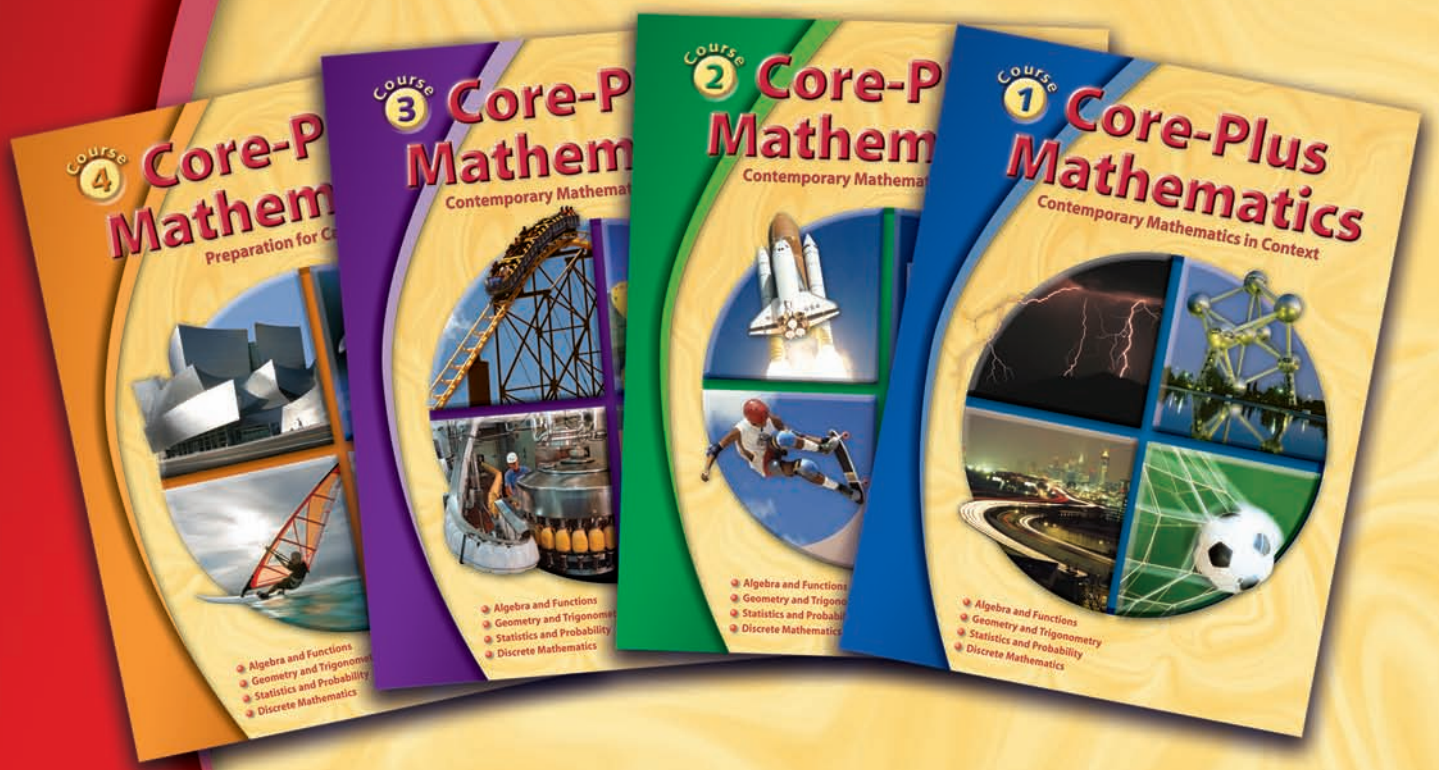
Core-Plus Mathematics

Contemporary Mathematics in Context



Scope and Sequence

- Algebra and Functions
- Geometry and Trigonometry
- Statistics and Probability
- Discrete Mathematics

Core-Plus Mathematics

Contemporary Mathematics in Context

Scope and Sequence

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About the Core-Plus Mathematics Project

The **Core-Plus Mathematics Project (CPMP)** was funded by the National Science Foundation to develop student and teacher materials for a comprehensive Standards-based high school mathematics curriculum. Courses 1–3 comprise a core program appropriate for *all* students. Course 4 continues the preparation of students for college mathematics and statistics.

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Core-Plus Mathematics 2 Field-Test Sites

Core-Plus Mathematics 2 builds on the strengths of the 1st edition which was shaped by multi-year field tests in 36 high schools in Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Ohio, South Carolina, and Texas. Each revised text is the product of a three-year cycle of research and development, pilot testing and refinement, and field testing and further refinement. Special thanks are extended to the following teachers and their students who participated in the testing and evaluation of the 2nd Edition materials.

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A Balanced and Unified Curriculum

Core-Plus Mathematics is a four-year unified curriculum that replaces the Algebra-Geometry-Advanced Algebra/Trigonometry-Precalculus sequence. Each course features interwoven strands of algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. Each of these strands is developed within coherent focused units connected by fundamental ideas such as symmetry, functions, matrices, and data analysis and curve-fitting. By actively investigating mathematics and its applications every year from an increasingly more mathematically sophisticated point of view, students' understanding of the mathematics in each strand deepens across the four-year curriculum. Mathematical connections between strands and ways of thinking mathematically that are common across strands are emphasized. These mathematical habits of mind include visual thinking, recursive thinking, searching for and explaining patterns, making and checking conjectures, reasoning with multiple representations, and providing convincing arguments and proofs.

Algebra and Functions

The Algebra and Functions strand develops student ability to recognize, represent, and solve problems involving relations among quantitative variables. Central to the development is the use of functions as mathematical models. The key algebraic models in the curriculum are linear, exponential, power, polynomial, logarithmic, rational, and trigonometric functions. Modeling with systems of equations, both linear and nonlinear, is developed. Attention is also given to symbolic reasoning and manipulation.

Geometry and Trigonometry

The primary goal of the Geometry and Trigonometry strand is to develop visual thinking and ability to construct, reason with, interpret, and apply mathematical models of patterns in visual and physical contexts. The focus is on describing patterns in shape, size, and location; representing patterns with drawings, coordinates, or vectors; predicting changes and invariants in shapes under transformations; and organizing geometric facts and relationships through deductive reasoning.

Statistics and Probability

The primary role of the Statistics and Probability strand is to develop student ability to analyze data intelligently, to recognize and measure variation, and to understand the patterns that underlie probabilistic situations. The ultimate goal is for students to understand how inferences can be made about a population by looking at a random sample from that population. Graphical methods of data analysis, simulations, sampling, and experience with the collection and interpretation of real data are featured.

Discrete Mathematics

The Discrete Mathematics strand develops student ability to solve problems using vertex-edge graphs, recursion, matrices, systematic counting methods (combinatorics), and mathematical methods for democratic decision making and information processing. Key themes are discrete mathematical modeling, optimization, and algorithmic problem solving.

Organization of the Curriculum

The first three courses in the *Core-Plus Mathematics* series provide a significant core of broadly useful mathematics for all students. They were developed to prepare students for success in college, in careers, and in daily life in contemporary society. Course 4 formalizes and extends the core program, with a focus on the mathematics needed to be successful in college mathematics and statistics courses. Unit titles for the four courses are given in the following table. Focus and content of these units are described on pages 3–7.

Course 1

1. Patterns of Change
2. Patterns in Data
3. Linear Functions
4. Vertex-Edge Graphs
5. Exponential Functions
6. Patterns in Shape
7. Quadratic Functions
8. Patterns in Chance

Course 2

1. Functions, Equations, and Systems
2. Matrix Methods
3. Coordinate Methods
4. Regression and Correlation
5. Nonlinear Functions and Equations
6. Network Optimization
7. Trigonometric Methods
8. Probability Distributions

Course 3

1. Reasoning and Proof
2. Inequalities and Linear Programming
3. Similarity and Congruence
4. Samples and Variation
5. Polynomial and Rational Functions
6. Circles and Circular Functions
7. Recursion and Iteration
8. Inverse Functions

Course 4

1. Families of Functions
2. Vectors and Motion
3. Algebraic Functions and Equations
4. Trigonometric Functions and Equations
5. Exponential Functions, Logarithms, and Data Modeling
6. Surfaces and Cross Sections
7. Concepts of Calculus
8. Counting Methods and Induction
9. Binomial Distributions and Statistical Inference
10. Mathematics of Information Processing and the Internet
11. Mathematics of Democratic Decision Making

Core-Plus Mathematics 2nd Edition

Course 1 Units

Unit 1	**Patterns of Change** develops student ability to recognize and describe important patterns that relate quantitative variables, to use data tables, graphs, words, and symbols to represent the relationships, and to use reasoning and calculating tools to answer questions and solve problems. *Topics include* variables and functions, algebraic expressions and recurrence relations, coordinate graphs, data tables and spreadsheets, and equations and inequalities.
Unit 2	**Patterns in Data** develops student ability to make sense of real-world data through use of graphical displays, measures of center, and measures of variability. *Topics include* distributions of data and their shapes, as displayed in dot plots, histograms, and box plots; measures of center including mean and median, and their properties; measures of variability including interquartile range and standard deviation, and their properties; and percentiles and outliers.
Unit 3	**Linear Functions** develops student ability to recognize and represent linear relationships between variables and to use tables, graphs, and algebraic expressions for linear functions to solve problems in situations that involve constant rate of change or slope. *Topics include* linear functions, slope of a line, rate of change, modeling linear data patterns, solving linear equations and inequalities, equivalent linear expressions.
Unit 4	**Vertex-Edge Graphs** develops student understanding of vertex-edge graphs and ability to use these graphs to represent and solve problems involving paths, networks, and relationships among a finite number of elements, including finding efficient routes and avoiding conflicts. *Topics include* vertex-edge graphs, mathematical modeling, optimization, algorithmic problem solving, Euler circuits and paths, matrix representation of graphs, vertex coloring and chromatic number.
Unit 5	**Exponential Functions** develops student ability to recognize and represent exponential growth and decay patterns, to express those patterns in symbolic forms, to solve problems that involve exponential change, and to use properties of exponents to write expressions in equivalent forms. *Topics include* exponential growth and decay functions, data modeling, growth and decay rates, half-life and doubling time, compound interest, and properties of exponents.
Unit 6	**Patterns in Shape** develops student ability to visualize and describe two- and three-dimensional shapes, to represent them with drawings, to examine shape properties through both experimentation and careful reasoning, and to use those properties to solve problems. *Topics include* Triangle Inequality, congruence conditions for triangles, special quadrilaterals and quadrilateral linkages, Pythagorean Theorem, properties of polygons, tilings of the plane, properties of polyhedra, and the Platonic solids.
Unit 7	**Quadratic Functions** develops student ability to recognize and represent quadratic relations between variables using data tables, graphs, and symbolic formulas, to solve problems involving quadratic functions, and to express quadratic polynomials in equivalent factored and expanded forms. *Topics include* quadratic functions and their graphs, applications to projectile motion and economic problems, expanding and factoring quadratic expressions, and solving quadratic equations by the quadratic formula and calculator approximation.
Unit 8	**Patterns in Chance** develops student ability to solve problems involving chance by constructing sample spaces of equally-likely outcomes or geometric models and to approximate solutions to more complex probability problems by using simulation. *Topics include* sample spaces, equally-likely outcomes, probability distributions, mutually exclusive (disjoint) events, Addition Rule, simulation, random digits, discrete and continuous random variables, Law of Large Numbers, and geometric probability.

Course 2 Units

Unit 1	**Functions, Equations, and Systems** reviews and extends student ability to recognize, describe, and use functional relationships among quantitative variables, with special emphasis on relationships that involve two or more independent variables. *Topics include* direct and inverse variation and joint variation; power functions; linear equations in standard form; and systems of two linear equations with two variables, including solution by graphing, substitution, and elimination.
Unit 2	**Matrix Methods** develops student understanding of matrices and ability to use matrices to represent and solve problems in a variety of real-world and mathematical settings. *Topics include* constructing and interpreting matrices, row and column sums, matrix addition, scalar multiplication, matrix multiplication, powers of matrices, inverse matrices, properties of matrices, and using matrices to solve systems of linear equations.
Unit 3	**Coordinate Methods** develops student understanding of coordinate methods for representing and analyzing properties of geometric shapes, for describing geometric change, and for producing animations. *Topics include* representing two-dimensional figures and modeling situations with coordinates, including computer-generated graphics; distance in the coordinate plane, midpoint of a segment, and slope; coordinate and matrix models of rigid transformations (translations, rotations, and line reflections), of size transformations, and of similarity transformations; animation effects.
Unit 4	**Regression and Correlation** develops student understanding of the characteristics and interpretation of the least squares regression equation and of the use of correlation to measure the strength of the linear association between two variables. *Topics include* interpreting scatterplots; least squares regression, residuals and errors in prediction, sum of squared errors, influential points; Pearson's correlation coefficient and its properties, lurking variables, and cause and effect.
Unit 5	**Nonlinear Functions and Equations** introduces function notation, reviews and extends student ability to construct and reason with functions that model parabolic shapes and other quadratic relationships in science and economics, with special emphasis on formal symbolic reasoning methods, and introduces common logarithms and algebraic methods for solving exponential equations. *Topics include* formalization of function concept, notation, domain and range; factoring and expanding quadratic expressions, solving quadratic equations by factoring and the quadratic formula, applications to supply and demand, break-even analysis; common logarithms and solving exponential equations using base 10 logarithms.
Unit 6	**Network Optimization** develops student understanding of vertex-edge graphs and ability to use these graphs to solve network optimization problems. *Topics include* optimization, mathematical modeling, algorithmic problem solving, digraphs, trees, minimum spanning trees, distance matrices, Hamilton circuits and paths, the Traveling Salesperson Problem, critical paths, and the PERT technique.
Unit 7	**Trigonometric Methods** develops student understanding of trigonometric functions and the ability to use trigonometric methods to solve triangulation and indirect measurement problems. *Topics include* sine, cosine, and tangent functions of measures of angles in standard position in a coordinate plane and in a right triangle; indirect measurement; analysis of variable-sided triangle mechanisms; Law of Sines and Law of Cosines.
Unit 8	**Probability Distributions** further develops student ability to understand and visualize situations involving chance by using simulation and mathematical analysis to construct probability distributions. *Topics include* Multiplication Rule, independent and dependent events, conditional probability, probability distributions and their graphs, waiting-time (or geometric) distributions, expected value, and rare events.

Course 3 Units

Unit 1	**Reasoning and Proof** develops student understanding of formal reasoning in geometric, algebraic, and statistical contexts and of basic principles that underlie those reasoning strategies. *Topics include* inductive and deductive reasoning strategies; principles of logical reasoning—Affirming the Hypothesis and Chaining Implications; relation among angles formed by two intersecting lines or by two parallel lines and a transversal; rules for transforming algebraic expressions and equations; design of experiments including the role of randomization, control groups, and blinding; sampling distribution, randomization test, and statistical significance.
Unit 2	**Inequalities and Linear Programming** develops student ability to reason both algebraically and graphically to solve inequalities in one and two variables, introduces systems of inequalities in two variables, and develops a strategy for optimizing a linear function in two variables within a system of linear constraints on those variables. *Topics include* inequalities in one and two variables, number line graphs, interval notation, systems of linear inequalities, and linear programming.
Unit 3	**Similarity and Congruence** extends student understanding of similarity and congruence and their ability to use those relations to solve problems and to prove geometric assertions with and without the use of coordinates. *Topics include* connections between Law of Cosines, Law of Sines, and sufficient conditions for similarity and congruence of triangles, centers of triangles, applications of similarity and congruence in real-world contexts, necessary and sufficient conditions for parallelograms, sufficient conditions for congruence of parallelograms, and midpoint connector theorems.
Unit 4	**Samples and Variation** extends student understanding of the measurement of variability, develops student ability to use the normal distribution as a model of variation, introduces students to the binomial distribution and its use in decision making, and introduces students to the probability and statistical inference involved in control charts used in industry for statistical process control. *Topics include* normal distribution, standardized scores, binomial distributions (shape, expected value, standard deviation), normal approximation to a binomial distribution, odds, statistical process control, control charts, and the Central Limit Theorem.
Unit 5	**Polynomial and Rational Functions** extends student ability to represent and draw inferences about polynomial and rational functions using symbolic expressions and manipulations. *Topics include* definition and properties of polynomials, operations on polynomials; completing the square, proof of the quadratic formula, solving quadratic equations (including complex number solutions), vertex form of quadratic functions; definition and properties of rational functions, operations on rational expressions.
Unit 6	**Circles and Circular Functions** develops student understanding of relationships among special lines, segments, and angles in circles and the ability to use properties of circles to solve problems; develops student understanding of circular functions and the ability to use these functions to model periodic change; and extends student ability to reason deductively in geometric settings. *Topics include* properties of chords, tangent lines, and central and inscribed angles of circles; linear and angular velocity; radian measure of angles; and circular functions as models of periodic change.
Unit 7	**Recursion and Iteration** extends student ability to represent, analyze, and solve problems in situations involving sequential and recursive change. *Topics include* iteration and recursion as tools to model and analyze sequential change in real-world contexts, including compound interest and population growth; arithmetic, geometric, and other sequences; arithmetic and geometric series; finite differences; linear and nonlinear recurrence relations; and function iteration, including graphical iteration and fixed points.
Unit 8	**Inverse Functions** develops student understanding of inverses of functions with a focus on logarithmic functions and their use in modeling and analyzing problem situations and data patterns. *Topics include* inverses of functions; logarithmic functions and their relation to exponential functions, properties of logarithms, equation solving with logarithms; and inverse trigonometric functions and their applications to solving trigonometric equations.

Course 4 Units

Unit 1

Families of Functions extends student understanding of linear, exponential, quadratic, power, and trigonometric functions to model data patterns whose graphs are transformations of basic patterns; and develops understanding of operations on functions useful in representing and reasoning about quantitative relationships.

Topics include linear, exponential, quadratic, power, and trigonometric functions; data modeling; translation, reflection, and stretching of graphs; and addition, subtraction, multiplication, division, and composition of functions.

Unit 2

Vectors and Motion develops student understanding of two-dimensional vectors and their use in modeling linear, circular, and other nonlinear motion.

Topics include concept of vector as a mathematical object used to model situations defined by magnitude and direction; equality of vectors, scalar multiples, opposite vectors, sum and difference vectors, dot product of two vectors, position vectors and coordinates; and parametric equations for motion along a line and for motion of projectiles and objects in circular and elliptical orbits.

Unit 3

Algebraic Functions and Equations reviews and extends student understanding of properties of polynomial and rational functions and skills in manipulating algebraic expressions and solving polynomial and rational equations, and develops student understanding of complex number representations and operations.

Topics include polynomials, polynomial division, factor and remainder theorems, operations on complex numbers, representation of complex numbers as vectors, solution of polynomial equations, rational function graphs and asymptotes, and solution of rational equations and equations involving radical expressions.

Unit 4

Trigonometric Functions and Equations extends student understanding of, and ability to reason with, trigonometric functions to prove or disprove two trigonometric expressions are identical and to solve trigonometric equations; to geometrically represent complex numbers and complex number operations and to find roots of complex numbers.

Topics include the tangent, cotangent, secant, and cosecant functions; fundamental trigonometric identities, sum and difference identities, double-angle identities; solving trigonometric equations and expression of periodic solutions; rectangular and polar representations of complex numbers, absolute value, DeMoivre's Theorem, and the roots of complex numbers.

Unit 5

Exponential Functions, Logarithms, and Data Modeling extends student understanding of exponential and logarithmic functions to the case of natural exponential and logarithmic functions, solution of exponential growth and decay problems, and use of logarithms for linearization and modeling of data patterns.

Topics include exponential functions with rules in the form $f(x) = Ae^{kx}$, natural logarithm function, linearizing bivariate data and fitting models using log and log-log transformations.

Unit 6

Surfaces and Cross Sections extends student ability to visualize and represent three-dimensional shapes using contours, cross sections, and reliefs, and to visualize and represent surfaces and conic sections defined by algebraic equations.

Topics include using contours to represent three-dimensional surfaces and developing contour maps from data; sketching surfaces from sets of cross sections; conics as planar sections of right circular cones and as locus of points in a plane; three-dimensional rectangular coordinate system; sketching surfaces using traces, intercepts and cross sections derived from algebraically-defined surfaces; and surfaces of revolution and cylindrical surfaces.

Unit 7

Concepts of Calculus develops student understanding of fundamental calculus ideas through explorations in a variety of applied problem contexts and their representations in function tables and graphs.

Topics include instantaneous rates of change, linear approximation, area under a curve, and applications to problems in physics, business, and other disciplines.

Unit 8	**Counting Methods and Induction** extends student ability to count systematically and solve enumeration problems, and develops understanding of, and ability to do, proof by mathematical induction. *Topics include* systematic listing and counting, counting trees, the Multiplication Principle of Counting, Addition Principle of Counting, combinations, permutations, selections with repetition; the binomial theorem, Pascal's triangle, combinatorial reasoning; the general multiplication rule for probability; and the Principle of Mathematical Induction.
Unit 9	**Binomial Distributions and Statistical Inference** extends student understanding of the binomial distribution, including its exact construction and how the normal approximation to the distribution of the sample proportion is used in statistical inference. *Topics include* binomial probability formula; shape, expected value, and standard deviation of the distribution of the sample proportion, $\hat{p}$; design of sample surveys including random sampling and stratified random sampling; measurement (response) bias; sample selection bias; variability in sampling; confidence intervals; margin of error; and test of significance of a proportion.
Unit 10	**Mathematics of Information Processing and the Internet** develops student understanding of the mathematical concepts and methods related to information processing, particularly on the Internet, focusing on the key issues of access, security, accuracy, and efficiency. *Topics include* elementary set theory and logic; modular arithmetic and number theory; secret codes, symmetric-key and private-key cryptosystems; error-detecting codes (including ZIP, UPC, and ISBN) and error-correcting codes (including Hamming distance); and trees and Huffman coding.
Unit 11	**Mathematics of Democratic Decision Making** develops student understanding of the mathematical concepts and methods needed to make decisions in a democratic society, as related to voting and fair division. *Topics include* preferential voting and associated vote-analysis methods such as majority, plurality, runoff, points-for-preferences (Borda method), and pairwise-comparison (Condorcet method); weighted voting; and fair division techniques, including apportionment methods.

Strand Charts

The following charts provide an overview of the mathematical content and flow of Courses 1–4 in the *Core-Plus Mathematics* curriculum. The charts are organized by mathematical strand: algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. Each of the four strands has been divided into major content categories, and under each of these categories you will find the key mathematical topics developed in the curriculum.

Many cells in the grid have either a "F" *or a* "C" to indicate the units in which each topic is treated. The "F" indicates *focus*; this means that the topic is initially developed or is extended beyond its initial development or use. The "C" indicates *connections*, which means that a conceptual basis for the topic is developed, the topic is informally introduced, or the topic is revisited and used without further development.

To help build and maintain proficiency with key topics in the charts, Review tasks in each lesson of each unit provide students distributed practice with related concepts and skills. These practice opportunities are not referenced in the following strand charts.

ALGEBRA AND FUNCTIONS

F : Focus **C :** Connections

Algebraic Representations

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Coordinate graphs	F	C	C		C		C	C	F	C	F	C	C		C	C	C	F		C	F	C	C	C	F		F	C	F	F	F				F
Tables and graphs	F	F	F		F	C	F	C	F	C	C	F	F	C	C	C		F		C	F	C	C	F	F		C	C	F	C	F				
Symbolic expressions	F	C	F		F	C	F	C	F	C		C	F		C	C	F	F		C	F	C	F	F	F	C	F	C	F		F	F	F	F	
Modeling data patterns	F	F	F		F		C	F	F	C		F	C		C	F				F	C	C	C					F							
Absolute value		C	C								C					C								F	F										
Error of prediction and residuals												F																C	F		C				
Matrices				F						F	F												C	C			C						C	
Coordinate models of transformations											F														F			C		C					
Summation notation		C										C			C	C				C			C					C			C	C			
Vectors																										F	C								
Parametric equations																										F									
Logarithmic scales													F											F					F						
Standard form of complex numbers																					F						F								
Polar coordinates																										C		F							
Polar form of complex numbers																												F							
Trigonometric form of complex numbers																												F							
Spreadsheets	C		C		C		C			C			C					C					F												C

Linear Expressions and Relations

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Modeling situations	C		F		C	C	C	C	F	C		F	C		C			F			C		C	C	C			F	F		F					
Symbolic forms and effects of parameters	C		F		C		C		F			C					C	F			C		F		C										
Graphs and equations of lines	C		F		C	C		C	F	C	F	C			C		C	F			C		C	C	C			C							
Solving equations	C		F			C			F	C		C			C			C			C		C	C											
Solving inequalities	C		F															F																	
Rates of change, slopes, and intercepts	C		F		C		C		F		C	F					C	C			C		F					C			F				
Parametric equations for linear motion																										F									

Exponential Expressions and Relations

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Modeling situations	C				F								C										C	C	C			F	F					C	
Symbolic forms and effects of parameters	C				F								C										C		C			C	F						
Graphs and intercepts	C				F								C												C			C	C						
Solving equations and inequalities	C				F								F										C					C	F						
Rates of change	C				F																										C				
Asymptotes													C												C										
Solving equations using logarithms													F											F					F						

ALGEBRA AND FUNCTIONS

F : Focus **C :** Connections

Power Expressions and Relations

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Symbolic forms and effects of parameters	C						F		F				C					C		C	C				C							C	C		
Graphs, intercepts, and zeroes	C						F		F				C					C			C				C						C				
Fractional and negative exponents					F								C																C						
Laws of exponents					F												C												C			C		C	
Modeling situations									F									C		C					C				F				C		
Inverse variation	C								F				C								C			C	C		C								
Asymptotes (inverse variation)									C												F						C								
Solving equations and inequalities							F		C									C						C			C								
Roots and radicals		C			F		C		C			C	C		C					C							F					C			
Rates of change	F						C		C				C					C			C			C					F						

Quadratic Expressions and Relations

Modeling situations	C	C					F	C	C			C	C					C			F		C		C					F						
Symbolic forms and effects of parameters	C	C					F		C			C	F					F			F		C		C		C			C	C				
Graphs, intercepts, and zeroes	C						F						F					F			F				C										
Rates of change	C						C		C																		C			C					
Solving equations and inequalities by graphic and numeric approximation	C						F		C				C					C			C														
Number of solutions							F						F					F			C						C								
Solving inequalities																		F																	
Higher-degree polynomials																					F			C			F								
Solving equations by factoring							C						F					C			F						C								
Solving equations by using the quadratic formula							F						F		C			C			F						C								
Parametric equations for projectile motion																										F									
Parametric equations for circular motion																						C				F									
Parametric equations for elliptical motion																										F									
Conic sections							C						C					C							C		C			F						

Polynomial Expressions and Relations

Modeling situations							C						C					C			F		C				F									
Symbolic forms and effects of parameters							C						C					C			F		C				F					C			
Graphs, intercepts, and zeroes							C						C					C			F				C		F								
Rates of change																					C						C								
End behavior																					C						F								
Solving equations and inequalities							C						C					C			C						C								
Number of solutions							C						C					C			F						F								

ALGEBRA AND FUNCTIONS

F : Focus **C :** Connections

Rational Expressions and Relations

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Proportions									C				C		C		C								C										C
Symbolic forms and effects of parameters	C								C												F				C		F								
Graphs, intercepts, and zeroes	C								C												F						F								
Rates of change									C												C						C								
Modeling situations									C									C			F						F								
Solving equations and inequalities																		C			C			C			F								
Asymptotes																					F						C								
Simplifying: adding subtracting, multiplying, and																					F						F								C

Periodic Relations

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Modeling situations															C							F			C										
Symbolic forms and effects of parameters															C							F			C										
Graphs, intercepts, and zeroes															C							C			C										
Rates of change																															C				
Inverse trigonometric functions															C							C		F				C							
Solving trigonometric equations															C							C		F				F							
Trigonometric identities															C							C						F							

Logarithmic Expressions and Relations

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Definition and notation													F											F					F							
Common logarithms													F											F					C							
Modeling situations													F											F					F							
Graphs and intercepts																								C					F							
Symbolic forms and effects of parameters																													F							
Properties of logarithms																								F					C							
Solving logarithmic equations and inequalities													C																C							
Evaluating logarithmic expressions													C											F					C							
Natural logarithms																													F							
Change of base for logarithms																								C					C							

Multivariable Expressions

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Formulas	C					C		C	C	C	C	C			C	C	C			C		C	C		C		C				F	C			
Pythagorean Theorem				C		F			F	C					C		C					C						C							
Law of Sines and Law of Cosines															F		C					F													
Modeling using symbolic equations and inequalities									F									F		C											C	C			
Solving for one variable in terms of others									F						C			C		C				C				C				C			
Effect of change among variables									F									C		C					C						C	C			

ALGEBRA AND FUNCTIONS

F : Focus **C :** Connections

Systems of Relations

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Graphing systems			F						F	C	F		C					F																	
Solving linear systems by graphical and numeric methods			F						F	C	F							C					C											
Solving linear systems by substitution			C						F	C	F							C					C						C				C	
Solving linear systems by matrix methods										F	C							C					C										C	
Solving linear systems by linear combinations									F	C								C					C										C	
Solving systems of linear inequalities																		F																
Linear programming																		F																
Linear-quadratic systems													F					C																
Linear-inverse systems													F																					
Systems of parametric equations																		C								C								

Functions

Definition and notation	C												F					C			C		C	C					C		C			
Domain and range					C		C						F		C			C			F		C	C					C					
Rates of change	C		F		C		C		C		C										C							F						
Transformations of graphs	C		C		C		C				C							C			C	C			F									
Composition of functions											C												C	F	F									
Zeroes of functions							C						C		C			C			F	C	C				F							
Sequences and series																	C						F						C					
Inverses of functions																C						C		F										

Algebraic Reasoning

Equivalent expressions			F		F		F		C				F				F				F						F				C			
Distributive property			C				C			C			C				C				C						C				C			
Commutative properties			C				C			F			C				C				C						C				C			
Properties of exponents					F					C			C				C							C			C				C		C	
Associative properties			C				C			C			C				C				C										C			
Properties of equations										C	C						F				C						C							
Additive inverse			C							C											C						C						C	
Multiplicative inverse			C							F																	C						C	
Closure property										C	C																						F	
Radicals and operations					F		C						C								C						F							
Algebraic proof																C	F			C	C		C							C	C	C		
Polynomials and binary operations																					F						F				C			
Factoring, expanding, and simplifying							F						F				C	C			F						C				C		C	
Binomial theorem																															F	C			
Pascal's triangle																															F				
Completing the square																					F														
Substitution of variable	C						C		F				C								C						C				C			
Proof by mathematical induction																															F				
De Moivre's Theorem																												F							

ALGEBRA AND FUNCTIONS

F : Focus **C :** Connections

Algebraic Systems

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Real numbers and their properties			C							F							C				C						C				F		C	C
Matrices and their properties										F														C									C	
Transformations and their properties	F										F	F								F					C				F					
Vectors and their properties																		F																
Complex numbers and their properties																					C						F							
Sets and set operations																		C												C		F	
Integers mod *n* and their properties																								C								F	

Calculus Underpinnings

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Rates of change	C		F		F						C							C			C		C								F				
Maxima and minima	C					C	F						F					C			F				C		F				F				
Limits																C					C		C								C	C			
Sequences and series																C	C						F									C			
Local linearity																																F			
Derivative functions																																F			
Accumulation of change																																F			
Area under a curve								C								C				F											F		F		
Definite integral																																F			

GEOMETRY AND TRIGONOMETRY

F : Focus **C :** Connections

Two-dimensional Figures

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Angle relations for two intersecting lines						C											F		C																
Parallel lines and angle relations																	F		C			C												
Triangles and their properties						F					F				C				F			C				C		C						
Quadrilaterals and their properties						F					C						C		F															
Parallelograms and their properties						F					F				C				F							C								
Triangular and quadrilateral linkages						F									F				C															
Polygons and their properties				C		F					F				C				C											C				
Geometric constructions						C											F		F			C												
Circles and their properties						C					F				C							F				C								
Conic sections and their properties							C						C													C				F				
Bilateral and rotational symmetry						F					C		C												F						C			
Tessellations						F																												
Translational symmetry						F					C											C			F						C			
Fractals					C																		C							C				
Congruence						F					F				C				F			F												
Congruence conditions for triangles						F					C				C		C		F			C												
Similarity						C					F				C				F			C												
Similarity conditions for triangles						C									C				F			C			C									

Three-dimensional Figures

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Right polygonal prisms and pyramids						F																			C										
Cones, cylinders, and spheres						F											C						C		C		C			C					
Sketching shapes						F																													
Rigidity						F																													
Bilateral and rotational symmetry						F																													
Regular (Platonic) solids				C		F		C								C																			
Cross sections						C																								F					
Contour diagrams																														F					
Surfaces																														F					
Surfaces of revolution																										F				F					
Cylindrical surfaces																									C					F					

Measurement

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Perimeter	C					F					C											C								C					
Area	C					F		C							C							C			C					C					
Pythagorean Theorem	C					F					C				C							C					C			C					
Surface area						F																			C		C								
Volume						F																			C										
Radian and degree measure						C									C							F				F		C							
Angular velocity																						F				F									
Linear velocity																						F				F									

GEOMETRY AND TRIGONOMETRY

F : Focus **C :** Connections

Coordinate Models

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Coordinate representation of figures											F				C		C		C			C													
Distance and midpoint formulas											F				C				C								C							
Slope of a line			F						C	C	C				C				C										F					
Equations of lines			F						F	F	F							C	C										C					
Slopes of parallel and perpendicular lines			C							C	F							C	C															
Equations of circles											F										C	C			C		C			C					
Coordinate representation of transformations											F														C										
Congruence and similarity						F					F				C				C			C													
Matrix representation of polygons				C							F																								
Coordinate proof											F								C			C							C					
Vectors																										F				F					
Polar coordinates																										C		F							
3-dimensional coordinate graphs																														F					
Sketching surfaces in 3-dimensional space						C																								F					
Equations of surfaces																														F					
Equations of planes																														F					
Equations of conic sections							C						C														C			F					
Traces of surfaces																														F					

Transformations

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Rigid transformations (isometries)						C					F				C				C						C		C								
Size transformations (dilations)											F				C				C			C			C										
Composition of transformations											F														C										
Similarity transformations											F								C			C			C										
Matrix representation of transformations											F				C							C					C								
Animation											F																								
Transformations of graphs and function rules											C											C			F		C								
Complex number operations and transformations																												F							

GEOMETRY AND TRIGONOMETRY

F : Focus **C :** Connections

Trigonometry

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Trigonometric ratios and functions (sine, cosine, and tangent)															F		C					F		C		F		F							
Law of Sines															F		C		F							C									
Law of Cosines															F		C		F							C									
Radian and degree measure						C									C							F		C	C	C		C							
Trigonometric graphs															C							F		F	F										
Period															C							F		F	F			F							
Amplitude															C							F		F	F										
Periodic models															C							F		F	F										
Inverse trigonometric functions															C							C		F				F							
Trigonometric identities															C							C						F							
Trigonometric ratios and functions (secant, cosecant, and cotangent)																												F							
Circular and periodic motion															C							F				F									
Component analysis of vectors																										F									
Indirect measurement (angles and lengths)															F				C			C													
Triangular linkages with one variable side															F																				

Reasoning and Proof

Inductive reasoning	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C		F	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
Counterexamples				C		C				C	C			C			F					C							C			C		C	
Deductive reasoning				C		C				C	F			C	C		F		F			F	C			C			C		F		F		C	
Designing and justifying algorithms				C		C					C			C			C										C							C	C
Coordinate and analytical proof											F				C		C		C			C													
Synthetic proof						C											F		F			F	C												
Assumptions in proof						C					C						F		C			C								F		C			
Principles of logic																	F													C		C			
Converse						C					C						F		C			C													
Vector proof																										F									

STATISTICS AND PROBABILITY

F : Focus **C :** Connections

Univariate Data

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Number line (dot) plots		F																																	
Measures of center and their properties: mean, median		F						C								C	C			C												C		
Basic shapes of distributions		F						C								C				C												C		
Skewness and symmetry		F						C								C				C												C		
Histograms		F						F								F				F												F		
Clusters, gaps, and outliers		F															C			F												C		
Box plots		F															C																		
Linear transformations of data		F										C					C			F												F		
Stem-and-leaf plots		F																																	
Range and interquartile range (IQR)		F																																	
Mean absolute deviation (MAD)		C																																	
Percentiles and quartiles		F															C			C												C		
Standard deviation and its properties		F						C								C				F						C						C		
Frequency tables		F						F								F	F			F												F		
Standardized or z-scores																				F												F		
Concept of distribution		F						F								F	F			F												F		

Bivariate Data

Scatterplots and association			F		F	C	C					F																F						
Plots and trends over time	F				C							C								F			C					C						
Sum of squared differences		F										F																						
Centroid												F																						
Least squares regression line			C									F																F						
Errors of prediction and residuals												F																F						
Nonlinear regression					C		C																											
Influential points												F																						
Spearman's rank correlation												F																						
Pearson's correlation *r*												F					F																	
Explanatory and response variables	C											F					F																	
Lurking variables	C											F					F																	
Cause-and-effect relationships	F											F												F										
Log and log-log transformations																												F						

STATISTICS AND PROBABILITY

F : Focus **C :** Connections

Probability

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Normal distribution	F																			F													F		
Simulation								F								C	C			C													C		
Random digit tables and random number generators								F												C															
Law of Large Numbers								F																									C		
Empirical (experimental) and theoretical probabilities								F								F				C															
Independent events																F																C	F		
Binomial distribution								C												F												C	F		
Waiting-time (geometric) distribution								C								F							C												
Multiplication Rule for independent events																F				C												C	C		
Geometric (area) models								F								C																			
Conditional probability																F																C			
Expected value																F				C			C										C		
Rare events																	C			F													F		
Sampling distributions																	F																F		
Mutually exclusive events								F												C													C		
Addition Rule for mutually exclusive events								F												C													C		
Central Limit Theorem																				F															
Probability distribution								F								F				F															
General multiplication rule																F																F			
Binomial probability formula																																C	F		
Normal approximation to a binomial																				F													F		

Inferential Statistics

Population vs. sample												F					C			F													F		
Confidence intervals																																	C		
Margin of error																																	C		
Control (run) charts																				F															
Tests of significance including Type I and Type II errors																	F			C													F		
Statistical significance																	F			C													F		
One sample z-test for a proportion																				C													F		
Generalizability of results												F					F			F													F		

Surveys

Characteristics of a well-designed survey																																	F		
Sample survey vs. census																																	F		
Sample size																	C																F		
Simple random sample																	C																F		
Bias in a survey method																																	F		
Response rate																																	F		

STATISTICS AND PROBABILITY

F : Focus **C :** Connections

Experiments

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Characteristics of a well-designed experiments: treatments, control groups, random assignment, and replication														C			F																		
Source of bias and confounding, including placebo effect and blinding																	F																		
Randomization test																	F																		

DISCRETE MATHEMATICS

F : Focus **C :** Connections

Vertex-Edge Graphs

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Vertex-edge graph models				F		C				C		C		F																C		C		
Digraphs				C						C				F																				
Adjacency matrices for vertex-edge graphs				F						F																								
Euler paths and circuits				F										C																				
Graph coloring				F																										C				
Critical path analysis and PERT charts														F									C											
Trees and minimum spanning trees														F		C														C		C		
Hamilton paths and circuits (including TSP)														F																				

Recursion and Iteration

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Informal representation with words like *NOW* and *NEXT*	F	C			F					C													F		C						C				
Recursive representation of linear functions		F			C																		F												
Recursive representation of exponential functions	F				F																		F												
Recursive representation of polynomial functions																							F												
Recurrence relations																							F								C				
Sequences and series																	C						F								C				
Finite differences																	C						F												
Function iteration																							F												
Fixed points																							F												
Graphical iteration																							F				C								
Fractals				C	C																		C								C				

Matrices

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Matrix models				C						F				C									C	C									C		
Row and column sums				C						F																									
Matrix addition										F																									
Scalar multiplication										F																									
Matrix multiplication										F														C			C						C		
Identity matrices										F																									
Inverse matrices										F														C									C		
Properties of matrices										F																							C		
Matrix solutions of linear systems										F													C										C		
Adjacency matrices for vertex-edge graphs				F						F																	C								
Transformation matrices															C												C								
Scatterplot matrices												F																								

DISCRETE MATHEMATICS

F : Focus **C :** Connections

Algorithms

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Algorithmic problem solving				F										F									C										C	C
Designing algorithms				F										F									C										C	C
Analyzing and comparing algorithms				C										F																			C	C
Programming algorithms				C										C																			C	C

Systematic Counting Methods

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Counting trees																F																F			
Multiplication Principle of counting				C										C		F																F			
Addition Principle of counting								F																								F			
Pigeonhole principle																																C			
Permutations																				C												F			
Combinations																																F	C	C	
Selections with repetition																				C												F	C		
Pascal's triangle																																F	C		
Binomial theorem																																F	C		
Combinatorial reasoning								F								F				C												F	F		

Information Processing

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Set operations																																C		F	
Logical (or Boolean) operators																																		F	
Venn diagrams								C																										F	
Modular arithmetic and number theory																								C										F	
Symmetric-key cryptosystems																																		F	
RSA public-key cryptosystems																																		F	
Error-detecting codes																																		F	
Error correcting codes																																		F	
Huffman code for data compression																																		F	

Democratic Decision Making

	Course 1								Course 2								Course 3								Course 4										
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11
Preferential voting analysis																																			F
Weighted voting analysis																																			F
Fair division and apportionment																																			F

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