructions of polygons similar to and with sides twice those of $VWXY$ are shown below. Notice that the placement of point O does not affect the size or shape of $V'W'X'Y'$, only its location.



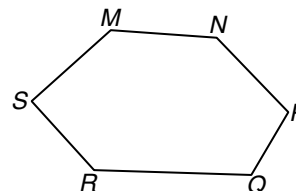
Trace each polygon. Then construct a similar polygon with sides twice as long as those of the given polygon.



3. Explain how to construct a similar polygon with sides three times the length of those of polygon $HJKLM$. Then do the construction.



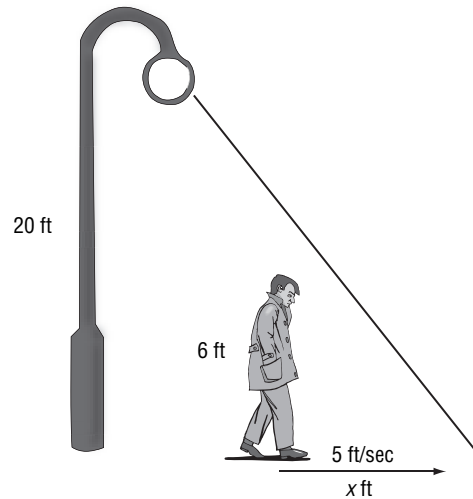
4. Explain how to construct a similar polygon $1\frac{1}{2}$ times the length of those of polygon $MNPQRS$. Then do the construction.



7-3 Enrichment

Moving Shadows

Have you ever watched your shadow as you walked along the street at night and observed how its shape changes as you move? Suppose a man who is 6 feet tall is standing below a lamppost that is 20 feet tall. The man is walking away from the lamppost at a rate of 5 feet per second.



1. If the man is moving at a rate of 5 feet per second, make a conjecture as to the rate that his shadow is moving.
2. How far away from the lamppost is the man after 8 seconds?
3. How far is the end of his shadow from the bottom of the lamppost after 8 seconds? Use similar triangles to solve this problem.
4. After 3 more seconds, how far from the lamppost is the man? How far from the lamppost is his shadow?
5. How many feet did the man move in 3 seconds? How many feet did the shadow move in 3 seconds?
6. The man is moving at a rate of 5 feet/second. What rate is his shadow moving? How does this rate compare to the conjecture you made in problem 1? Make a conjecture as to why the results are like this.

7-4

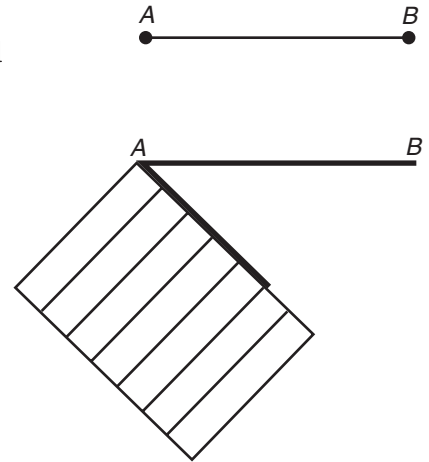
Enrichment

Parallel Lines and Congruent Parts

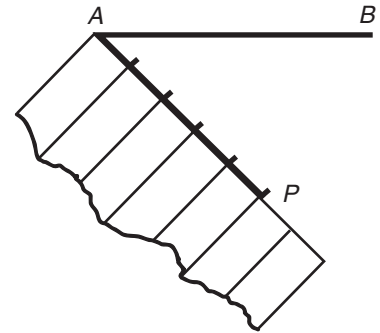
There is a theorem stating that if three parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on any transversal. This can be shown for any number of parallel lines. The following drafting technique uses this fact to divide a segment into congruent parts.

\overline{AB} is to be separated into five congruent parts. This can be done very accurately without using a ruler. All that is needed is a compass and a piece of notebook paper.

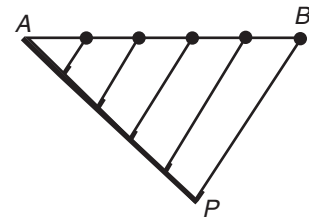
Step 1 Hold the corner of a piece of notebook paper at point A .



Step 2 From point A , draw a segment along the paper that is five spaces long. Mark where the lines of the notebook paper meet the segment. Label the fifth point, P .



Step 3 Draw \overline{PB} . Through each of the other marks on \overline{AP} , construct a line parallel to \overline{BP} . The points where these lines intersect \overline{AB} will divide \overline{AB} into five congruent segments.



Use a compass and a piece of notebook paper to divide each segment into the given number of congruent parts.

1. six congruent parts



2. seven congruent parts

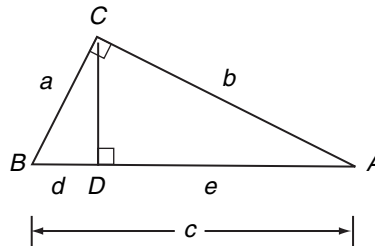


7-5

Enrichment

Another Proof of Pythagorean Theorem

1. For right triangle ABC with right angle C , and altitude \overline{CD} as shown at the right, name three similar triangles.



2. List the three similar triangles as headings in the table below. Use the figure to complete the table to list the corresponding parts of the three similar right triangles.

| | | | |
|------------|--|--|--|
| | | | |
| Short Leg | | | |
| Long Leg | | | |
| Hypotenuse | | | |

3. Use the corresponding parts of these similar triangles and their proportions to complete the statements in the proof and algebraically prove the Pythagorean Theorem.

| Statements | Reasons |
|---|---|
| 1. Right triangle ABC with altitude \overline{CD} . | 1. Given |
| 2. _____ | 2. Corresponding parts of similar triangles are in the same ratio |
| 3. _____ | 3. Cross Products |
| 4. _____ | 4. Addition Property of Equality |
| 5. _____ | 5. Substitution |
| 6. _____ | 6. Distribution Property |
| 7. _____ | 7. Segment addition |
| 8. $a^2 + b^2 = c^2$ | 8. Substitution |

8-1

Enrichment

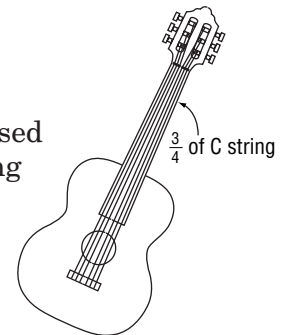
Mathematics and Music

Pythagoras, a Greek philosopher who lived during the sixth century B.C., believed that all nature, beauty, and harmony could be expressed by whole-number relationships. Most people remember Pythagoras for his teachings about right triangles. (The sum of the squares of the legs equals the square of the hypotenuse.) But Pythagoras also discovered relationships between the musical notes of a scale. These relationships can be expressed as ratios.

| | | | | | | | |
|---------------|---------------|---------------|---------------|---------------|---------------|----------------|---------------|
| C | D | E | F | G | A | B | C' |
| $\frac{1}{1}$ | $\frac{8}{9}$ | $\frac{4}{5}$ | $\frac{3}{4}$ | $\frac{2}{3}$ | $\frac{3}{5}$ | $\frac{8}{15}$ | $\frac{1}{2}$ |

When you play a stringed instrument, you produce different notes by placing your finger on different places on a string. This is the result of changing the length of the vibrating part of the string.

The C string can be used to produce F by placing a finger $\frac{3}{4}$ of the way along the string.



Lesson 8-1

Suppose a C string has a length of 16 inches. Write and solve proportions to determine what length of string would have to vibrate to produce the remaining notes of the scale.

- | | | |
|-------|------|------|
| 1. D | 2. E | 3. F |
| 4. G | 5. A | 6. B |
| 7. C' | | |

8. Complete to show the distance between finger positions on the 16-inch

C string for each note. For example, $C(16) - D\left(14\frac{2}{9}\right) = 1\frac{7}{9}$.

C $1\frac{7}{9}$ in. D _____ E _____ F _____ G _____ A _____ B _____ C'

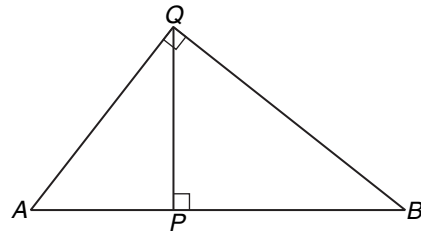
9. Between two consecutive musical notes, there is either a whole step or a half step. Using the distances you found in Exercise 8, determine what two pairs of notes have a half step between them.

8-2 Enrichment

Converse of a Right Triangle Theorem

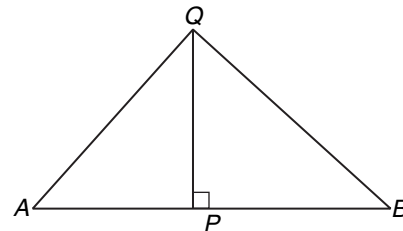
You have learned that the measure of the altitude from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse. Is the converse of this theorem true? In order to find out, it will help to rewrite the original theorem in if-then form as follows.

If $\triangle ABQ$ is a right triangle with right angle at Q , then QP is the geometric mean between AP and PB , where P is between A and B and \overline{QP} is perpendicular to \overline{AB} .



1. Write the converse of the if-then form of the theorem.

2. Is the converse of the original theorem true? Refer to the figure at the right to explain your answer.

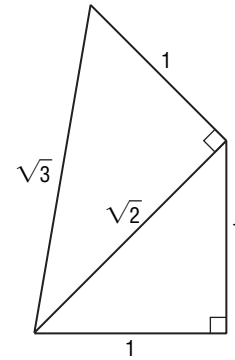


You may find it interesting to examine the other theorems in Chapter 7 to see whether their converses are true or false. You will need to restate the theorems carefully in order to write their converses.

8-3 Enrichment

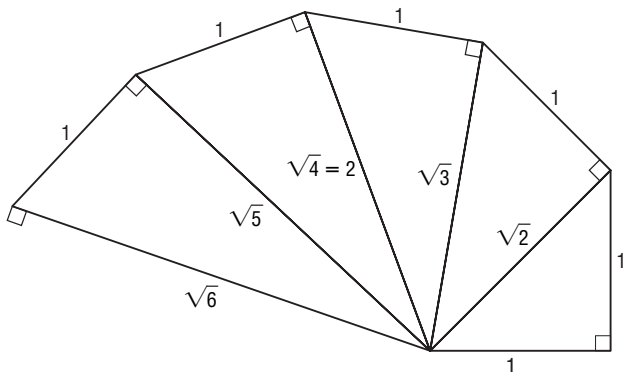
Constructing Values of Square Roots

The diagram at the right shows a right isosceles triangle with two legs of length 1 inch. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{2}$ inches. By constructing an adjacent right triangle with legs of $\sqrt{2}$ inches and 1 inch, you can create a segment of length $\sqrt{3}$.



By continuing this process as shown below, you can construct a “wheel” of square roots. This wheel is called the “Wheel of Theodorus” after a Greek philosopher who lived about 400 B.C.

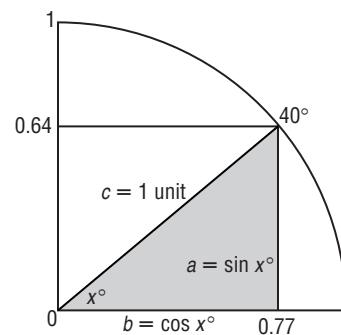
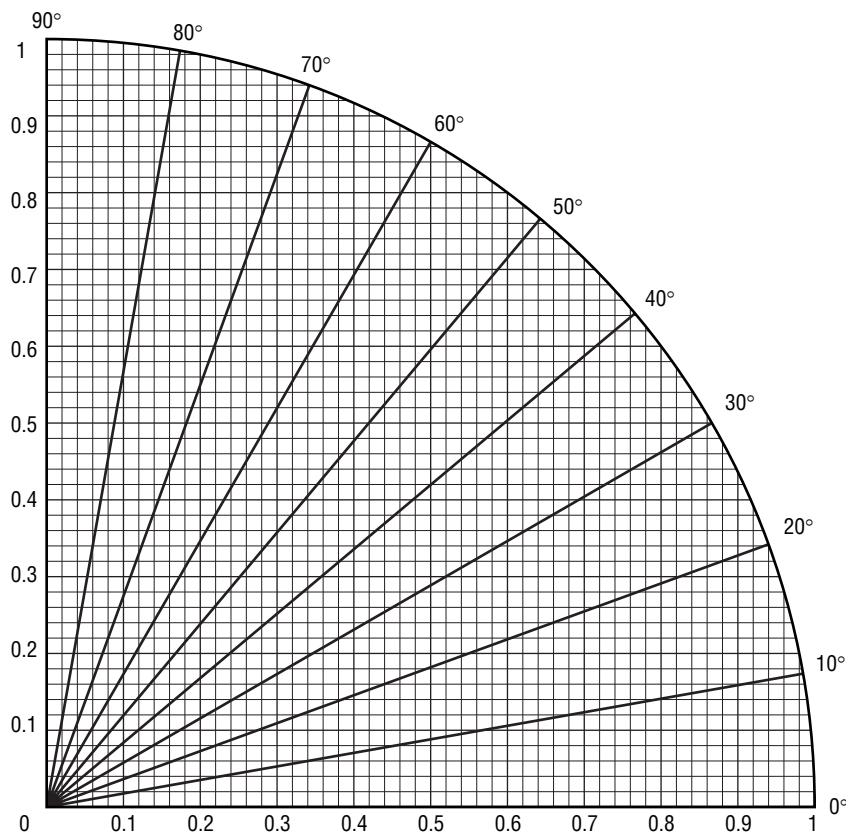
Continue constructing the wheel until you make a segment of length $\sqrt{18}$.



8-4 Enrichment

Sine and Cosine of Angles

The following diagram can be used to obtain approximate values for the sine and cosine of angles from 0° to 90°. The radius of the circle is 1. So, the sine and cosine values can be read directly from the vertical and horizontal axes.



Example Find approximate values for $\sin 40^\circ$ and $\cos 40^\circ$. Consider the triangle formed by the segment marked 40° , as illustrated by the shaded triangle at right.

$$\sin 40^\circ = \frac{a}{c} \approx \frac{0.64}{1} \text{ or } 0.64 \quad \cos 40^\circ = \frac{b}{c} \approx \frac{0.77}{1} \text{ or } 0.77$$

1. Use the diagram above to complete the chart of values.

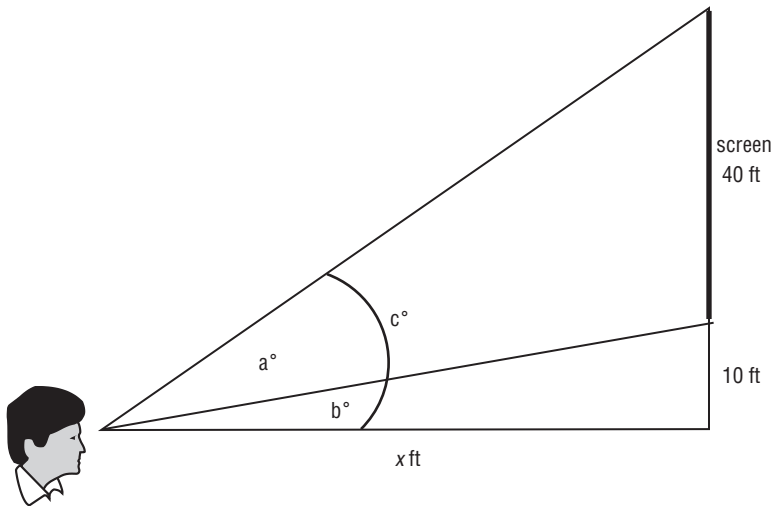
| x° | 0° | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° |
|----------------|----|-----|-----|-----|------|-----|-----|-----|-----|-----|
| $\sin x^\circ$ | | | | | 0.64 | | | | | |
| $\cos x^\circ$ | | | | | 0.77 | | | | | |

2. Compare the sine and cosine of two complementary angles (angles with a sum is 90°). What do you notice?

8-5 Enrichment

Best Seat in the House

Most people want to sit in the best seat in the movie theater. The best seat could be defined as the seat that allows you to see the maximum amount of screen. The picture below represents this situation.



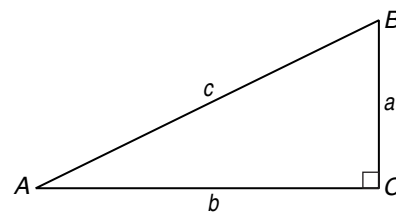
To determine the best seat in the house, you want to find what value of x allows you to see the maximum amount of screen. The value of x is how far from the screen you should sit.

- To maximize the amount of screen viewed, which angle value needs to be maximized? Why?
- What is the value of a if $x = 10$ feet?
- What is the value of a if $x = 20$ feet?
- What is the value of a if $x = 25$ feet?
- What is the value of a if $x = 35$ feet?
- What is the value of a if $x = 55$ feet?
- Which value of x gives the greatest value of a ? So, where is the best seat in the movie theater?

8-6 Enrichment

Identities

An **identity** is an equation that is true for all values of the variable for which both sides are defined. One way to verify an identity is to use a right triangle and the definitions for trigonometric functions.



Example 1 Verify that $(\sin A)^2 + (\cos A)^2 = 1$ is an identity.

$$\begin{aligned} (\sin A)^2 + (\cos A)^2 &= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 \\ &= \frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1 \end{aligned}$$

To check whether an equation *may* be an identity, you can test several values. However, since you cannot test all values, you cannot be *certain* that the equation is an identity.

Example 2 Test $\sin 2x = 2 \sin x \cos x$ to see if it could be an identity.

Try $x = 20$. Use a calculator to evaluate each expression.

$$\begin{array}{ll} \sin 2x = \sin 40 & 2 \sin x \cos x = 2 (\sin 20)(\cos 20) \\ \approx 0.643 & \approx 2(0.342)(0.940) \\ & \approx 0.643 \end{array}$$

Since the left and right sides seem equal, the equation may be an identity.

Exercises

Use triangle ABC shown above. Verify that each equation is an identity.

1. $\frac{\cos A}{\sin A} = \frac{1}{\tan A}$

2. $\frac{\tan B}{\sin B} = \frac{1}{\cos B}$

3. $\tan B \cos B = \sin B$

4. $1 - (\cos B)^2 = (\sin B)^2$

Try several values for x to test whether each equation could be an identity.

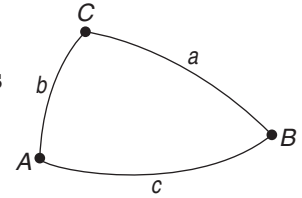
5. $\cos 2x = (\cos x)^2 - (\sin x)^2$

6. $\cos(90 - x) = \sin x$

8-7 Enrichment

Spherical Triangles

Spherical trigonometry is an extension of plane trigonometry. Figures are drawn on the surface of a sphere. Arcs of great circles correspond to line segments in the plane. The arcs of three great circles intersecting on a sphere form a spherical triangle. Angles have the same measure as the tangent lines drawn to each great circle at the vertex. Since the sides are arcs, they too can be measured in degrees.



The sum of the sides of a spherical triangle is less than 360° .
 The sum of the angles is greater than 180° and less than 540° .
 The Law of Sines for spherical triangles is as follows.

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

There is also a Law of Cosines for spherical triangles.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

Example

Solve the spherical triangle given $a = 72^\circ$, $b = 105^\circ$, and $c = 61^\circ$.

Use the Law of Cosines.

$$0.3090 = (-0.2588)(0.4848) + (0.9659)(0.8746) \cos A$$

$$\cos A = 0.5143$$

$$A = 59^\circ$$

$$-0.2588 = (0.3090)(0.4848) + (0.9511)(0.8746) \cos B$$

$$\cos B = -0.4912$$

$$B = 119^\circ$$

$$0.4848 = (0.3090)(-0.2588) + (0.9511)(0.9659) \cos C$$

$$\cos C = 0.6148$$

$$C = 52^\circ$$

Check by using the Law of Sines.

$$\frac{\sin 72^\circ}{\sin 59^\circ} = \frac{\sin 105^\circ}{\sin 119^\circ} = \frac{\sin 61^\circ}{\sin 52^\circ} = 1.1$$

Exercises

Solve each spherical triangle.

1. $a = 56^\circ, b = 53^\circ, c = 94^\circ$

2. $a = 110^\circ, b = 33^\circ, c = 97^\circ$

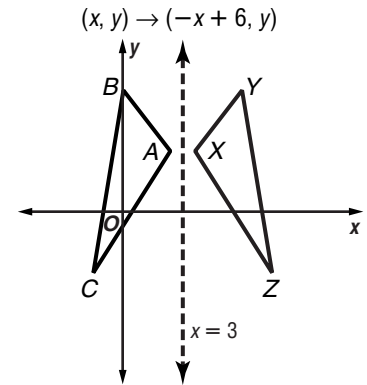
3. $a = 76^\circ, b = 110^\circ, C = 49^\circ$

4. $b = 94^\circ, c = 55^\circ, A = 48^\circ$

9-1 Enrichment

Reflections in the Coordinate Plane

Study the diagram at the right. It shows how the triangle ABC is mapped onto triangle XYZ by the transformation $(x, y) \rightarrow (-x + 6, y)$. Notice that $\triangle XYZ$ is the reflection image with respect to the vertical line with equation $x = 3$.

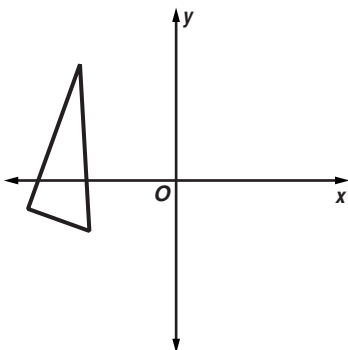


1. Prove that the vertical line with equation $x = 3$ is the perpendicular bisector of the segment with endpoints (x, y) and $(-x + 6, y)$. (Hint: Use the midpoint formula.)

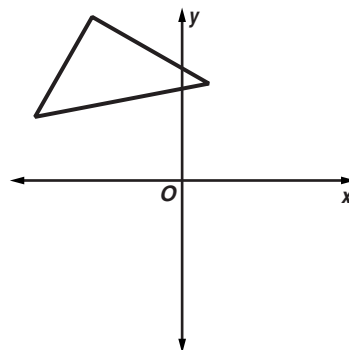
2. Every transformation of the form $(x, y) \rightarrow (-x + 2h, y)$ is a reflection with respect to the vertical line with equation $x = h$. What kind of transformation is $(x, y) \rightarrow (x, -y + 2k)$?

Draw the transformation image for each figure and the given transformation. Is it a reflection transformation? If so, with respect to what line?

3. $(x, y) \rightarrow (-x - 4, y)$



4. $(x, y) \rightarrow (x, -y + 8)$



9-2 Enrichment

Translations in The Coordinate Plane

You can use algebraic descriptions of reflections to show that the composite of two reflections with respect to parallel lines is a translation (that is, a slide).

1. Suppose a and b are two different real numbers. Let S and T be the following reflections.

$$S: (x, y) \rightarrow (-x + 2a, y)$$

$$T: (x, y) \rightarrow (-x + 2b, y)$$

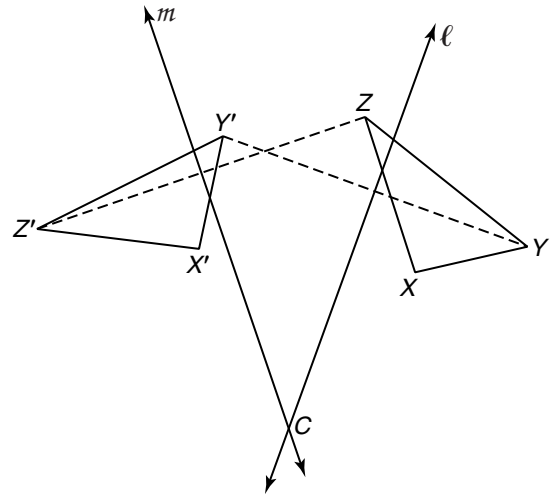
S is reflection with respect to the line with equation $x = a$, and T is reflection with respect to the line with equation $x = b$.

- a. Find an algebraic description (similar to those above for S and T) to describe the composite transformation “ S followed by T .”
- b. Find an algebraic description for the composite transformation “ T followed by S .”
2. Think about the results you obtained in Exercise 1. What do they tell you about how the distance between two parallel lines is related to the distance between a preimage and image point for a composite of reflections with respect to these lines?
3. Illustrate your answers to Exercises 1 and 2 with sketches. Use a separate sheet if necessary.

9-3 Enrichment

Finding the Center of Rotation

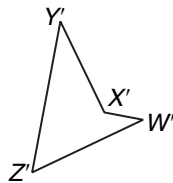
Suppose you are told that $\triangle X'Y'Z'$ is the rotation image of $\triangle XYZ$, but you are not told where the center of rotation is nor the measure of the angle of rotation. Can you find them? Yes, you can. Connect two pairs of corresponding vertices with segments. In the figure, the segments YY' and ZZ' are used. Draw the perpendicular bisectors, ℓ and m , of these segments. The point C where ℓ and m intersect is the center of rotation.



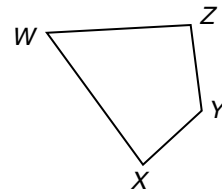
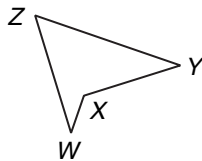
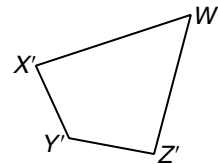
- How can you find the measure of the angle of rotation in the figure above?

Locate the center of rotation for the rotation that maps $WXYZ$ onto $W'X'Y'Z'$. Then find the measure of the angle of rotation.

2.



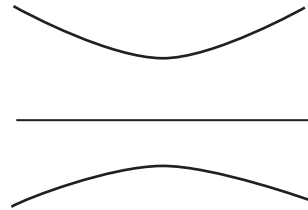
3.



9-4 Enrichment

Hyperbolic Geometry

When asked to define parallel lines, most people will answer that parallel lines are non-intersecting lines. The definition also states that there is only one unique line through a given point parallel to another line. This definition is only true in Euclidean Geometry. In hyperbolic geometry, parallel lines are defined only as non-intersecting. This means that the lines at the right are considered parallel in hyperbolic geometry.



1. M.C. Escher is known for his artwork involving tessellations. Go to the Escher Web site at www.mcescher.com to view his 1959 woodcut *Circle Limit III*. Do the fish appear to be changing size as they move from the inside of the circle to the outside? Explain.
2. In hyperbolic space, objects are drawn on a disc and repeating objects have to be drawn smaller so that they fit on the disc. However, in hyperbolic space, the objects are all the same size. Based on the definition of hyperbolic space, reconsider your answer to Exercise 1. Is your answer the same or have you changed it? Explain.
3. Create your own tessellation in a hyperbolic space. Explain your drawing.

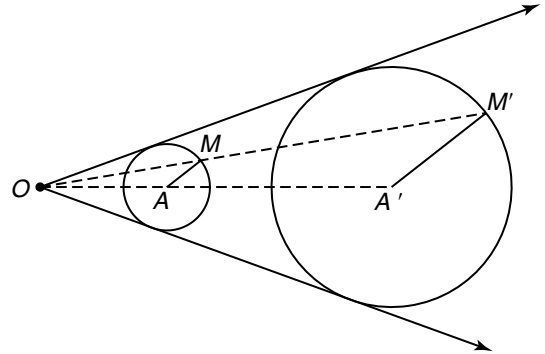
9-5

Enrichment

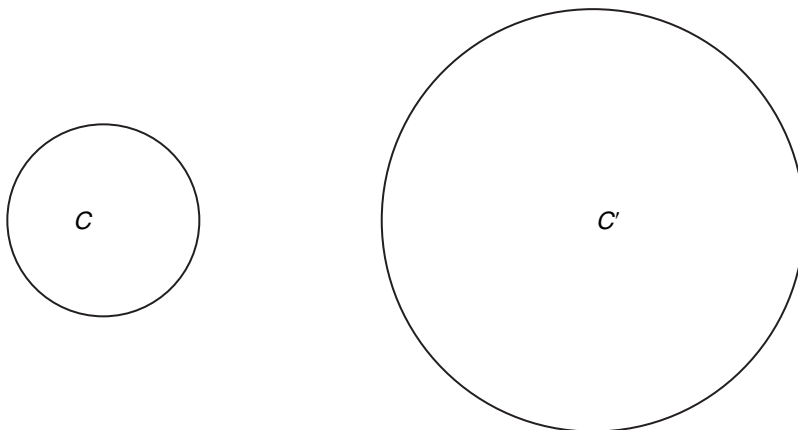
Similar Circles

You may be surprised to learn that two noncongruent circles that lie in the same plane and have no common interior points can be mapped one onto the other by more than one dilation.

- Here is diagram that suggests one way to map a smaller circle onto a larger one using a dilation. The circles are given. The lines suggest how to find the center for the dilation. Describe how the center is found. Use segments in the diagram to name the scale factor.



- Here is another pair of noncongruent circles with no common interior point. From Exercise 1, you know you can locate a point off to the left of the smaller circle that is the center for a dilation mapping $\odot C$ onto $\odot C'$. Find another center for another dilation that maps $\odot C$ onto $\odot C'$. Mark and label segments to name the scale factor.



9-6 Enrichment

Dot Product

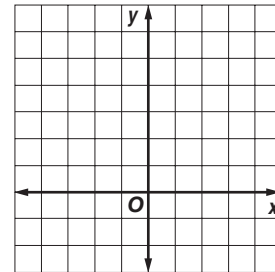
The dot product of two vectors represents how much the vectors point in the direction of each other. If \vec{v} is a vector represented by $\langle a, b \rangle$ and \vec{u} is a vector represented by $\langle c, d \rangle$, the formula to find the dot product is:

$$\vec{v} \cdot \vec{u} = ac + bd$$

Look at the following example:

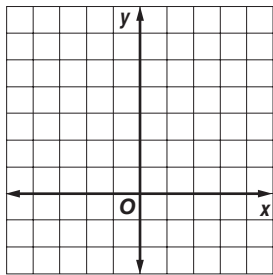
Graph the vectors and find the dot product of \vec{v} and \vec{u} if $\vec{v} = \langle 3, -1 \rangle$ and $\vec{u} = \langle 2, 5 \rangle$.

$$\vec{v} \cdot \vec{u} = (3)(2) + (-1)(5) \text{ or } 1$$

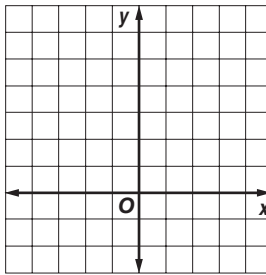


Graph the vectors and find the dot products.

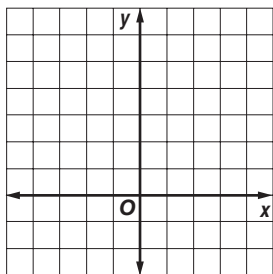
1. $\vec{v} = \langle 2, 1 \rangle$ and $\vec{u} = \langle -4, 2 \rangle$
The dot product is _____



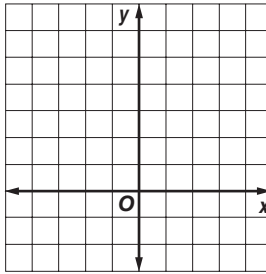
2. $\vec{v} = \langle 3, -2 \rangle$ and $\vec{u} = \langle 1, 4 \rangle$
The dot product is _____



3. $\vec{v} = \langle 0, 3 \rangle$ and $\vec{u} = \langle 2, 4 \rangle$
The dot product is _____



4. $\vec{v} = \langle -1, 4 \rangle$ and $\vec{u} = \langle -4, 2 \rangle$
The dot product is _____

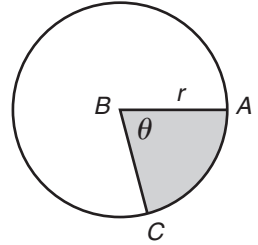


5. Notice the angle formed by the two vectors and the corresponding dot product. Is there any relationship between the type of angle between the two vectors and the sign of the dot product? Make a conjecture.

10-1 Enrichment

Sectors

The area of a circle is found by using the formula $A = \pi r^2$. A sector is a pie-shaped portion of the circle enclosed by 2 radii and the edge of the circle. A central angle of a sector is an angle whose vertex is at the center of the circle and crosses the circle.



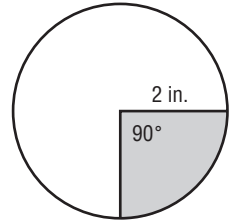
The area of a circle is represented by the formula $A = \pi r^2$. The area of the sector θ is proportional to the part that the central angle is of 360° .

$$\frac{\text{area of sector}}{\text{area of the circle}} = \frac{\theta}{360} \quad \text{or} \quad \text{area of sector} = \frac{\theta}{360} \pi r^2.$$

Example

Find the area of the sector shown at the right.

$$\begin{aligned} A &= \frac{\theta}{360} \pi r^2 \\ A &= \frac{90}{360} \pi 2^2 \quad r = 2, \theta = 90 \\ &= \frac{1}{4} (4\pi) \text{ or } \pi \end{aligned}$$



So the area of the sector is $\pi \text{ in}^2$ or approximately 3.14 square inches.

Exercises

- Find the area of a sector if the circle has a radius of 10 centimeters and the central angle measures 72° .
- Find the area of a sector if the circle has a radius of 5 inches and the central angle measures 60° .
- If the area of a sector is 15π square centimeters and the radius of the circle is 5 centimeters, find the measure of the central angle.
- Find the measure of the central angle that intercepts a sector that is $\frac{1}{3}$ the area of the circle.

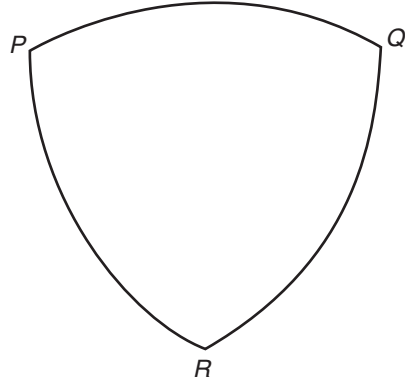
10-2 Enrichment

Curves of Constant Width

A circle is called a curve of constant width because no matter how you turn it, the greatest distance across it is always the same. However, the circle is not the only figure with this property.

The figure at the right is called a Reuleaux triangle.

1. Use a metric ruler to find the distance from P to any point on the opposite side.
2. Find the distance from Q to the opposite side.
3. What is the distance from R to the opposite side?



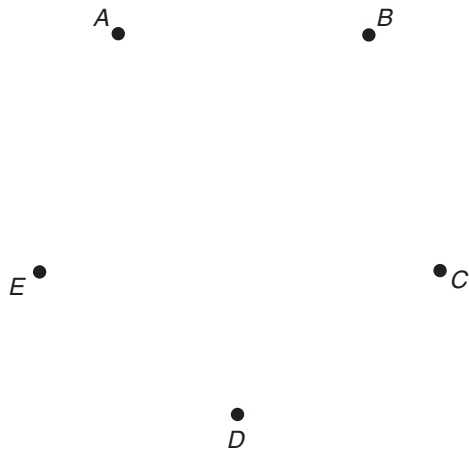
The Reuleaux triangle is made of three arcs. In the example shown, \overline{PQ} has center R , \overline{QR} has center P , and \overline{PR} has center Q .

4. Trace the Reuleaux triangle above on a piece of paper and cut it out. Make a square with sides the length you found in Exercise 1. Show that you can turn the triangle inside the square while keeping its sides in contact with the sides of the square.
5. Make a different curve of constant width by starting with the five points below and following the steps given.

Step 1: Place the point of your compass on D with opening DA . Make an arc with endpoints A and B .

Step 2: Make another arc from B to C that has center E .

Step 3: Continue this process until you have five arcs drawn.



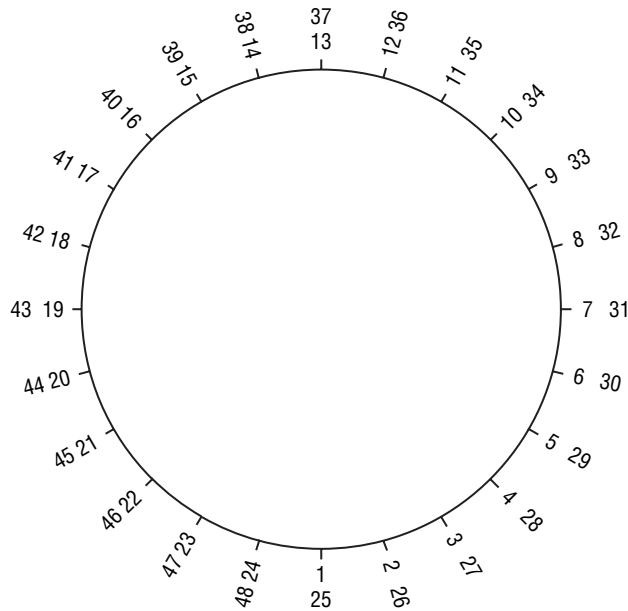
Some countries use shapes like this for coins. They are useful because they can be distinguished by touch, yet they will work in vending machines because of their constant width.

6. Measure the width of the figure you made in Exercise 5. Draw two parallel lines with the distance between them equal to the width you found. On a piece of paper, trace the five-sided figure and cut it out. Show that it will roll between the lines drawn.

10-3 Enrichment

Patterns from Chords

Some beautiful and interesting patterns result if you draw chords to connect evenly spaced points on a circle. On the circle shown below, 24 points have been marked to divide the circle into 24 equal parts. Numbers from 1 to 48 have been placed beside the points. Study the diagram to see exactly how this was done.

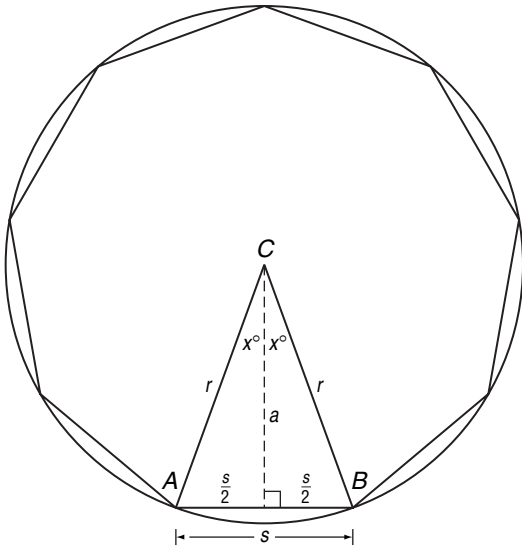


- Use your ruler and pencil to draw chords to connect numbered points as follows: 1 to 2, 2 to 4, 3 to 6, 4 to 8, and so on. Keep doubling until you have gone all the way around the circle. What kind of pattern do you get?
- Copy the original circle, points, and numbers. Try other patterns for connecting points. For example, you might try tripling the first number to get the number for the second endpoint of each chord. Keep special patterns for a possible class display.

10-4 Enrichment

Formulas for Regular Polygons

Suppose a regular polygon of n sides is inscribed in a circle of radius r . The figure shows one of the isosceles triangles formed by joining the endpoints of one side of the polygon to the center C of the circle. In the figure, s is the length of each side of the regular polygon, and a is the length of the segment from C perpendicular to \overline{AB} .

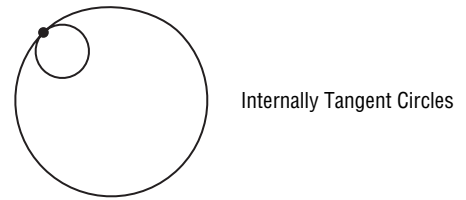
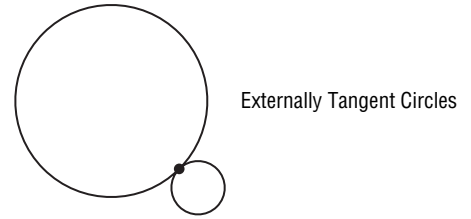


Use your knowledge of triangles and trigonometry to solve the following problems.

1. Find a formula for x in terms of the number of sides n of the polygon.
2. Find a formula for s in terms of the number of n and r . Use trigonometry.
3. Find a formula for a in terms of n and r . Use trigonometry.
4. Find a formula for the *perimeter* of the regular polygon in terms of n and r .

10-5 Enrichment***Tangent Circles***

Two circles in the same plane are **tangent circles** if they have exactly one point in common. Tangent circles with no common interior points are **externally tangent**. If tangent circles have common interior points, then they are **internally tangent**. Three or more circles are **mutually tangent** if each pair of them is tangent.

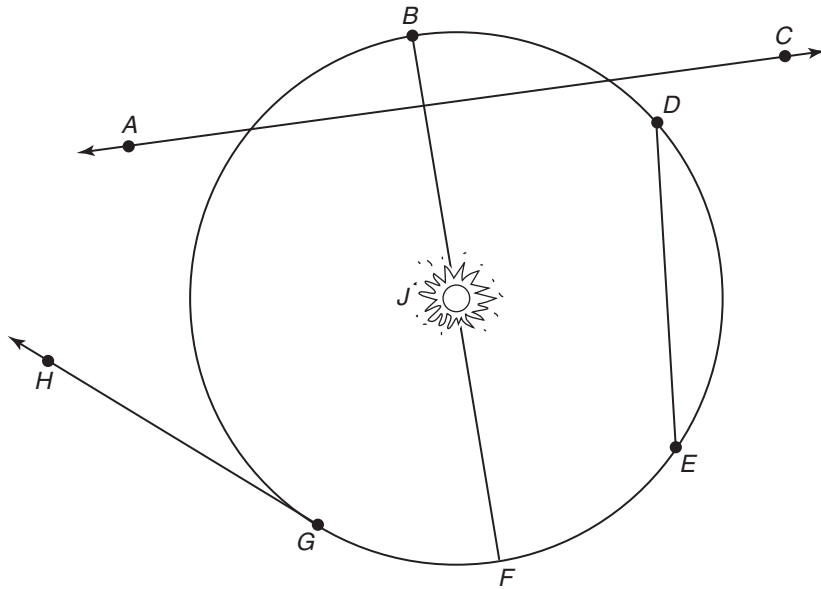


1. Make sketches to show all possible positions of three mutually tangent circles.
2. Make sketches to show all possible positions of four mutually tangent circles.
3. Make sketches to show all possible positions of five mutually tangent circles.
4. Write a conjecture about the number of possible positions for n mutually tangent circles if n is a whole number greater than four.

10-6 Enrichment

Orbiting Bodies

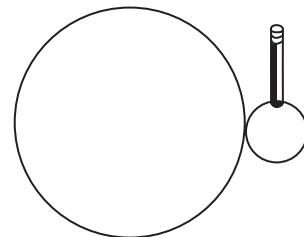
The path of the Earth's orbit around the sun is elliptical. However, it is often viewed as circular.



Use the drawing above of the Earth orbiting the sun to name the line or segment described. Then identify it as a *radius*, *diameter*, *chord*, *tangent*, or *secant* of the orbit.

1. the path of an asteroid
2. the distance between the Earth's position in July and the Earth's position in October
3. the distance between the Earth's position in December and the Earth's position in June
4. the path of a rocket shot toward Saturn
5. the path of a sunbeam
6. If a planet has a moon, the moon circles the planet as the planet circles the sun. To visualize the path of the moon, cut two circles from a piece of cardboard, one with a diameter of 4 inches and one with a diameter of 1 inch.

Tape the larger circle firmly to a piece of paper. Poke a pencil point through the smaller circle, close to the edge. Roll the small circle around the outside of the large one. The pencil will trace out the path of a moon circling its planet. This kind of curve is called an epicycloid. To see the path of the planet around the sun, poke the pencil through the center of the small circle (the planet), and roll the small circle around the large one (the sun).

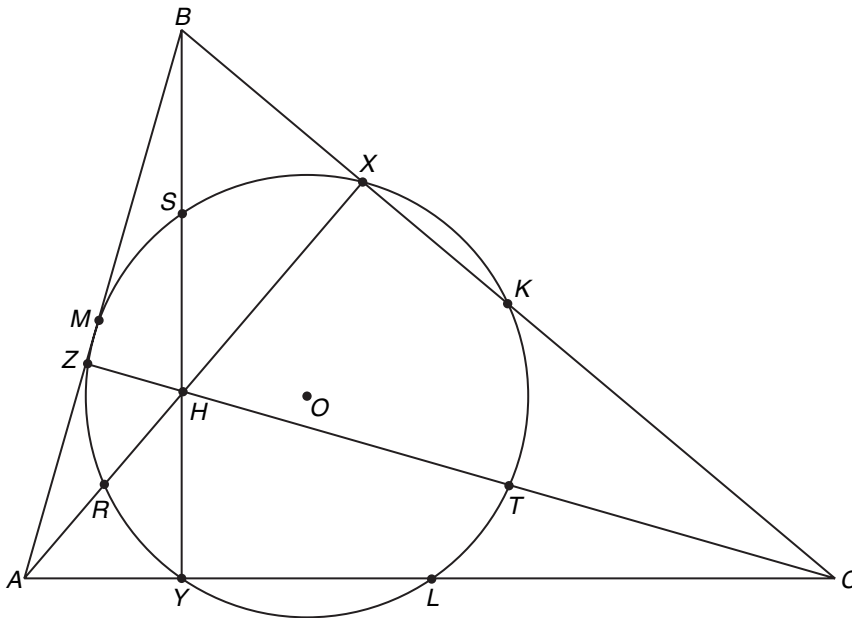


10-7 Enrichment

The Nine-Point Circle

The figure below illustrates a surprising fact about triangles and circles. Given any $\triangle ABC$, there is a circle that contains all of the following nine points:

- (1) the midpoints K , L , and M of the sides of $\triangle ABC$
- (2) the points X , Y , and Z , where \overline{AX} , \overline{BY} , and \overline{CZ} are the altitudes of $\triangle ABC$
- (3) the points R , S , and T which are the midpoints of the segments \overline{AH} , \overline{BH} , and \overline{CH} that join the vertices of $\triangle ABC$ to the point H where the lines containing the altitudes intersect.

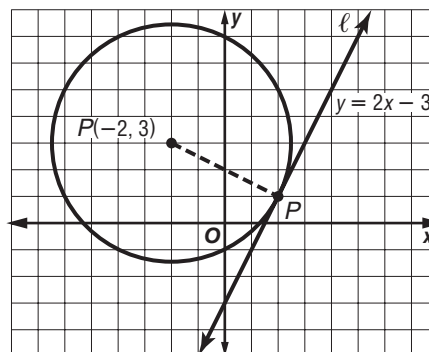


1. On a separate sheet of paper, draw an obtuse triangle ABC . Use your straightedge and compass to construct the circle passing through the midpoints of the sides. Be careful to make your construction as accurate as possible. Does your circle contain the other six points described above?
2. In the figure you constructed for Exercise 1, draw \overline{RK} , \overline{SL} , and \overline{TM} . What do you observe?

10-8 Enrichment

Equations of Circles and Tangents

Recall that the circle whose radius is r and whose center has coordinates (h, k) is the graph of $(x - h)^2 + (y - k)^2 = r^2$. You can use this idea and what you know about circles and tangents to find an equation of the circle that has a given center and is tangent to a given line.



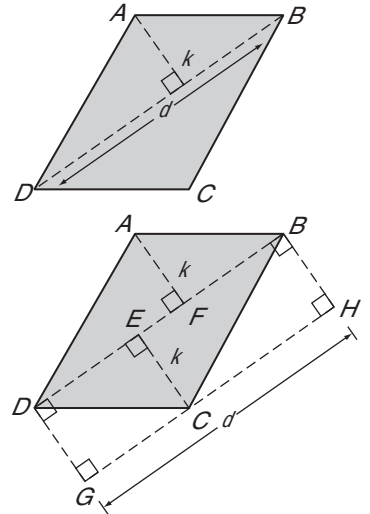
Use the following steps to find an equation for the circle that has center $C(-2, 3)$ and is tangent to the graph $y = 2x - 3$. Refer to the figure.

1. State the slope of the line ℓ that has equation $y = 2x - 3$.
2. Suppose $\odot C$ with center $C(-2, 3)$ is tangent to line ℓ at point P . What is the slope of radius \overline{CP} ?
3. Find an equation for the line that contains \overline{CP} .
4. Use your equation from Exercise 3 and the equation $y = 2x - 3$. At what point do the lines for these equations intersect? What are its coordinates?
5. Find the measure of radius \overline{CP} .
6. Use the coordinate pair $C(-2, 3)$ and your answer for Exercise 5 to write an equation for $\odot C$.

11-1 Enrichment

Area of a Parallelogram

You can prove some interesting results using the formula you have proved for the area of a parallelogram by drawing auxiliary lines to form congruent regions. Consider the top parallelogram shown at the right. In the figure, d is the length of the diagonal \overline{BD} , and k is the length of the perpendicular segment from A to \overline{BD} . Now consider the second figure, which shows the same parallelogram with a number of auxiliary perpendiculars added. Use what you know about perpendicular lines, parallel lines, and congruent triangles to answer the following.



1. What kind of figure is $DBHG$?

2. If you moved $\triangle AFB$ to the lower-left end of figure $DBHG$, would it fit perfectly on top of $\triangle DGC$? Explain your answer.

3. Which two triangular pieces of $\square ABCD$ are congruent to $\triangle CBH$?

4. The area of $\square ABCD$ is the same as that of figure $DBHG$, since the pieces of $\square ABCD$ can be rearranged to form $DBHG$. Express the area of $\square ABCD$ in terms of the measurements k and d .

11-2 Enrichment

Areas of Similar Triangles

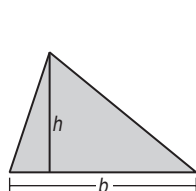
You have learned that if two triangles are similar, the ratio of the lengths of corresponding altitudes is equal to the ratio of the lengths of a pair of corresponding sides. However, there is a different relationship between the areas of the two triangles.

Theorem If two triangles are similar, the ratio of their areas is the square of the ratio of the lengths of a pair of corresponding sides.

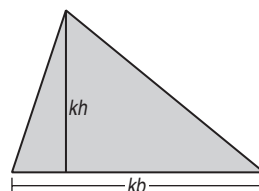
Triangle II is k times larger than Triangle I. Thus, its base is k times as large as that of Triangle I and its height is k times as large as that of Triangle I.

$$\frac{\text{side of } \triangle II}{\text{side of } \triangle I} = \frac{kb}{b} \text{ or } \frac{k}{1}$$

$$\frac{\text{area of } \triangle II}{\text{area of } \triangle I} = \frac{\frac{1}{2}k^2bh}{\frac{1}{2}bh} \text{ or } \frac{k^2}{1}$$



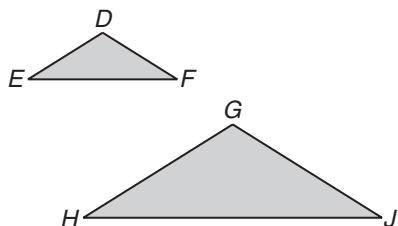
Triangle I
area $\triangle I = \frac{1}{2}bh$



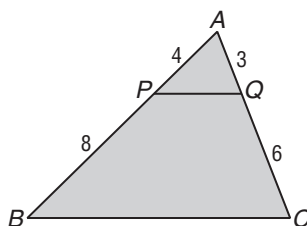
Triangle II
area $\triangle II = \frac{1}{2}(kb)(kh)$
 $= \frac{1}{2}k^2bh$

Solve.

1. $\triangle DEF \sim \triangle GHJ$, $HJ = 16$, and $EF = 8$. The area of $\triangle GHJ$ is 40. Find the area of $\triangle DEF$.



2. In the figure below, $\overline{PQ} \parallel \overline{BC}$. The area of $\triangle ABC$ is 72. Find the area of $\triangle APQ$.



3. Two similar triangles have areas of 16 and 36. The length of a side of the smaller triangle is 10 feet. Find the length of the corresponding side of the larger triangle.
4. Find the ratio of the areas of two similar triangles if the lengths of two corresponding sides of the triangles are 3 centimeters and 5 centimeters.

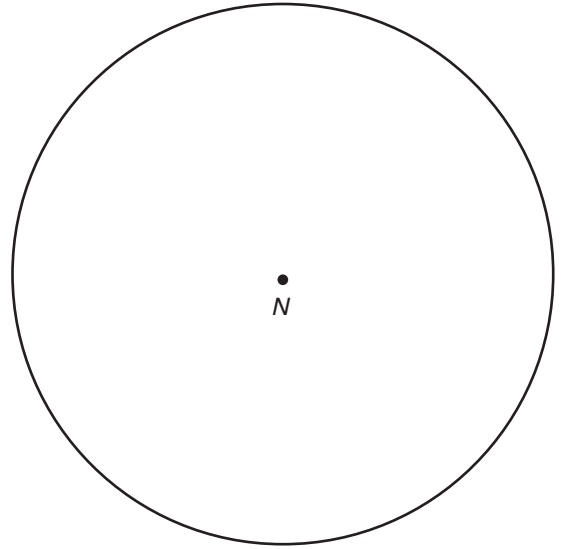
11-3 Enrichment***Areas of Inscribed Polygons***

A protractor can be used to inscribe a regular polygon in a circle. Follow the steps below to inscribe a regular nonagon in $\odot N$.

Step 1 Find the degree measure of each of the nine congruent arcs.

Step 2 Draw 9 radii to form 9 angles with the measure you found in Step 1. The radii will intersect the circle in 9 points.

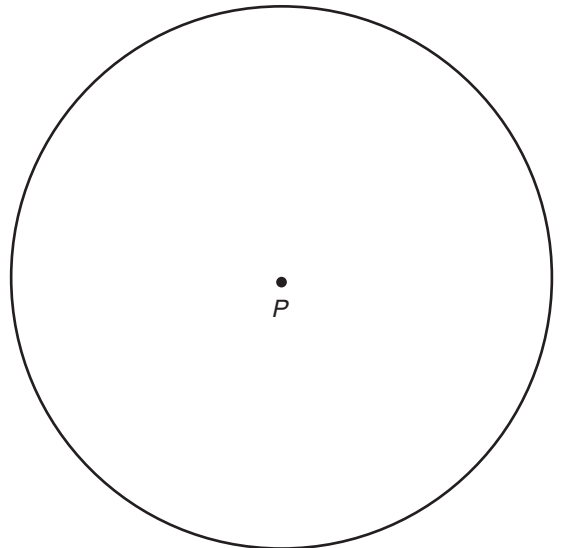
Step 3 Connect the nine points to form the nonagon.



1. Find the length of one side of the nonagon to the nearest tenth of a centimeter. What is the perimeter of the nonagon?
2. Measure the distance from the center perpendicular to one of the sides of the nonagon.
3. What is the area of one of the nine triangles formed?
4. What is the area of the nonagon?

Make the appropriate changes in Steps 1–3 above to inscribe a regular pentagon in $\odot P$. Answer each of the following.

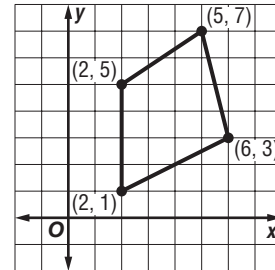
5. Use a protractor to inscribe a regular pentagon in $\odot P$.
6. What is the measure of each of the five congruent arcs?
7. What is the perimeter of the pentagon to the nearest tenth of a centimeter?
8. What is the area of the pentagon to the nearest tenth of a centimeter?



11-4 Enrichment

Aerial Surveyors and Area

Many land regions have irregular shapes. Aerial surveyors often use coordinates when finding areas of such regions. The coordinate method described in the steps below can be used to find the area of *any* polygonal region. Study how this method is used to find the area of the region at the right.



Step 1 List the ordered pairs for the vertices in counter-clockwise order, repeating the first ordered pair at the bottom of the list.

Step 2 Find D , the sum of the downward diagonal products (from left to right).

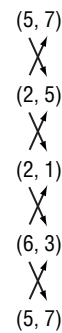
$$D = (5 \cdot 5) + (2 \cdot 1) + (2 \cdot 3) + (6 \cdot 7) \\ = 25 + 2 + 6 + 42 \text{ or } 75$$

Step 3 Find U , the sum of the upward diagonal products (from left to right).

$$U = (2 \cdot 7) + (2 \cdot 5) + (6 \cdot 1) + (5 \cdot 3) \\ = 14 + 10 + 6 + 15 \text{ or } 45$$

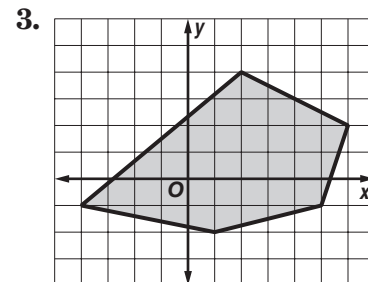
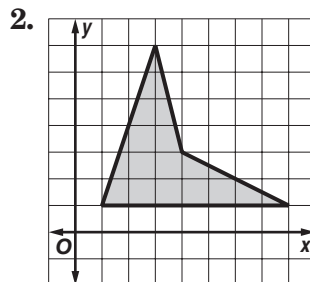
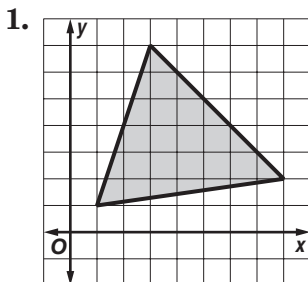
Step 4 Use the formula $A = \frac{1}{2}(D - U)$ to find the area.

$$A = \frac{1}{2}(D - U) \\ = \frac{1}{2}(75 - 45) \\ = \frac{1}{2}(30) \text{ or } 15$$



The area is 15 square units. Count the number of square units enclosed by the polygon. Does this result seem reasonable?

Use the coordinate method to find the area of each region in square units.

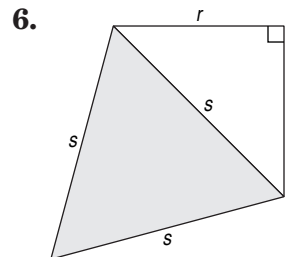
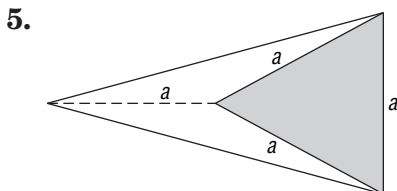
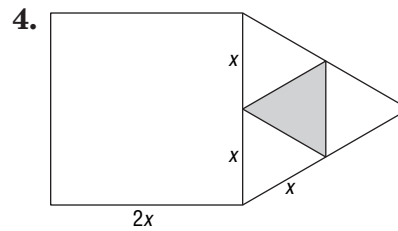
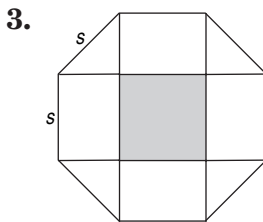
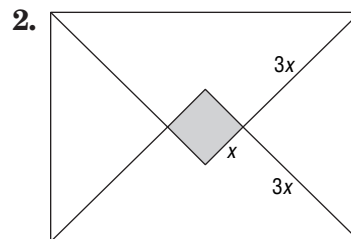
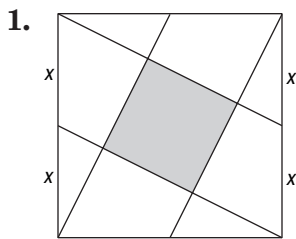


11-5 Enrichment

Polygon Probability

Each problem on this page involves one or more regular polygons. To find the probability of a point chosen at random being in the shaded region, you need to find the ratio of the shaded area to the total area. If you wish, you may substitute numbers for the variables.

Find the probability that a point chosen at random in each figure is in the shaded region. Assume polygons that appear to be regular are regular. Round your answer to the nearest hundredth.

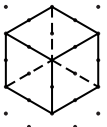


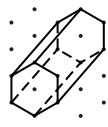
12-1 Enrichment


Drawing Solids on Isometric Dot Paper


Isometric dot paper is helpful for drawing solids. Remember to use dashed lines for hidden edges.

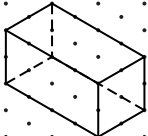
For each solid shown, draw another solid whose dimensions are twice as large.

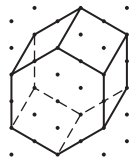
1. 

2. 

3. 

4. 

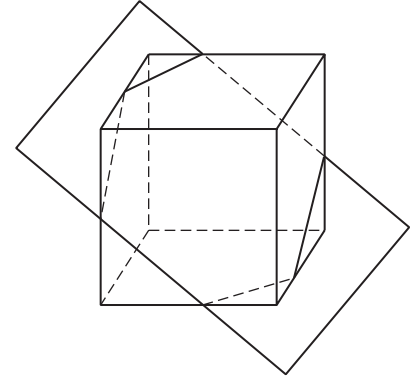
5. 

6. 

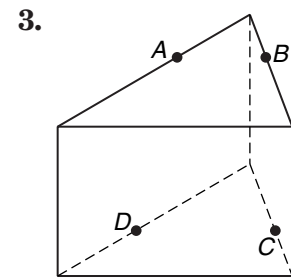
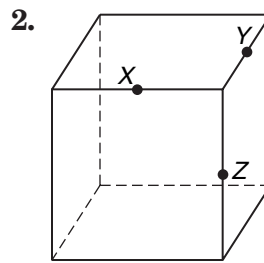
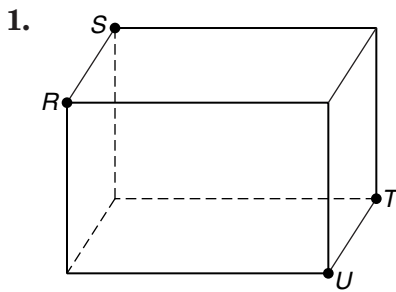
12-2 Enrichment

Cross Sections of Prisms

When a plane intersects a solid figure to form a two-dimensional figure, the result is called a **cross section**. The figure at the right shows a plane intersecting a cube. The cross section is a hexagon.

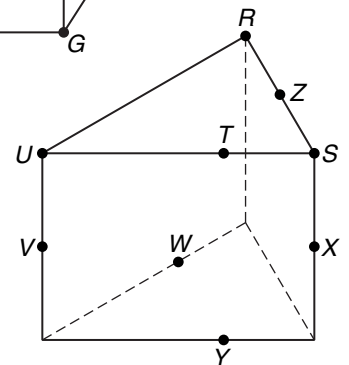
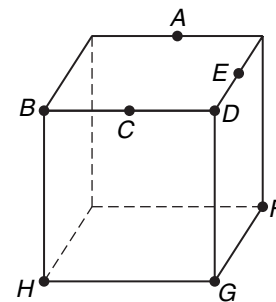


For each right prism, connect the labeled points in alphabetical order to show a cross section. Then identify the polygon.



Refer to the right prisms shown at the right. In the rectangular prism, *A* and *C* are midpoints. Identify the cross-section polygon formed by a plane containing the given points.

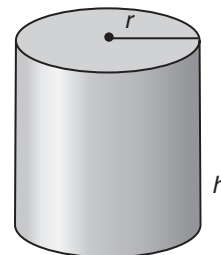
4. *A, C, H*
5. *C, E, G*
6. *H, C, E, F*
7. *H, A, E*
8. *B, D, F*
9. *V, X, R*
10. *R, T, Y*
11. *R, S, W*



12-3 Enrichment

Minimizing Cost in Manufacturing

Suppose that a manufacturer wants to make a can that has a volume of 40 cubic inches. The cost to make the can is 3 cents per square inch for the top and bottom and 1 cent per square inch for the side.



- Write the value of h in terms of r .
- Write a formula for the cost in terms of r .
- Use a graphing calculator to graph the formula, letting Y_1 represent the cost and X represent r . Use the graph to estimate the point at which the cost is minimized.
- Repeat the procedure using 2 cents per square inch for the top and bottom and 4 cents per square inch for the top and bottom.
- What would you expect to happen as the cost of the top and bottom increases?
- Compute the table for the cost value given. What happens to the height of the can as the cost of the top and bottom increases?

| Cost Top & Bottom | Cost Cylinder | Minimum h |
|-------------------|---------------|-------------|
| 2 cents | 1 cent | |
| 3 cents | 1 cent | |
| 4 cents | 1 cent | |
| 5 cents | 1 cent | |
| 6 cents | 1 cent | |

12-4 Enrichment

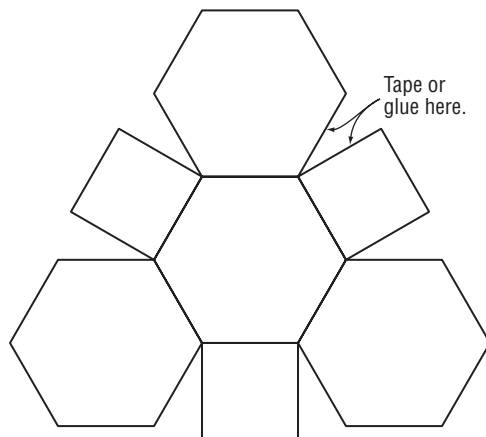
Two Truncated Solids

To create a truncated solid, you could start with an ordinary solid and then cut off the corners. Another way to make such a shape is to use the patterns on this page.

The Truncated Octahedron

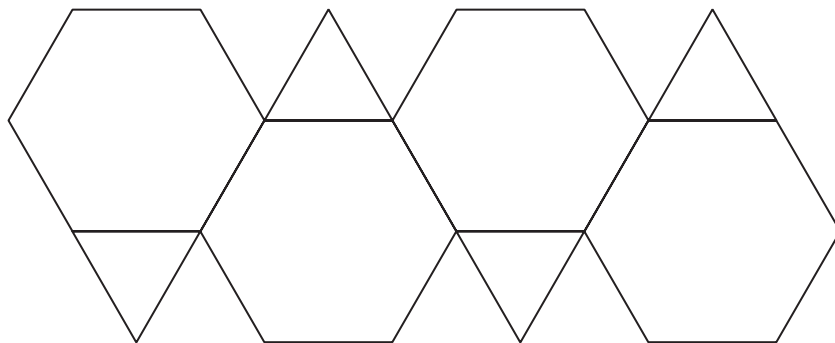
- Two copies of the pattern at the right can be used to make a *truncated octahedron*, a solid with 6 square faces and 8 regular hexagonal faces.

Each pattern makes half of the truncated octahedron. Attach adjacent faces using glue or tape to make a cup-shaped figure.



The Truncated Tetrahedron

- The pattern below will make a *truncated tetrahedron*, a solid with 8 polygonal faces: 4 hexagons and 4 equilateral triangles.



Solve.

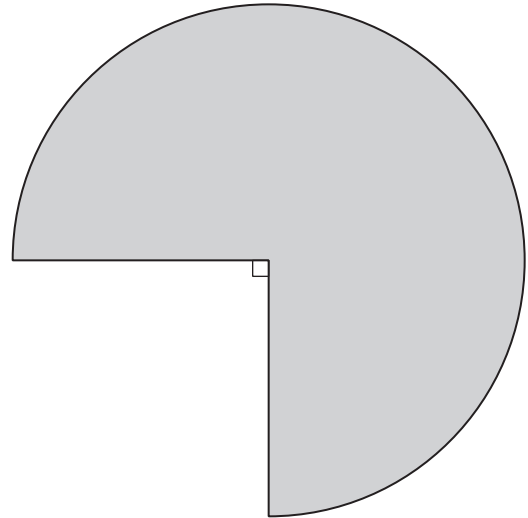
- Find the surface area of the truncated octahedron if each polygon in the pattern has sides of 3 inches.
- Find the surface area of the truncated tetrahedron if each polygon in the pattern has sides of 3 inches.

| Area Formulas for Regular Polygons (s is the length of one side) | |
|--|------------------------------|
| triangle | $A = \frac{s^2}{4}\sqrt{3}$ |
| hexagon | $A = \frac{3s^2}{2}\sqrt{3}$ |
| octagon | $A = 2s^2(\sqrt{2} + 1)$ |

12-5 Enrichment

Cone Patterns

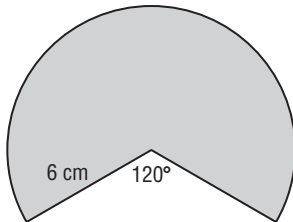
The pattern at the right is made from a circle. It can be folded to make a cone.



1. Measure the radius of the circle to the nearest centimeter.
2. The pattern is what fraction of the complete circle?
3. What is the circumference of the complete circle?
4. How long is the circular arc that is the outside of the pattern?
5. Cut out the pattern and tape it together to form a cone.
6. Measure the diameter of the circular base of the cone.
7. What is the circumference of the base of the cone?
8. What is the slant height of the cone?
9. Use the Pythagorean Theorem to calculate the height of the cone. Use a decimal approximation. Check your calculation by measuring the height with a metric ruler.
10. Find the lateral area.
11. Find the total surface area.

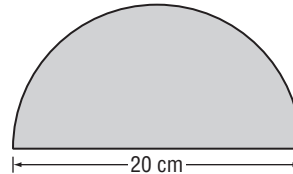
Make a paper pattern for each cone with the given measurements. Then cut the pattern out and make the cone. Find the measurements.

12.



diameter of base =
 lateral area =
 height of cone =
 (to nearest tenth of a centimeter)

13.



diameter of base =
 lateral area =
 height of cone =
 (to nearest tenth of a centimeter)

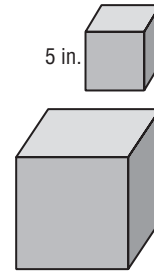
12-6 Enrichment

Doubling Sizes

Consider what happens to surface area when the sides of a figure are doubled.

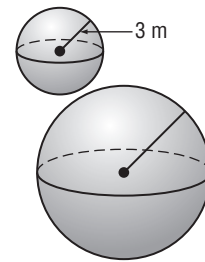
The sides of the large cube are twice the size of the sides of the small cube.

1. How long are the edges of the large cube?
2. What is the surface area of the small cube?
3. What is the surface area of the large cube?
4. The surface area of the large cube is how many times greater than that of the small cube?



The radius of the large sphere at the right is twice the radius of the small sphere.

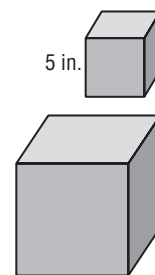
5. What is the surface area of the small sphere?
6. What is the surface area of the large sphere?
7. The surface area of the large sphere is how many times greater than the surface area of the small sphere?
8. It appears that if the dimensions of a solid are doubled, the surface area is multiplied by _____.



Now consider how doubling the dimensions affects the volume of a cube.

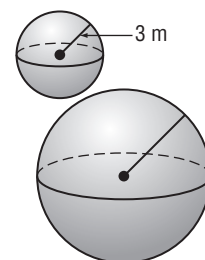
The sides of the large cube are twice the size of the sides of the small cube.

9. How long are the edges of the large cube?
10. What is the volume of the small cube?
11. What is the volume of the large cube?
12. The volume of the large cube is how many times greater than that of the small cube?



The large sphere at the right has twice the radius of the small sphere.

13. What is the volume of the small sphere?
14. What is the volume of the large sphere?
15. The volume of the large sphere is how many times greater than the volume of the small sphere?
16. It appears that if the dimensions of a solid are doubled, the volume is multiplied by _____.

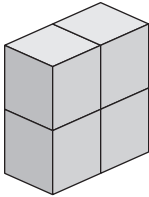


13-1 Enrichment

Visible Surface Area

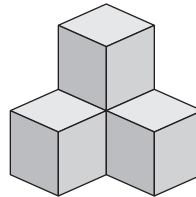
Use paper, scissors, and tape to make five cubes that have one-inch edges. Arrange the cubes to form each shape shown. Then find the volume and the visible surface area. In other words, do not include the area of surface covered by other cubes or by the table or desk.

1.



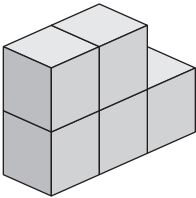
volume =
surface area =

2.



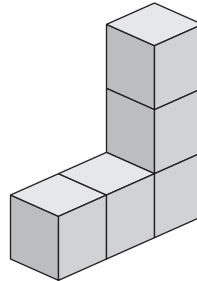
volume =
surface area =

3.



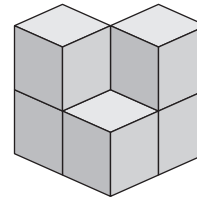
volume =
surface area =

4.



volume =
surface area =

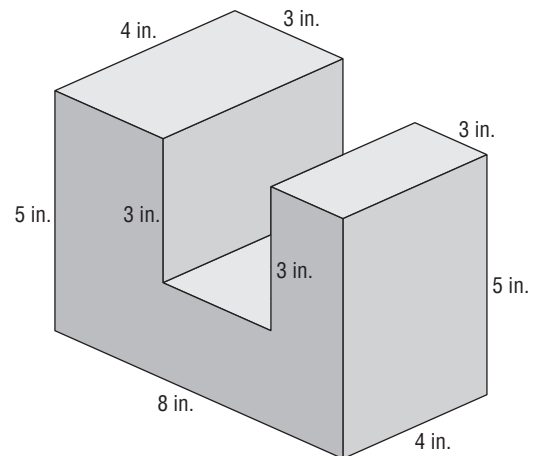
5.



volume =
surface area =

6. Find the volume and the visible surface area of the figure at the right.

volume =
surface area =



13-2 Enrichment

Frustums

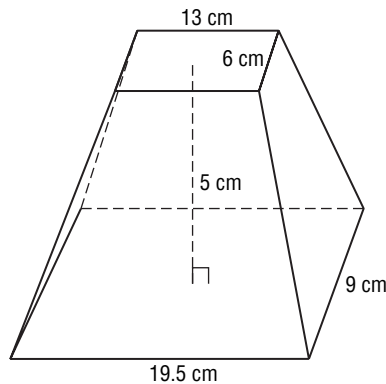
A **frustum** is a figure formed when a plane intersects a pyramid or cone so that the plane is parallel to the solid's base. The frustum is the part of the solid between the plane and the base. To find the volume of a frustum, the areas of both bases must be calculated and used in the formula

$$V = \frac{1}{3}h(B_1 + B_2 + \sqrt{B_1B_2}),$$

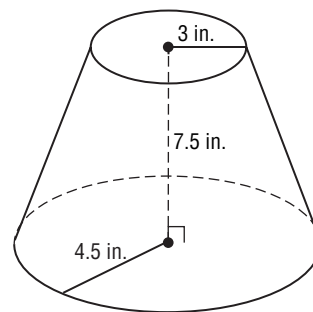
where h = height (perpendicular distance between the bases),
 B_1 = area of top base, and B_2 = area of bottom base.

Describe the shape of the bases of each frustum. Then find the volume. Round to the nearest tenth.

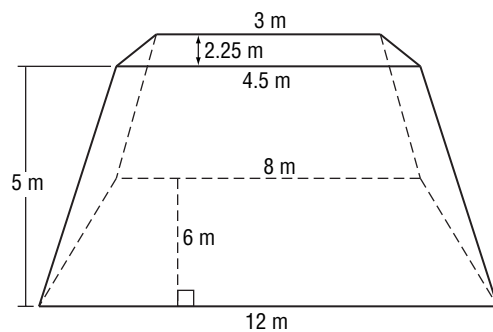
1.



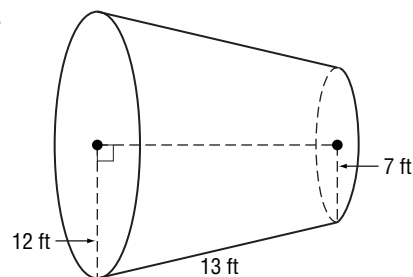
2.



3.



4.



13-3 Enrichment**Spheres and Density**

The **density** of a metal is a ratio of its mass to its volume. For example, the mass of aluminum is 2.7 grams per cubic centimeter. Here is a list of several metals and their densities.

| | | | |
|----------|-------------------------|----------|-------------------------|
| Aluminum | 2.7 g/cm ³ | Copper | 8.96 g/cm ³ |
| Gold | 19.32 g/cm ³ | Iron | 7.874 g/cm ³ |
| Lead | 11.35 g/cm ³ | Platinum | 21.45 g/cm ³ |
| Silver | 10.50 g/cm ³ | | |

To calculate the mass of a piece of metal, multiply volume by density.

Example

Find the mass of a silver ball that is 0.8 cm in diameter.

$$\begin{aligned}
 M &= D \cdot V \\
 &= 10.5 \cdot \frac{4}{3}\pi(0.4)^3 \\
 &\approx 10.5 (0.27) \\
 &\approx 2.81
 \end{aligned}$$

The mass is about 2.81 grams.

Exercises

Find the mass of each metal ball described. Assume the balls are spherical. Round your answers to the nearest tenth.

- a copper ball 1.2 cm in diameter
- a gold ball 0.6 cm in diameter
- an aluminum ball with radius 3 cm
- a platinum ball with radius 0.7 cm

Solve. Assume the balls are spherical. Round your answers to the nearest tenth.

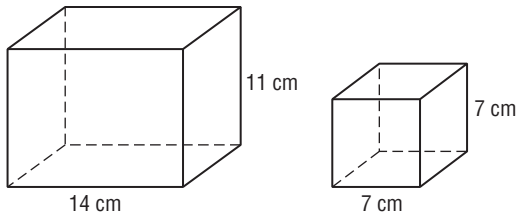
- A lead ball weighs 326 g. Find the radius of the ball to the nearest tenth of a centimeter.
- An iron ball weighs 804 g. Find the diameter of the ball to the nearest tenth of a centimeter.
- A silver ball and a copper ball each have a diameter of 3.5 cm. Which weighs more? How much more?
- An aluminum ball and a lead ball each have a radius of 1.2 cm. Which weighs more? How much more?

13-4 Enrichment

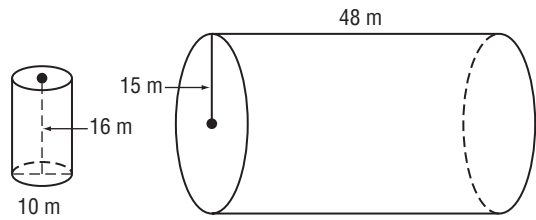
Congruent and Similar Solids

Determine whether each pair of solids is *similar*, *congruent*, or *neither*.

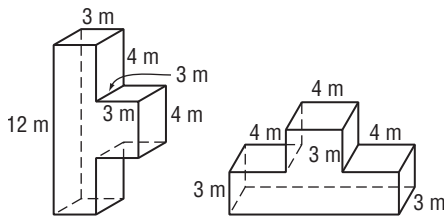
1.



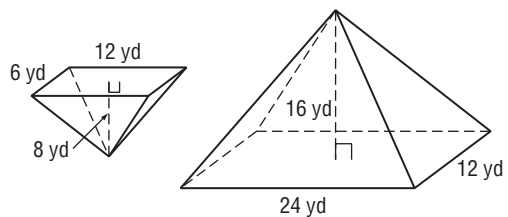
2.



3.



4.

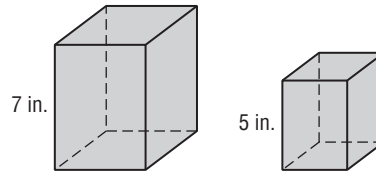


The two rectangular prisms shown at the right are similar.

5. Find the ratio of the perimeters of the bases.

6. What is the ratio of the surface areas?

7. Suppose the volume of the smaller prism is 60 in^3 . Find the volume of the larger prism.



Determine whether each statement is *true* or *false*. If the statement is false, rewrite it so that it is true.

8. If two cylinders are similar, then their volumes are equal.

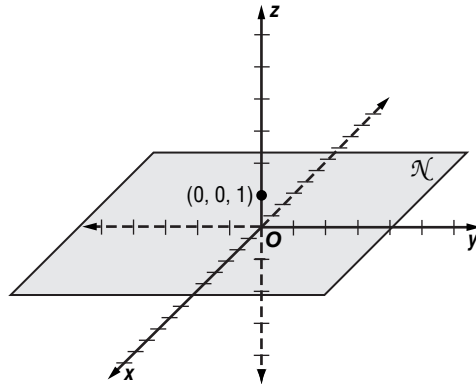
9. Doubling the height of a cylinder doubles the volume.

10. Two solids are congruent if they have the same shape.

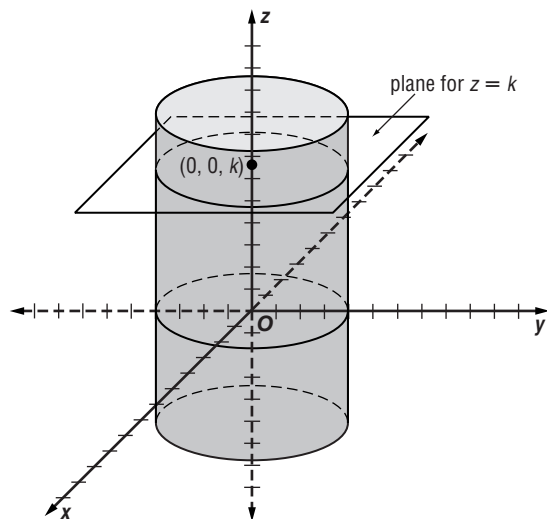
13-5 Enrichment

Planes and Cylindrical Surfaces

Consider the points (x, y, z) in space whose coordinates satisfy the equation $z = 1$. Since x and y do not occur in the equation, any point with its z -coordinate equal to 1 has coordinates that satisfy the equation. These are the points in the plane 1 unit above the xy -plane. This plane is perpendicular to the z -axis at $(0, 0, 1)$.



Next consider the points (x, y, z) whose coordinates satisfy $x^2 + y^2 = 16$. In the xy -plane, all points on the circle with center $(0, 0, 0)$ and radius 4 have coordinates that satisfy the equation. In the plane perpendicular to the z -axis at $(0, 0, k)$, the points that satisfy the equation are those on the circle with center $(0, 0, k)$ and radius 4. The graph in space of $x^2 + y^2 = 16$ is an infinite cylindrical surface whose axis is the z -axis and whose radius is 4.



Describe the graph in space of each equation. You may find it helpful to make sketches on a separate sheet.

1. $x = 5$
2. $y = -2$
3. $x + y = 7$
4. $z^2 + y^2 = 25$
5. $(x - 2)^2 + (y - 5)^2 = 1$
6. $x^2 + y^2 + z^2 = 0$