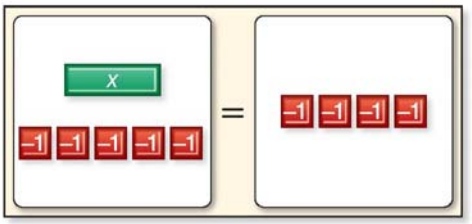
Core-Plus Mathematics, Course 3

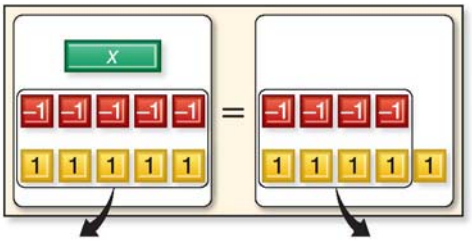
MA.912.A.3.In.e **Solve real-world equations and inequalities with one unknown (variable) using visual models to represent the procedure.**

Algebra Tiles Have students use algebra tiles to model and solve equations, such as $x - 5 = -4$.

Step 1: Model $x - 5 = -4$ using tiles.



Step 2: To isolate the x-tile, add 5 positive tiles to the left mat to make 5 zero pairs. Add 5 positive tiles to the right mat.



Step 3: There are 5 zero pairs on the left mat and 4 zero pairs on the right mat. Remove the zero pairs.

Step 4: Count the tile(s) on the right mat. So, $x = 1$.

MA.912.A.3.In.f **Create function tables and simple graphs that show the mathematical relationship between number pairs.**

Multiple Representations Have students work in pairs. Give each pair of students two equations. One student creates a function table for the first equation while the other student creates the graph. Students compare representations and check each other's work. Then students switch roles to create a table and graph for the second equation. Have students present their representations.

MA.912.A.3.Su.c **Use the concepts of equality and inequality as strategies to solve problems involving real-world situations.**

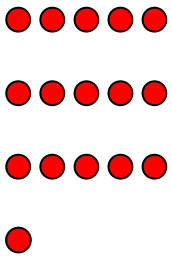
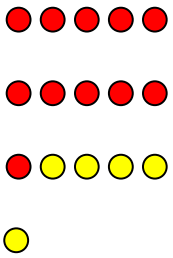
Problem-Solving Work Mats, Counters, and Equality Create and make photocopies of a problem-solving work mat, such as the one shown below, from an 8.5" by 11" sheet of paper.

Left Side	Right Side

Provide two-color counters and a work mat to each student. Students place counters, with the red-side facing up, to model real-world situations, such as the following "School Supply" problem.

SCHOOL SUPPLIES Mr. Lopez is buying school supplies for his class. He wants each of his 16 students to have a ruler. He currently has 11 rulers. How many rulers does he need to buy?
5 rulers

To solve this problem, students would label the two sides as shown. Students add counters, with the yellow-side facing up, to the left or right side of the mat in order to create an equal number of counters on both sides. Have students check their solutions with a partner and present their solutions to the class.

Left Side: Students	Right Side: Rulers
	

MA.912.A.3.Su.e **Identify the mathematical relationship between number pairs in function tables, such as +2 or –3.**

Function Tables Present students with two sequences of numbers, such as 1, 3, 5, 7, 9 and 30, 27, 24, 21, 18. Have students identify the change between numbers in each sequence. Then present the same values in tables, such as the ones shown below, and have students identify the change between numbers.

1	3	5	7	9

30	27	24	21	18

Next, present students with the same values in a function table, such as the one shown below. Have students identify the changes in both rows of the table.

x	1	3	5	7	9
y	30	27	24	21	18

Repeat the process with other pairs of sequences until students can work directly with a function table.

MA.912.A.3.Pa.c **Identify quantities to 10 as equal or unequal.**

Connecting Cubes Connect cubes to form two sets of 2 cubes, two sets of 4 cubes, two sets of 6 cubes, two sets of 8 cubes, and two sets of 10 cubes. You may wish to form one of each set using red cubes and one of each set using blue cubes. Place one set of each quantity in an opaque bag, such as a brown lunch bag. Place the remaining sets in a second bag. Have volunteer students randomly select one set from each bag. Students determine whether selected sets represent equal or unequal quantities. Repeat until all students have had at least one opportunity to select the sets and make comparison statements.

MA.912.A.3.Pa.d **Sort sets of objects to 10 into groups by quantity.**

Manipulatives Give each student 10 connecting cubes. Prompt students to count out various numbers of cubes and connect them into groups of specified quantities. Possibilities include: 4 cubes in groups of 2, 6 cubes in groups of 2, 6 cubes in groups of 3, 8 cubes in groups of 2, 8 cubes in groups of 4, 9 cubes in groups of 3, 10 cubes in groups of 2, and 10 cubes in groups of 5.

MA.912.A.4.In.d **Identify factors of expressions with whole numbers by dividing.**

Binomials and Trinomials Have students use a table, like the one shown below, to identify greatest common factors when factoring binomial expressions. Structure the table so that students supply more information as they gain fluency with the process, as shown in the first and second rows, the third and fourth rows, and the fifth row. Allow students to use a calculator when finding the greatest common factor (GCF). Continue to add binomials to the table until students have mastered working with binomials.

Binomial	GCF	Factored Binomial
$2x + 8$	2	2 $(x + 4)$
$3y - 6$	3	3 $(y - 2)$
$4z + 20$	4	4 $(z +$ 5 $)$
$7d - 49$	7	7 $(d -$ 7 $)$
$10m + 80$	10	10 $($ m $+$ 8 $)$

Once students have mastered factoring out the greatest common factor for binomials, use a similar table to factor out greatest common factors for trinomials.

MA.912.A.4.Su.c **Identify factors of whole numbers by using division facts.**

Factor Challenge Game Divide an 8.5” by 11” sheet of paper into 16 equal-sized sections. Write one of the following numbers in each section.

12, 16, 18, 20, 24, 36, 40, 42, 48, 50, 54, 60, 66, 70, 80, 100

Photocopy and cut the paper to create sets of cards. Arrange students in groups of 3 or 4. Give a set of cards and a calculator to each group of students.

To start play, students “shuffle” the cards and place them face down in the middle of the group. In each round of the game, one student per group serves as the referee. The referee is responsible for selecting that round’s card and, as needed, verifying student responses. The referee may use a calculator to verify answers.

Competing students write down as many whole number factors as they can within a designated period of time. Points are awarded to all competing students for all correct factor pairs. Consider the number 24. A student who writes 2×12 and 3×8 receives two points and a student who writes 2×12, 4×6 and 3×8 receives 3 points.

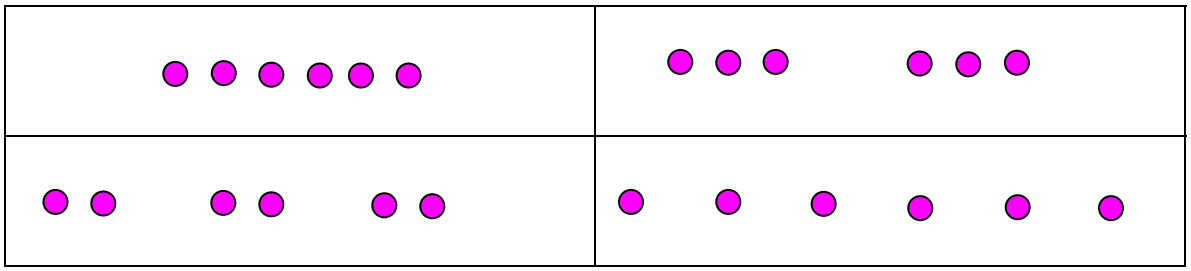
Students take turns serving as the referee. The game continues until time expires or all numbers have been factored. The winner is the student with the greatest number of total points.

MA.912.A.4.Pa.c **Separate groups of objects to 10 into sets with the same quantity.**

Manipulatives Provide each student with 10 counters or cubes and an 8.5" by 11" sheet of paper divided in fourths as shown below.

Have students separate 6 counters/cubes into the following equal groups and draw an illustration of each grouping in one of the sections.

one group of six
three groups of two
two groups of three
six groups of one



Repeat with 8, 9, and 10 counters/cubes.

MA.912.A.10.In.a **Use a variety of problem-solving strategies, such as finding key information to determine the correct operation and using graphic representations for numbers, to solve real-world problems.**

Problem-Solving Strategies Have students use the four-step problem solving plan when solving real-world problems.

Step 1: Students **understand** key words and identify unnecessary information.

Step 2: Students **plan** to solve the problem using a problem-solving strategy. Strategies include:

Act it out.
Solve a simpler problem.
Draw a diagram.
Make a model.
Make a table.
Use a graph.
Work backward.
Use an equation or formula.

Step 3: Students **solve** the problem using the selected strategy.

Step 4: Students **check** the solution using a different strategy.

MA.912.A.10.In.b **Use estimation strategies, such as rounding, grouping, and comparing, to determine if answers are reasonable.**

Estimation Work with students to estimate using several different strategies. Teach them how to round fractions to 0, $\frac{1}{2}$, or 1, how to round whole numbers to the largest place value, and how to round decimals to the nearest whole number. Provide students with real-world problems, such as the one below. Encourage them to determine if their answer is reasonable by estimating the answer and comparing it to their exact answer.

GUITAR PRACTICE Maria practiced her guitar for $\frac{1}{4}$ hour on Monday, $\frac{5}{8}$ hour on Tuesday, and $\frac{8}{12}$ hour on Wednesday. So far this week, how much time has Maria spent practicing?

Exact answer: $\frac{1}{4}+\frac{5}{8}+\frac{8}{12}=\frac{37}{24}=1\frac{13}{24}$

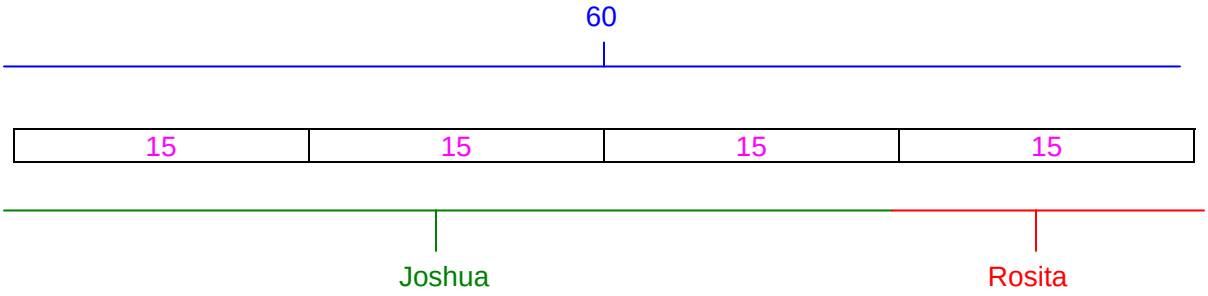
Estimated answer: 0 + $\frac{1}{2}$ + 1 = $1\frac{1}{2}$

The exact answer is the very close to the estimate.
The exact answer is reasonable.

MA.912.A.10.Su.a **Use visual and physical models as strategies for solving real-world mathematical problems.**

Bar Diagrams Have students use bar diagrams when solving real-world problems, as shown in the following example.

HOBBIES Joshua and Rosita collect trading cards. Joshua has three times as many trading cards as Rosita. They have 60 cards altogether. How many cards does Joshua have? 45 cards



MA.912.A.10.Su.b **Use a variety of problem-solving strategies, such as finding key information to determine the correct operation and using graphic representations for numbers, to solve real-world problems.**

Problem-Solving Strategies Many times students want to rush through real-world problem-solving exercises. Encourage students to follow the four-step problem-solving strategy, and emphasize the importance of checking their answers.

Step 1: Students **understand** the information provided in the exercise, as well as the information needed to supply the correct response.

Step 2: Students **plan** to solve the problem using a problem-solving strategy. Strategies include:

Act it out.
Solve a simpler problem.
Draw a diagram.
Make a model.
Make a table.
Use a graph.
Work backward.
Use an equation or formula.

Step 3: Students **solve** the problem.

Step 4: Students **check** their answers.

MA.912.A.10.Pa.a **Solve real-world problems involving quantities to 10, matching the result to the correct answer to determine accuracy.**

Classroom Routines Establish a procedure for solving real-world problems using play money. Provide opportunities for students to "earn" the play money for "a job well-done." Give them one dollar at a time for following established procedures, such as turning in their work on time, following classroom rules, or showing respect to other students. Once students have earned ten dollars, allow them to trade the money for a privilege, such as being the class leader for a week. Ask students to use a money chart to match their "dollars" to the number of dollars required to earn the classroom privilege.

Adapt this idea in accordance with classroom and school requirements in order to meet the needs of the students.

MA.912.G.7.In.a **Identify and describe three-dimensional solids, including sphere, cylinder, rectangular prism, and cone, in the environment using mathematical names.**

Figures in the Environment Provide a variety of real-world objects with different three-dimensional shapes. Use items that students can easily identify, such as cylindrical storage containers, rectangular tissue boxes, dice, and baseballs.

Have students divide a piece of paper into two columns. Ask students to name the object in the left column and then to describe the solid using its mathematical name in the right column.

Students can work at a learning center in pairs or small groups to complete this activity.

MA.912.G.7.In.b **Identify a line that divides a sphere in half.**

Great Circles Introduce students to the idea of hemispheres using a globe. Explain that the equator is a line that runs around the circumference of the globe, and it indicates where the two hemispheres meet. Other lines, such as the Prime Meridian, or other lines of latitude and longitude can be used to show that there is more than one line which indicates the meeting of two hemispheres.

Provide students with several plastic foam spheres which have been divided into two parts. If possible, color code the spheres before dividing them so that students can easily determine which parts are from the same sphere. Divide each sphere differently and provide only one sphere that is divided into hemispheres.

Show students the various ways that a plane can intersect a sphere by using a piece of paper to represent a plane. Demonstrate that the plane can intersect in a circle, by placing the paper between the two parts of the sphere.

Allow students to explore the similarities and differences among the spheres. They should notice that every sphere is cut into two parts, and that the plane intersects, or transects, the sphere in a circle.

However, only one sphere has been divided into two equal parts, or hemispheres. Students can test this conjecture by comparing the height of each hemisphere. The plane which divides the sphere in half contains the center of the sphere. This intersection is called a great circle.

MA.912.G.7.In.c **Measure rectangular prisms to find the volume using the literal formula: length × width × height.**

Cubic Centimeters Use cubic centimeters to build rectangular prisms. Provide students with the dimensions for length, width, and height. Ask them to create a rectangular prism using these dimensions. Next, ask students to determine the volume by using the formula length × width × height. Finally, have students confirm the volume by counting the number of cubes they used to create the prism.

MA.912.G.7.In.d **Compare volumes of three-dimensional solids using physical and visual models.**

Explore Volume Provide a variety of three-dimensional solids with one open end, such as a cylindrical container without the lid. Allow students to measure the dimensions of the containers using rulers. Have them make predictions about which containers have a greater volume. Finally, ask students to test their conjectures by measuring the volume using water or rice. Students can compare two solids at a time or use measuring cups to compare the measurements. Ask them to review their predictions, describe any differences, and explain their results in a math journal.

MA.912.G.7.In.e **Identify the effect of changes in the lengths of the sides of cubes or rectangular prisms on the volume using physical and visual models.**

Cubic Centimeters Use cubic centimeters to build rectangular prisms. Provide students with the dimensions for length, width, and height, such as 2 × 3 × 4. Ask them to create a rectangular prism using these dimensions. Next, ask students to determine the volume by using the formula length × width × height.

Have students change the rectangular prism so that the length is now 3, instead of 2. Ask students to determine the new volume, and then compare the new volume with the previous volume measure. Repeat this activity several times until students can see a pattern. Encourage students to create a table like the one shown below.

Length	2	3	4	5
Width	3	3	3	3
Height	4	4	4	4
Volume	**24**	**36**	**48**	**60**

MA.912.G.7.Su.a **Identify properties of three-dimensional solids, such as sphere, cylinder, cube, and cone, in the environment, when given the common name.**

Figures in the Environment Provide a variety of real-world objects with different three-dimensional shapes. Use items that students can easily identify, such as cylindrical storage containers, dice, and baseballs.

Have students divide a piece of paper into three columns. Ask students to name the object in the left column and then to describe the solid using its mathematical name in the middle column. In the right column, ask students to describe the attributes of the solids. For example, they might note the two-dimensional shape of the base, whether or not the object can roll or stack, or any other defining property.

Students can work at a learning center in pairs or small groups to complete this activity.

MA.912.G.7.Su.b **Compare volumes of three-dimensional solids in real-world situations.**

Compare Volume Provide a variety of three-dimensional solids with one open end, such as a cylindrical container without the lid. Allow students to measure the dimensions of the containers using rulers. Have them make predictions about which containers have a greater volume. Finally, ask students to test their conjectures by measuring the volume using water or rice. Students can compare two solids at a time or use measuring cups to compare the measurements. Ask them to review their predictions, describe any differences, and explain their results in a math journal.

Provide several real-world situations in which finding the volume would be relevant. Storage containers, for example, are often used to store food items, such as rice, to prevent food from spoiling.

MA.912.G.7.Su.c **Identify that changes in the lengths of sides of cubes or rectangular prisms will make the volume smaller or larger using physical models.**

Cubic Centimeters Use cubic centimeters to build rectangular prisms. Provide students with the dimensions for length, width, and height, such as 2 × 3 × 4. Ask them to create a rectangular prism using these dimensions. Next, ask students to determine the volume by using the formula length × width × height.

Have students change the rectangular prism so that the length is now 3, instead of 2. Ask students to determine the new volume, and then compare the new volume with the previous volume measure. Repeat this activity several times until students can see a pattern. Encourage students to create a table like the one shown below.

Length	2	3	4	5
Width	3	3	3	3
Height	4	4	4	4
Volume	**24**	**36**	**48**	**60**

MA.912.G.7.Pa.a **Identify objects or pictures with three-dimensional solids in real-world situations.**

Figures in the Environment Provide a variety of real-world objects with different three-dimensional shapes. Use items that students can easily identify, such as cylindrical storage containers, dice, and baseballs.

Provide students with a table divided into three columns. In the left column, provide a description of the attributes of different solids, such as cylinders, cubes, and rectangular prisms. Ask students to name the solid using its mathematical name, such as “cylinder,” in the middle column. Ask students to name or draw a picture of a real-world object in the right column.

Students can work at a learning center in pairs or small groups to complete this activity.

MA.912.G.7.Pa.b **Match two or more objects with three-dimensional solids based on a given feature, such as the number of faces or overall size, in real-world situations.**

Materials: a variety of three-dimensional solids from a math manipulative kit, real-world objects

Match Solids Distribute a variety of three-dimensional solids from a math manipulative kit, and several different real-world objects to each pair of students. Use real-world objects that have the attributes of common three-dimensional solids, such as canned food, baseballs, or tissue boxes.

Students should compare the number of faces of each math manipulative to the number of faces of the real-world object. Assist students in creating a table that lists the number of sides for each three-dimensional solid and the shape of each face. Students should match each math manipulative to a real-world object.

MA.912.S.3.In.c **Determine the mode by identifying the number that occurs most often and the mean by finding the average.**

Represent Data Have students use connecting cubes when finding mode and mean to solve real-world problems, such as the following.

SPORTS During four cross-country practices, Mitch ran 7 miles, 3 miles, 4 miles, and 6 miles. What is the mean number of miles that Mitch ran? 5 miles

To solve the problem, students can use cubes to represent the number of miles completed during each practice. Then students move the cubes until each stack has the same number of cubes. There will be 5 unit cubes in each of the four stacks.

Ask students: **If Mitch runs 3 miles and 1 mile during his next two practices, how does the mean change?** Have students use the cubes to provide support for their answers. The mean becomes 4 miles per practice. There are now 6 stacks groups, with 4 centimeter cubes in each stack.

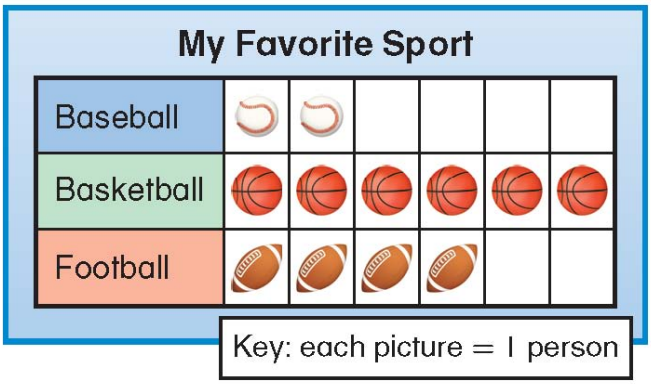
You can also use connecting cubes to help students determine the mode. Use the cubes to represent the number of miles completed during each practice by connecting a line of cubes for each practice's distance. Have students sort the lines of cubes by their lengths. The category that has the most lines of connecting cubes is the mode. For the set of data representing 4 practices, there was not a mode. For the set of data representing 5 practices, the mode is 3 miles.

MA.912.S.3.Su.c **Identify the number that occurs most frequently (mode) in a set of data with up to nine numbers.**

Data Sets Have students work in small groups. Distribute a deck of playing cards to each group, and have them remove the face cards. Tell students to randomly draw seven cards to create a data set. Students should determine the mode of the data set. Ask students to explain how to find the mode. Have groups record each data set and the mode on a piece of paper.

MA.912.S.3.Pa.a **Identify quantity in data sets of 10 by counting objects, pictures, or symbols and identify which category has more, less, or none.**

Pictographs



Step 1: Display the pictograph.

Step 2: Have students copy the pictograph. Have students place one counter on top of each symbol.

Step 3: Have students verbally explain how many counters they placed for each sport. 2 counters for baseball, 6 counters for basketball, 4 counters for football

Step 4: Have students compare the number of people who chose baseball as their favorite sport to the number of people who chose basketball. Sample answer: 4 more people chose football than baseball.

MA.912.T.2.In.a **Compare the length of the straight sides in a right triangle with the length of the side opposite the right angle (hypotenuse) by measuring the sides.**

Materials: grid paper, rulers, right triangle cut-outs

Right Triangles Distribute rulers, grid paper, and cut-outs of various sizes of right triangles. Use common right triangles that are easily measured, such as a triangle with sides that measure 3 inches, 4 inches, and 5 inches.

Emphasize to students that grid paper is a good manipulative for identifying angles as acute, obtuse, or right. Show students how to align one corner of a grid box with each vertex of the triangle, and how to align one side of the triangle with a line of the grid (which includes the vertex). Students can use the grid boxes to determine which angle is the right angle.

Once students have identified the right angle, have them find the hypotenuse. Provide a model of a right triangle which shows the right angle and the hypotenuse labeled. Ask students to identify the hypotenuse and measure its length using the ruler. Next, have students measure the length of the other two sides (the legs). Students should compare the length of the hypotenuse with the lengths of the other two sides of each triangle. Have them note any patterns they find.

MA.912.T.2.Su.a **Measure the sides of a right triangle to determine which side is the longest.**

Materials: grid paper, rulers, right triangle cut-outs

Right Triangles Distribute rulers, grid paper, and cut-outs of various sizes of right triangles. Use common right triangles that are easily measured, such as a triangle with sides that measure 3 inches, 4 inches, and 5 inches.

Emphasize to students that grid paper is a good manipulative for identifying angles as acute, obtuse, or right. Show students how to align one corner of a grid box with each vertex of the triangle, and how to align one side of the triangle with a line of the grid (which includes the vertex). Students can use the grid boxes to determine which angle is the right angle.

Once students have identified the right angle, have them find the hypotenuse. Provide a model of a right triangle which shows the right angle and the hypotenuse labeled. Ask students to identify the hypotenuse and measure its length using the ruler. Next, have students measure the length of the other two sides (the legs). Students should compare the length of the hypotenuse with the lengths of the other two sides of each triangle. Have them note any patterns they find. They should determine that the hypotenuse is always longer than either of the other two legs of a right triangle.

MA.912.T.2.Pa.a **Recognize a right triangle in objects, pictures, or signs in real-world situations.**

Materials: paper, scissors, glue, magazines, protractor or grid paper

Shape Search

- Have students look through magazines to find pictures of objects containing right triangles.

- When students find a right triangle, have them cut it out and glue it on their paper. They can use a protractor or grid paper to determine which angle is the right angle.

- Encourage students to explain their choices and allow them to share their work with a partner.