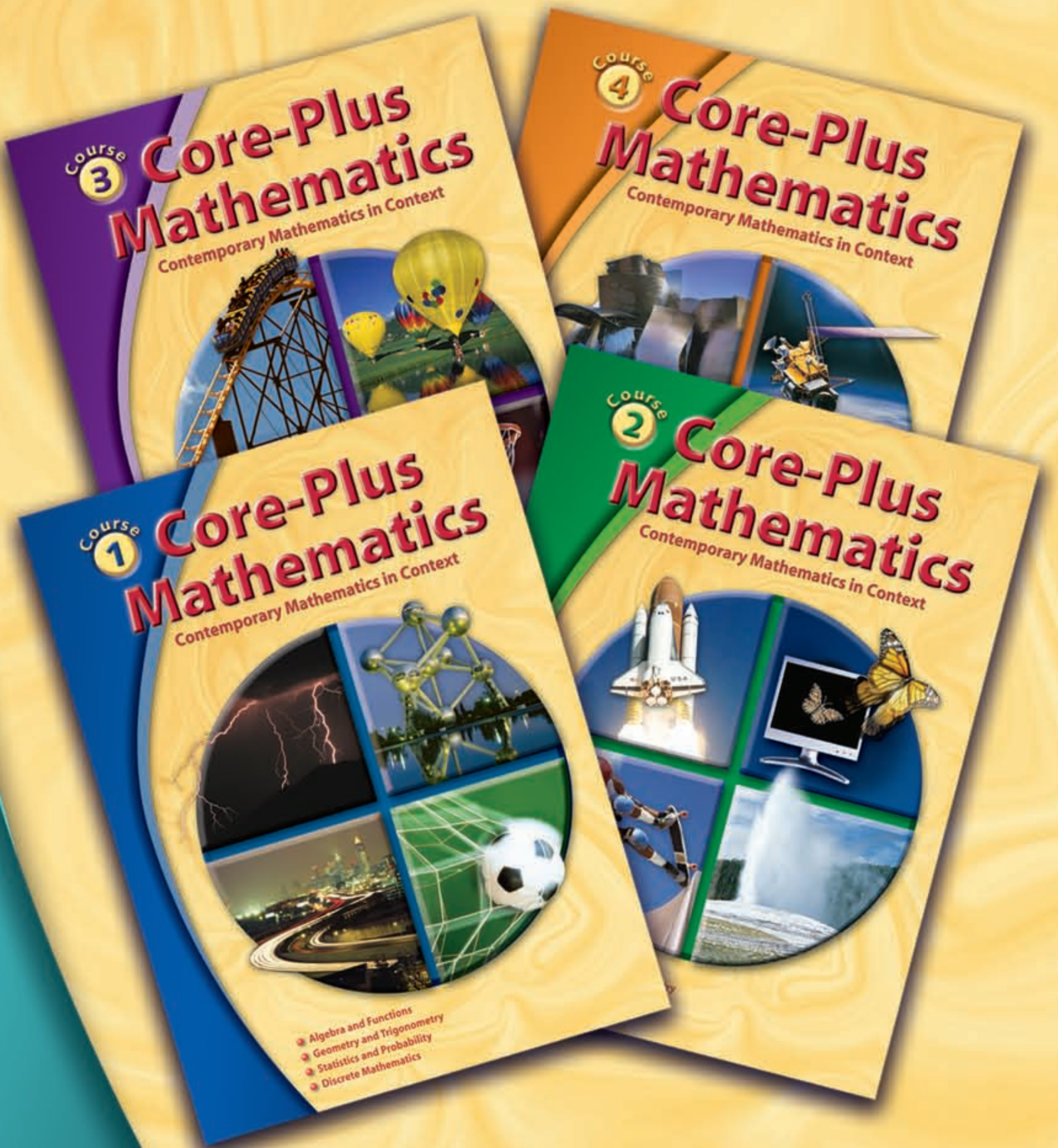
Implementing Core-Plus Mathematics

Contemporary Mathematics in Context



CORE-PLUS MATHEMATICS PROJECT

Implementing Core-Plus Mathematics

Christian R. Hirsch • James T. Fey • Eric W. Hart
Harold L. Schoen • Ann E. Watkins
with
Beth E. Ritsema • Rebecca K. Walker • Sabrina Keller
Robin Marcus • Arthur F. Coxford • Gail Burrill

McGraw Hill Glencoe

New York, New York Columbus, Ohio Chicago, Illinois Woodland Hills, California

Glencoe

The McGraw-Hill Companies

This project was supported, in part, by the National Science Foundation. Opinions expressed are those of the authors and not necessarily those of the Foundation.

Send all inquiries to:
The McGraw-Hill Companies
8787 Orion Place
Columbus, OH 43240-4027

ISBN: 978-0-07-893587-9
MHID: 0-07-893587-3

Implementing Core-Plus Mathematics

Printed in the United States of America.

2 3 4 5 6 7 8 9 10 MAL 18 17 16 15 14 13 12 11

Contents

Core-Plus Mathematics 2 Development Team

Senior Curriculum Developers

Christian R. Hirsch (Director)
Western Michigan University

James T. Fey
University of Maryland

Eric W. Hart
Maharishi University of Management

Harold L. Schoen
University of Iowa

Ann E. Watkins
California State University, Northridge

Contributing Curriculum Developers

Beth E. Ritsema
Western Michigan University

Rebecca K. Walker
Grand Valley State University

Sabrina Keller
Michigan State University

Robin Marcus
University of Maryland

Arthur F. Coxford (deceased)
University of Michigan

Gail Burrill
Michigan State University
(First edition only)

Principal Evaluator

Steven W. Ziebarth
Western Michigan University

Advisory Board

Diane Briars
Pittsburgh Public Schools

Jeremy Kilpatrick
University of Georgia

Robert E. Megginson
University of Michigan

Kenneth Ruthven
University of Cambridge

David A. Smith
Duke University

Mathematical Consultants

Deborah Hughes-Hallett
University of Arizona / Harvard University

Stephen B. Maurer
Swarthmore College

William McCallum
University of Arizona

Doris Schattschneider
Moravian College

Richard Scheaffer
University of Florida

Evaluation Consultant

Norman L. Webb
University of Wisconsin-Madison

Technical Coordinator

James Laser
Western Michigan University

Collaborating Teachers

Mary Jo Messenger
Howard County Public Schools, Maryland

Jacqueline Stewart
Okemos, Michigan

Graduate Assistants

Allison BrckaLorenz
Christopher Hlas
University of Iowa

Madeline Ahearn
Geoffrey Birky
Kyle Cochran
Michael Conklin
Brandon Cunningham
Tim Fukawa-Connelly
University of Maryland

Dana Cox
Jodi Edington
Karen Fonkert
Dana Grosser
Rob Kipka
Anna Kruizenga
Nicole Lanie
Diane Moore
Western Michigan University

Undergraduate Assistants

Cassie Durgin
University of Maryland

Rachael Kaluzny
Megan O'Connor
Jessica Tucker
Ashley Wiersma
Western Michigan University

Core-Plus Mathematics 2 Field-Test Sites

Core-Plus Mathematics 2 builds on the strengths of the 1st edition which was shaped by multi-year field tests in 36 high schools in Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Ohio, South Carolina, and Texas. Each revised text is the product of a four-year cycle of research and development, pilot testing and refinement, and field testing and further refinement. Special thanks are extended to the following teachers and their students who participated in the testing and evaluation of the 2nd edition.

Hickman High School
Columbia, Missouri
Sandra Baker
Lindsay Carlson
Peter Doll
Melissa Hundley
Michelle Johnson
Stephanie Krawczyk
Rachelle Kunce
Cheryl Lightner
Amy McCann
Tiffany McCracken
Dana Meyer
Ryan Pingrey
Michael Westcott

Holland Christian High School
Holland, Michigan
Jeff Goorhouse
Tim Laverell
Brian Lemmen
Betsi Roelofs
John Timmer
Mike Verkaik

Jefferson Junior High School
Columbia, Missouri
Marla Clowe
Lori Kilfoil
Martha McCabe
Paul Rahmoeller
Evan Schilling

Malcolm Price Lab School
Cedar Falls, Iowa
Megan Balong
Dennis Kettner
James Maltas
Josh Wilkinson

North Shore Middle School
Holland, Michigan
Brenda Katerberg
Sheila Schippers

Oakland Junior High School
Columbia, Missouri
Teresa Barry
Erin Little
Christine Sedgwick
Dana Sleeth

Riverside University High School
Milwaukee, Wisconsin
Cheryl Brenner
Dave Cusma
Scott Hanson
Ela Kiblawi
Alice Lanphier
Ulices Sepulveda

Rock Bridge High School
Columbia, Missouri
Patricia Avery
Cynthia Francisco
Nancy Hanson
Emily Hawn
Lisa Holt
Betsy Launder
Donna Lillard
Linda Shumate

Sauk Prairie High School
Prairie du Sac, Wisconsin
Joel Amidon
Shane Been
Kent Jensen
Joan Quenan
Scott Schutt
Dan Tess
Mary Walz

Sauk Prairie Middle School
Sauk City, Wisconsin
Julie Dahlman
Janine Jorgensen

South Shore Middle School
Holland, Michigan
Lynn Schipper

Washington High School
Milwaukee, Wisconsin
Anthony Amoroso
Debbie French

West Junior High School
Columbia, Missouri
Katie Bihr
Josephus Johnson
Rachel Lowery
Mike Rowson
Amanda Schoenfeld
Patrick Troup

Acknowledgments

Development and evaluation of the student text materials, teacher materials, assessments, and computer software for *Core-Plus Mathematics 2nd Edition* was funded through a grant from the National Science Foundation to the Core-Plus Mathematics Project (CPMP). We express our appreciation to NSF and, in particular, to our program officer John Bradley for his long-term trust, support, and input.

We are also grateful to Texas Instruments and in particular, Dave Santucci for collaborating with us by providing classroom sets of graphing calculators to field-test schools.

As seen on page iv, CPMP has been a collaborative effort that has drawn on the talents and energies of teams of mathematics educators at several institutions. This diversity of experiences and ideas has been a particular strength of the project. Special thanks is owed to the exceptionally capable support staff at these institutions, particularly to Angela Reiter, Matthew Tuley, and Teresa Ziebarth at Western Michigan University.

We are grateful to our Advisory Board, Diane Briars (Pittsburgh Public Schools), Jeremy Kilpatrick (University of Georgia), Robert E. Megginson (University of Michigan), Kenneth Ruthven (University of Cambridge), and David A. Smith (Duke University) for their ongoing guidance and advice. We also acknowldedge and thank Norman L. Webb (University of Wisconsin-Madison) for his advice on the design and conduct of our field-test evaluations.

Special thanks are owed to the following mathematicians: Deborah Hughes-Hallett (Univeristy of Arizona/Harvard University), Stephen B. Maurere (Swarthmore College), William McCallum (University of Arizona), Doris Schattschneider (Moravian College), and to statistician Richard Scheaffer (University of Florida) who reviewed and commented on units as they were being developed, tested, and refined.

Our gratitude is expressed to the teachers and students in our thirteen evaluation sites listed on page v. Their experiences using the revised *Core-Plus Mathematics* units provided constructive feedback and suggested improvements that were immensely helpful.

Finally, we want to acknowledge Lisa Carmona, James Matthews, Carrie Mollette, William Sellers, Karen Vujnovic, and their colleagues at Glencoe/ McGraw-Hill who contributed to the publication of this program.

Curriculum Overview

Introduction

Core-Plus Mathematics is a four-year integrated mathematics program developed by the **Core-Plus Mathematics Project (CPMP)**. The program includes student and teacher materials for a three-year core curriculum for all students and for a flexible fourth-year course that continues the preparation of students for college mathematics. All of the materials were designed to implement the vision of high school mathematics portrayed in the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Professional Standards for Teaching Mathematics* (1991). The completed curriculum and the instructional and assessment practices it supports align well with NCTM's *Principles and Standards for School Mathematics* (2000).

The CPMP curriculum builds upon the theme of *mathematics as sense-making*. Through investigations of real-life contexts, students develop a rich understanding of important mathematics that makes sense to them and that, in turn, enables them to make sense out of new situations and problems. This theme of sense-making, as well as the pervasive expectation that students reason about mathematics, aligns with the recently released NCTM document "Focus in High School Mathematics: Reasoning and Sense Making" (in press Fall 2009).

Each course in *Core-Plus Mathematics* shares the following mathematical and instructional features.

Integrated Content

Each year, the curriculum advances students' understanding of mathematics along interwoven strands of algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. These strands are unified by fundamental themes, by common topics, and by mathematical habits of mind or ways of thinking.

Mathematical Modeling

The curriculum emphasizes mathematical modeling including the processes of data collection, representation, interpretation, prediction, and simulation.

Access and Challenge

The curriculum is designed to make mathematics accessible to more students, while at the same time challenging the most able students. Differences in students' performance and interest can be accommodated by the depth and level of abstraction to which core topics are pursued, by the nature and degree of difficulty of applications, and by providing opportunities for student choice of homework tasks and projects.

Technology

Numeric, graphic, and symbolic manipulation capabilities such as those found on many graphing calculators are assumed and appropriately used throughout the curriculum. The curriculum materials also include a suite of computer software called *CPMP-Tools®* that provide powerful aids to learning mathematics and solving mathematical problems. (See pages 14–20 for further details.) This use of technology permits the curriculum and instruction to emphasize multiple representations (verbal, numerical, graphical, and symbolic) and to focus on goals in which mathematical thinking and problem solving are central.

Active Learning

Instructional materials promote active learning and teaching centered around collaborative investigations of problem situations followed by teacher-led whole-class summarizing activities that lead to analysis, abstraction, and further application of underlying mathematical ideas and principles. Students are actively engaged in exploring, conjecturing, verifying, generalizing, applying, proving, evaluating, and communicating mathematical ideas.

Multi-dimensional Assessment

Comprehensive assessment of student understanding and progress through both curriculum-embedded assessment opportunities and supplementary assessment tasks supports instruction and enables monitoring and evaluation of each student's performance in terms of mathematical processes, content, and dispositions.

Core-Plus Mathematics is designed to make mathematics accessible and more meaningful to more students. Developing mathematics along multiple strands nurtures the differing strengths and talents of students and simultaneously helps them to develop diverse mathematical insights. Developing mathematics from a modeling perspective permits students to experience mathematics as a means of making sense of data and problems that arise in diverse contexts within and across cultures. Engaging students in collaborating on tasks in small groups develops their ability to both deal with, and find commonality in, diversity of ideas. Using technology as a means for learning and doing mathematics enables students to develop versatile ways of dealing with realistic situations and reduces the manipulative skill filter which has prevented large numbers of students from continuing their study of significant mathematics. In addition, technology-produced graphics offer powerful new ways of visualizing mathematics across each of the strands.

The *Core-Plus Mathematics* program has been designated an Exemplary Program by the U.S. Department of Education, is recommended as a program that works by the Business-Higher Education Forum, and served as a model for the 2007 College Board Standards for College Success as adapted for integrated secondary school mathematics curricula.

Integrated Mathematics

Core-Plus Mathematics is a four-year integrated curriculum that replaces an Algebra-Geometry-Advanced Algebra/Trigonometry-Precalculus sequence of high-school mathematics courses. Each course features concurrent and connected development of important mathematics drawn from four strands: algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics.

Each of these strands is developed within focused units connected by fundamental ideas such as symmetry, matrices, functions, and data analysis and curve fitting. The strands also are connected across units by mathematical habits of mind such as visual thinking, recursive thinking, searching for and explaining patterns, making and checking conjectures, reasoning with multiple representations, describing and using algorithms, and providing convincing arguments and proofs.

The strands are unified further by fundamental themes of data, representation, shape, and change. Important mathematical ideas are frequently revisited through this attention to connections within and across strands, enabling students to develop a robust and connected view of mathematics.

Algebra and Functions

The Algebra and Functions strand develops student ability to recognize, represent, and solve problems involving relations among quantitative variables. Central to the development is the use of functions as mathematical models. The key algebraic models in the curriculum are linear, exponential, power, polynomial, logarithmic, rational, and trigonometric functions. Each algebraic model is investigated in at least four linked representations—verbal, graphic, numeric, and symbolic—with the aid of technology. Modeling with systems of equations, both linear and nonlinear, is developed. Attention is also given to symbolic reasoning and manipulation.

Geometry and Trigonometry

The primary goal of the Geometry and Trigonometry strand is to develop visual thinking and the ability to construct, reason with, interpret, and apply mathematical models of patterns in visual and physical contexts. The focus is on describing patterns with regard to shape, size, and location; representing patterns with drawings, coordinates, or vectors; predicting changes and invariants in figures under transformations; and organizing geometric facts and relationships through deductive reasoning.

Statistics and Probability

The primary role of the Statistics and Probability strand is to develop student ability to analyze data intelligently, to recognize and measure variation, and to understand the patterns that underlie probabilistic situations. The ultimate goal is for students to understand how inferences can be made about a population by looking at a sample from that population. Graphical methods of data analysis, simulations, sampling, and experience with the collection and interpretation of real data are featured.

Discrete Mathematics

The Discrete Mathematics strand develops student ability to model and solve problems using vertex-edge graphs, recursion, matrices, and systematic counting methods (combinatorics). Key themes are discrete mathematical modeling, optimization, and algorithmic problem solving.

Organization of the Curriculum

Courses 1–3

Each of the first three courses of *Core-Plus Mathematics* consists of eight units. Each unit contains two to four multi-day lessons in which major mathematical ideas are developed through investigations focused on sense-making and reasoning. Most investigations are developed from rich applied problems. Some are developed from examining mathematical patterns and procedures. Each unit also includes a "Looking Back" lesson to help students review and organize their thinking related to the mathematics learned in the unit. The time needed to complete a unit (in class periods of about 55 minutes) ranges from approximately two to six weeks. Unit titles for the three-year core curriculum are listed in the following table. Goals and mathematical topics for each of these units can be found on pages 7–9.

Unit Titles for Courses 1–3

Unit	Course 1	Unit	Course 2	Unit	Course 3
1	Patterns of Change	1	Functions, Equations, and Systems	1	Reasoning and Proof
2	Patterns in Data	2	Matrix Methods	2	Inequalities and Linear Programming
3	Linear Functions	3	Coordinate Methods	3	Similarity and Congruence
4	Vertex-Edge Graphs	4	Regression and Correlation	4	Samples and Variation
5	Exponential Functions	5	Nonlinear Functions and Equations	5	Polynomial and Rational Functions
6	Patterns in Shape	6	Network Optimization	6	Circles and Circular Functions
7	Quadratic Functions	7	Trigonometric Methods	7	Recursion and Iteration
8	Patterns in Chance	8	Probability Distributions	8	Inverse Functions

In developing *Core-Plus Mathematics*, the developers chose mathematical content that they believe is the most important mathematics that all high school students should have the opportunity to learn. In particular, the content of the last units in the texts is not viewed as optional. The developers believe that this content is so important and so broadly useful that it should be completed by students before embarking on the next course of the program.

The organization of the student texts differs in several other ways from most conventional textbooks. Boxed-off definitions, "worked out" examples, and content summaries are not as prevelant as in conventional texts. Students learn mathematics by doing mathematics. Concept images are developed as students complete investigations; later concept definitions are formalized. Mathematical ideas are developed and then shared by groups of students at strategically placed points, called Summarize the Mathematics, within the lessons. These class discussions then lead to a class summary of shared understandings.

It is important that each student constructs a Math Toolkit consisting of mathematical concepts, methods, and theorems as they are developed. By organizing these important class-generated ideas from a unit, from occasional lesson summary responses or from a unit summary in the student's own words, the student will gather a valuable set of tools that can be used throughout the course and in subsequent courses.

Course 4 *Core-Plus Mathematics* Course 4 formalizes and extends the core program with a focus on the mathematics needed to be successful in undergraduate programs requiring calculus. This course shares many of the design features of Courses 1–3. Course 4 unit titles are given in the following table. Goals and mathematical topics for each of these units can be found on pages 10–11.

Unit Titles for Course 4	
1	Families of Functions
2	Vectors and Motion
3	Algebraic Functions and Equations
4	Trigonometric Functions and Equations
5	Exponential Functions, Logarithms, and Data Modeling
6	Surfaces and Cross Sections
7	Concepts of Calculus
8	Counting Methods and Induction

Alignment with the Common Core State Standards for Mathematics

In June 2010, the *Common Core State Standards for Mathematics* (CCSSM) were released and, to date, adopted by 43 states and the District of Columbia. Included are standards for mathematical content to be learned and for mathematical practices (outlined below) to be cultivated.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The *Core-Plus Mathematics* curriculum, by design, incorporates the Mathematical Practices into each lesson. A document with page references that aligns the CCSSM to the *Core-Plus Mathematics* texts is available at: www.wmich.edu/cpmp/programoverview.html

In addition to the strong alignment with the CCSSM, *Core-Plus Mathematics* is designed to promote the CCSS for English Language Arts & Literacy, particularly the anchor standards for reading, writing, and speaking and listening related to informational texts.

Informational Text
Literary Nonfiction and Historical, Scientific, and Technical Texts
Includes the subgenres of exposition, argument, and functional text in the form of personal essays, speeches, opinion pieces, essays about art or literature, biographies, memoirs, journalism, and historical, scientific, technical, or economic accounts (including digital sources) written for a broad audience

The Common Core State Standards can be found at: www.corestandards.org/the-standards.

Alignment with Other National and State High School Recommendations

The mathematical content developed in *Core-Plus Mathematics* aligns well with the content described in documents such as NCTM's *Principles and Standards for School Mathematics* (NCTM, 2000), the *College Board Standards for College Success: Mathematics and Statistics* (2006, www.collegeboard.com), the American Diploma Project, Achieve Secondary Mathematics Standards (www.achieve.org/node969), and most state mathematics standards and expectations. More detail on topic alignment to *Principles and Standards for School Mathematics* is available in the front matter of each *Core-Plus Mathematics Teacher's Guide* and on the *TeacherWorks* CDs. Alignments by unit to the College Board and Achieve standards adapted for integrated curricula are available at www.wmich.edu/cpmp/programoverview.html. You may be able to obtain correlations of *Core-Plus Mathematics* to your state standards from the Glencoe Web site or from your local sales representative.

Alignment with College Recommendations

Curriculum recommendations of the Mathematical Association of America (MAA) and the American Mathematical Association of Two-Year Colleges (AMATYC) align well with the goals of the *Core-Plus Mathematics* program.

You may wish to review the MAA Curriculum Foundations Project recommendations *MAA Curriculum Foundations Project Core Mathematics in the First Two Years* at www.maa.org/features/currfound.html. You may also wish to read the booklet *Changing Core Mathematics* (Small, D., MAA 2002) that outlines a revitalization of undergraduate mathematics in the first two years through integrated courses with a focus on inquiry and mathematical modeling.

The American Mathematical Association of Two-Year Colleges (AMATYC) has developed a second standards document entitled *Beyond Crossroads: Implementing Standards in the First Two Years of College*. This is available at www.beyondcrossroads.com.

Course 1 Units and Descriptions

Unit 1	**Patterns of Change** develops student ability to recognize and describe important patterns that relate quantitative variables; to use data tables, graphs, words, and symbols to represent the relationships; and to use reasoning and calculating tools to answer questions and solve problems. *Topics include* variables and functions, algebraic expressions and recurrence relations, coordinate graphs, data tables and spreadsheets, and equations and inequalities.
Unit 2	**Patterns in Data** develops student ability to make sense of real-world data through use of graphical displays, measures of center, and measures of variability. *Topics include* distributions of data and their shapes, as displayed in dot plots, histograms, and box plots; measures of center including mean and median and their properties; measures of variability including interquartile range and standard deviation and their properties; and percentiles and outliers.
Unit 3	**Linear Functions** develops student ability to recognize and represent linear relationships between variables and to use tables, graphs, and algebraic expressions for linear functions to solve problems in situations that involve constant rate of change or slope. *Topics include* linear functions, slope of a line, rate of change, modeling linear data patterns, solving linear equations and inequalities, and equivalent linear expressions.
Unit 4	**Vertex-Edge Graphs** develops student understanding of vertex-edge graphs and ability to use these graphs to represent and solve problems involving paths, networks, and relationships among a finite number of elements, including finding efficient routes and avoiding conflicts. *Topics include* vertex-edge graphs, mathematical modeling, optimization, algorithmic problem solving, Euler circuits and paths, matrix representation of graphs, and vertex coloring and chromatic number.
Unit 5	**Exponential Functions** develops student ability to recognize and represent exponential growth and decay patterns, to express those patterns in symbolic forms, to solve problems that involve exponential change, and to use properties of exponents to write expressions in equivalent forms. *Topics include* exponential growth and decay functions, data modeling, growth and decay rates, half-life and doubling time, compound interest, and properties of exponents.
Unit 6	**Patterns in Shape** develops student ability to visualize and describe two- and three-dimensional shapes, to represent them with drawings, to examine shape properties through both experimentation and careful reasoning, and to use those properties to solve problems. *Topics include* Triangle Inequality, congruence conditions for triangles, special quadrilaterals and quadrilateral linkages, Pythagorean Theorem, properties of polygons, tilings of the plane, properties of polyhedra, and the Platonic solids.
Unit 7	**Quadratic Functions** develops student ability to recognize and represent quadratic relations between variables using data tables, graphs, and symbolic formulas, to solve problems involving quadratic functions, and to express quadratic polynomials in equivalent factored and expanded forms. *Topics include* quadratic functions and their graphs, applications to projectile motion and economic problems, expanding and factoring quadratic expressions, and solving quadratic equations by the quadratic formula and calculator approximation.
Unit 8	**Patterns in Chance** develops student ability to solve problems involving chance by constructing sample spaces of equally likely outcomes or geometric models and to approximate solutions to more complex probability problems by using simulation. *Topics include* sample spaces, equally likely outcomes, probability distributions, mutually exclusive (disjoint) events, Addition Rule, simulation, random digits, discrete and continuous random variables, Law of Large Numbers, and geometric probability.

Course 2 Units and Descriptions	
Unit 1	**Functions, Equations, and Systems** reviews and extends student ability to recognize, describe, and use functional relationships among quantitative variables with special emphasis on relationships that involve two or more independent variables. *Topics include* direct and inverse variation and joint variation; power functions; linear equations in standard form; and systems of two linear equations with two variables, including solution by graphing, substitution, and elimination.
Unit 2	**Matrix Methods** develops student understanding of matrices and ability to use matrices to represent and solve problems in a variety of real-world and mathematical settings. *Topics include* constructing and interpreting matrices, row and column sums, matrix addition, scalar multiplication, matrix multiplication, powers of matrices, inverse matrices, properties of matrices, and using matrices to solve systems of linear equations.
Unit 3	**Coordinate Methods** develops student understanding of coordinate methods for representing and analyzing properties of geometric shapes, for describing geometric change, and for producing animations. *Topics include* representing two-dimensional figures and modeling situations with coordinates, including computer-generated graphics; distance in the coordinate plane, midpoint of a segment, and slope; coordinate and matrix models of rigid transformations (translations, rotations, and line reflections), size transformations, and similarity transformations; animation effects.
Unit 4	**Regression and Correlation** develops student understanding of the characteristics and interpretation of the least squares regression equation and of the use of correlation to measure the strength of the linear association between two variables. *Topics include* interpreting scatterplots; least squares regression, residuals and errors in prediction, sum of squared errors, influential points; Pearson's correlation coefficient and its properties, lurking variables, and cause and effect.
Unit 5	**Nonlinear Functions and Equations** introduces function notation, reviews and extends student ability to construct and reason with functions that model parabolic shapes and other quadratic relationships in science and economics with special emphasis on formal symbolic reasoning methods, and introduces common logarithms and algebraic methods for solving exponential equations. *Topics include* formalization of function concept, notation, domain and range; factoring and expanding quadratic expressions, solving quadratic equations by factoring and the quadratic formula, applications to supply and demand, break-even analysis; common logarithms and solving exponential equations using base 10 logarithms.
Unit 6	**Network Optimization** develops student understanding of vertex-edge graphs and ability to use these graphs to solve network optimization problems. *Topics include* optimization, mathematical modeling, algorithmic problem solving, digraphs, trees, minimum spanning trees, distance matrices, Hamilton circuits and paths, the Traveling Salesperson Problem, critical paths, and the PERT technique.
Unit 7	**Trigonometric Methods** develops student understanding of trigonometric functions and the ability to use trigonometric methods to solve triangulation and indirect measurement problems. *Topics include* sine, cosine, and tangent functions of measures of angles in standard position in a coordinate plane and in a right triangle; indirect measurement; analysis of variable-sided triangle mechanisms; and Law of Sines and Law of Cosines.
Unit 8	**Probability Distributions** further develops student ability to understand and visualize situations involving chance by using simulation and mathematical analysis to construct probability distributions. *Topics include* Multiplication Rule, independent and dependent events, conditional probability, probability distributions and their graphs, waiting-time (or geometric) distributions, expected value, and rare events.

Course 3 Units and Descriptions	
Unit 1	**Reasoning and Proof** develops student understanding of formal reasoning in geometric, algebraic, and statistical contexts and of basic principles that underlie those reasoning strategies. *Topics include* inductive and deductive reasoning strategies; principles of logical reasoning—Affirming the Hypothesis and Chaining Implications; relation among angles formed by two intersecting lines or by two parallel lines and a transversal; rules for transforming algebraic expressions and equations; design of experiments including the role of randomization, control groups, and blinding; sampling distribution, randomization test, and statistical significance.
Unit 2	**Inequalities and Linear Programming** develops student ability to reason both algebraically and graphically to solve inequalities in one and two variables, introduces systems of inequalities in two variables, and develops a strategy for optimizing a linear function in two variables within a system of linear constraints on those variables. *Topics include* inequalities in one and two variables, number line graphs, interval notation, systems of linear inequalities, and linear programming.
Unit 3	**Similarity and Congruence** extends student understanding of similarity and congruence and their ability to use those relations to solve problems and to prove geometric assertions with and without the use of coordinates. *Topics include* connections between Law of Cosines, Law of Sines, and sufficient conditions for similarity and congruence of triangles, centers of triangles, applications of similarity and congruence in real-world contexts, necessary and sufficient conditions for parallelograms, sufficient conditions for congruence of parallelograms, and midpoint connector theorems.
Unit 4	**Samples and Variation** extends student understanding of the measurement of variability, develops student ability to use the normal distribution as a model of variation, introduces students to the binomial distribution and its use in decision making, and introduces students to the probability and statistical inference involved in control charts used in industry for statistical process control. *Topics include* normal distribution, standardized scores, binomial distributions (shape, expected value, standard deviation), normal approximation to a binomial distribution, odds, statistical process control, control charts, and the Central Limit Theorem.
Unit 5	**Polynomial and Rational Functions** extends student ability to represent and draw inferences about polynomial and rational functions using symbolic expressions and manipulations. *Topics include* definition and properties of polynomials, operations on polynomials; completing the square, proof of the quadratic formula, solving quadratic equations (including complex number solutions), vertex form of quadratic functions; definition and properties of rational functions, operations on rational expressions.
Unit 6	**Circles and Circular Functions** develops student understanding of relationships among special lines, segments, and angles in circles and the ability to use properties of circles to solve problems; develops student understanding of circular functions and the ability to use these functions to model periodic change; and extends student ability to reason deductively in geometric settings. *Topics include* properties of chords, tangent lines, and central and inscribed angles of circles; linear and angular velocity; radian measure of angles; and circular functions as models of periodic change.
Unit 7	**Recursion and Iteration** extends student ability to represent, analyze, and solve problems in situations involving sequential and recursive change. *Topics include* iteration and recursion as tools to model and analyze sequential change in real-world contexts, including compound interest and population growth; arithmetic, geometric, and other sequences; arithmetic and geometric series; finite differences; linear and nonlinear recurrence relations; and function iteration, including graphical iteration and fixed points.
Unit 8	**Inverse Functions** develops student understanding of inverses of functions with a focus on logarithmic functions and their use in modeling and analyzing problem situations and data patterns. *Topics include* inverses of functions; logarithmic functions and their relation to exponential functions, properties of logarithms, equation solving with logarithms; and inverse trigonometric functions and their applications to solving trigonometric equations.

Course 4 Units and Descriptions

Unit 1	**Families of Functions** extends student understanding of linear, exponential, quadratic, power, and trigonometric functions to model data patterns whose graphs are transformations of basic patterns; and develops understanding of operations on functions useful in representing and reasoning about quantitative relationships. *Topics include* linear, exponential, quadratic, power, and trigonometric functions; data modeling; translation, reflection, and stretching of graphs; and addition, subtraction, multiplication, division, and composition of functions.
Unit 2	**Vectors and Motion** develops student understanding of two-dimensional vectors and their use in modeling linear, circular, and other nonlinear motion. *Topics include* concept of vector as a mathematical object used to model situations defined by magnitude and direction; equality of vectors, scalar multiples, opposite vectors, sum and difference vectors, dot product of two vectors, position vectors and coordinates; and parametric equations for motion along a line and for motion of projectiles and objects in circular and elliptical orbits.
Unit 3	**Algebraic Functions and Equations** reviews and extends student understanding of properties of polynomial and rational functions and skills in manipulating algebraic expressions and solving polynomial and rational equations, and develops student understanding of complex number representations and operations. *Topics include* polynomials, polynomial division, factor and remainder theorems, operations on complex numbers, representation of complex numbers as vectors, solution of polynomial equations, rational function graphs and asymptotes, and solution of rational equations and equations involving radical expressions.
Unit 4	**Trigonometric Functions and Equations** extends student understanding of, and ability to reason with, trigonometric functions to prove or disprove two trigonometric expressions are identical and to solve trigonometric equations; to geometrically represent complex numbers and complex number operations and to find roots of complex numbers. *Topics include* the tangent, cotangent, secant, and cosecant functions; fundamental trigonometric identities, sum and difference identities, double-angle identities; solving trigonometric equations and expression of periodic solutions; rectangular and polar representations of complex numbers, absolute value, DeMoivre's Theorem, and the roots of complex numbers.
Unit 5	**Exponential Functions, Logarithms, and Data Modeling** extends student understanding of exponential and logarithmic functions to the case of natural exponential and logarithmic functions, solution of exponential growth and decay problems, and use of logarithms for linearization and modeling of data patterns. *Topics include* exponential functions with rules in the form $f(x) = Ae^{kx}$, natural logarithm function, linearizing bivariate data, and fitting models using log and log-log transformations.
Unit 6	**Surfaces and Cross Sections** extends student ability to visualize and represent three-dimensional shapes using contours, cross sections, and reliefs and to visualize and represent surfaces and conic sections defined by algebraic equations. *Topics include* using contours to represent three-dimensional surfaces and developing contour maps from data; sketching surfaces from sets of cross sections; conics as planar sections of right circular cones and as locus of points in a plane; three-dimensional rectangular coordinate system; sketching surfaces using traces, intercepts and cross sections derived from algebraically-defined surfaces; and surfaces of revolution and cylindrical surfaces.
Unit 7	**Concepts of Calculus** develops student understanding of fundamental calculus ideas through explorations in a variety of applied problem contexts and their representations in function tables and graphs. *Topics include* instantaneous rates of change; linear approximation; area under a curve; and applications to problems in physics, business, and other disciplines.

Course 4 Units and Descriptions (continued)	
Unit 8	**Counting Methods and Induction** extends student ability to count systematically and solve enumeration problems and develops understanding of and ability to do proof by mathematical induction. *Topics include* systematic listing and counting, counting trees, the Multiplication Principle of Counting, Addition Principle of Counting, combinations, permutations, selections with repetition; the binomial theorem, Pascal's triangle, combinatorial reasoning; the general multiplication rule for probability; the Principle of Mathematical Induction; and the Least Number Principle.

Instructional Model

The manner in which students encounter mathematical ideas can contribute significantly to the quality of their learning and the depth of their understanding. *Core-Plus Mathematics* units are designed around multi-day lessons centered on big ideas. Each lesson includes 2–4 focused mathematical investigations that engage students in a four-phase cycle of classroom activities, described in the following paragraphs—*Launch, Explore, Share and Summarize,* and *Apply*. This cycle is designed to engage students in investigating and making sense of problem situations, in constructing important mathematical concepts and methods, in generalizing and proving mathematical relationships, and in communicating, both orally and in writing, their thinking and the results of their efforts. Most classroom activities are designed to be completed by students working collaboratively in groups of two to four students.

LAUNCH class discussion

Think About This Situation

The lesson launch promotes a teacher-led discussion of a problem situation and of related questions to think about. This discussion sets the context for the student work to follow and helps to generate student interest. It also provides an opportunity for the teacher to assess student knowledge and to clarify directions for the investigation to follow.

EXPLORE group investigation

Investigation

Classroom activity then shifts to investigating focused problems and questions related to the launching situation by gathering data, looking for and explaining patterns, constructing models and meanings, and making and verifying conjectures. As students collaborate in pairs or small groups, the teacher circulates among students providing guidance and support, clarifying or asking questions, giving hints, providing encouragement, and drawing group members into the discussion to help groups collaborate more effectively. The investigations and related questions posed by students and teachers drive the learning.

SHARE AND SUMMARIZE class discussion

Summarize the Mathematics

This investigative work is followed by a teacher-led class discussion (referred to as Summarize the Mathematics) in which students summarize and explain the reasoning supporting mathematical ideas developed in their groups, providing an opportunity to construct a shared understanding of important concepts, methods, and approaches. This discussion leads to a class summary of important ideas or to further exploration of a topic if competing perspectives remain. Varying points of view and differing conclusions that can be justified should be encouraged.

APPLY individual tasks

Check Your Understanding

Students are given a task to complete on their own to check and reinforce their initial understanding of concepts and methods.

Homework

In addition to the classroom investigations, *Core-Plus Mathematics* provides sets of On Your Own tasks, which are designed to engage students in applying, connecting, reflecting on, extending, and reviewing their evolving mathematical knowledge. On Your Own tasks are provided for each lesson in the materials and are central to the learning goals of each lesson. These tasks are intended primarily for individual work outside of class. Selection of homework tasks should be based on student performance and the availability of time and technology. Also, students should exercise some choice of tasks to pursue, and, at times should be given the opportunity to pose their own problems and questions to investigate. The chart below describes the types of tasks in a typical On Your Own set.

On Your Own: Homework Tasks	
Applications	These tasks provide opportunities for students to use and strengthen their understanding of the ideas they have learned in the lesson.
Connections	These tasks help students to build links between mathematical topics they have studied in the lesson and to connect those topics with other mathematics that they know.
Reflections	These tasks provide opportunities for students to re-examine their thinking about ideas in the lesson.
Extensions	These tasks provide opportunities for students to explore further or more deeply the mathematics they are learning.
Review	These tasks provide opportunities for just-in-time review and distributed practice of mathematical skills to maintain procedural fluency.

Review and Practice

Core-Plus Mathematics includes review tasks in the homework sets. The purpose of the review tasks is two-fold. Some tasks are **just-in-time review** of concepts and skills needed in the following lesson. These tasks will be designated by a clock icon near the solution. Some tasks provide **distributed practice** of mathematical skills to maintain procedural fluency. These tasks should be completed outside of class by students. If a few students are identified as needing additional assistance with specific skills, they should be given additional assistance outside of class.

Practicing for Standardized Tests

Opportunities for additional review and practice are provided in the Practicing for Standardized Tests masters in each Unit Resources booklet. Each Practicing for Standardized Tests master presents 10 questions that draw on all content strands. The questions are presented in the form of test items similar to how they often appear in standardized tests such as state assessment tests, the Preliminary Scholastic Aptitude Test (PSAT), SAT, ACT PLAN, or ACT. We suggest using these practice sets following the unit assessment so students can become familiar with the formats of standardized tests and develop effective test-taking strategies for performing well on such tests.

Additional Summarizing Activities

In *Core-Plus Mathematics*, students learn mathematics by doing mathematics. However, it is important that students prepare and maintain summaries of important concepts and methods that are developed. Students should create a Math Toolkit that organizes important class-generated ideas and selected Summarize the Mathematics responses as they complete investigations. Math Toolkit Prompts are provided in this *Teacher's Guide* to assist in identifying and summarizing key concepts and methods as they are developed by students.

In addition, the final lesson in each unit is a Looking Back lesson that helps students review and synthesize the key mathematical concepts and techniques developed in the unit. The Summarize the Mathematics questions in this lesson are focused on key ideas of the unit. The Check Your Understanding asks students to prepare a summary of the important concepts and skills developed in the unit. Templates to guide preparation of these unit summaries can be found in the *Unit Resource Masters.* Completed Unit Summaries should become part of students' Math Toolkits.

Students should retain their Math Toolkits as they continue on to later courses. In some districts, teachers collect these resources at the end of the school year and return them to students in the fall.

Technology

In the 21st century, anyone who faces the challenge of learning mathematics or using mathematics to solve problems can draw on the resources of powerful information technology tools. Calculators and computers can help with calculations and table-building, graphing and equation solving, drawing and measuring, and data analysis.

Graphing Calculators

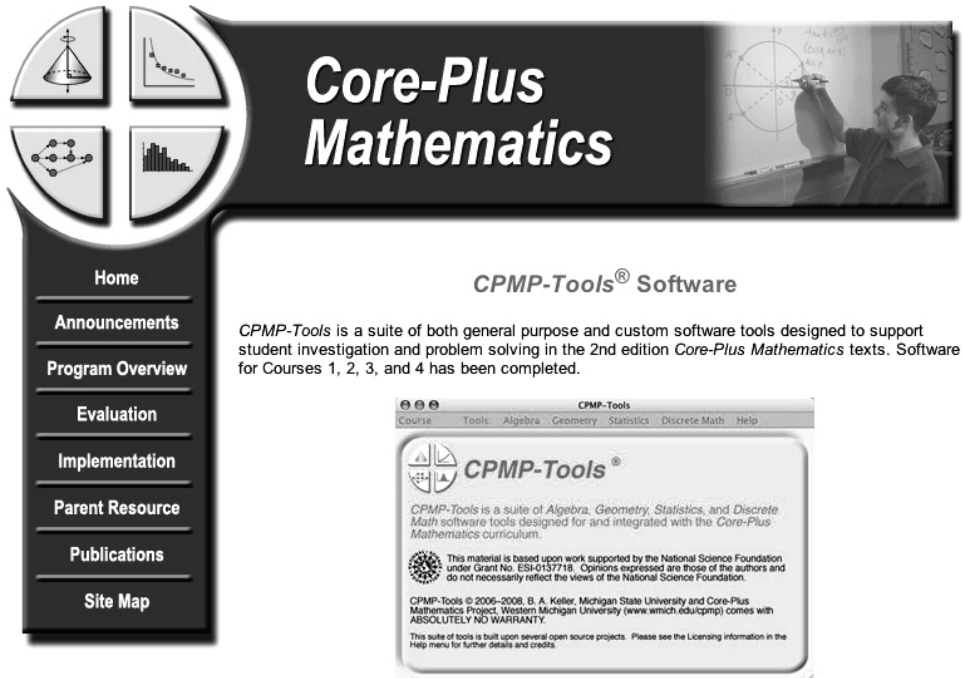
Graphing calculators with iteration capabilities are assumed for class work and homework. Technology tips are available in the *Unit Resource Masters*. For schools using the TI-Nspire™, a keystroke specific booklet of tips is available. (See page 30.)

Computers

Periodically, it would be valuable to have one classroom computer for whole class discussions, 4–6 classroom computers for groups to use at stations during investigations, portable classroom sets of computers, or computer lab access. For some homework tasks, school or home computer availability is also desirable.

Computer Software

Appropriate use of spreadsheet, interactive geometry, data analysis, and vertex-edge graph software, and computer algebra systems (CAS) is incorporated into *Core-Plus Mathematics* units. The curriculum materials include computer software called *CPMP-Tools* specifically designed to support student learning and problem solving. *CPMP-Tools* is available under a Gnu-public license at www.wmich.edu/cpmp/CPMP-Tools. A version of *CPMP-Tools* that does not require Internet access is on each *Core-Plus Mathematics StudentWorks™ Plus* and *TeacherWorks™ Plus* CD produced by Glencoe/McGraw-Hill. An extensive Help system is built into the software.



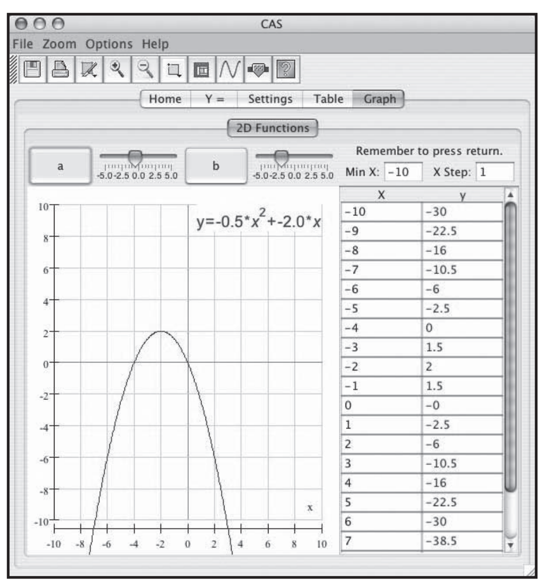
The *CPMP-Tools* software toolkit was developed from the ground up with specific curriculum applications in mind. Tools were developed for each strand of the curriculum.

With the goal of reducing the steepness of the learning curve for students, the tools and their functionality are organized by course to focus on the intended mathematics of each of the four courses of *Core-Plus Mathematics*. To promote learning transfer from one tool to another, the tools share similar menu screens and, in some cases, interface with each other.

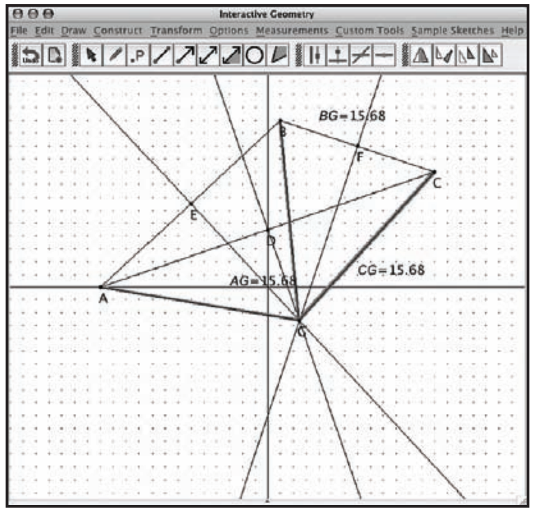
General All Purpose Tools

The suite of Java-based mathematical software includes four families of programs.

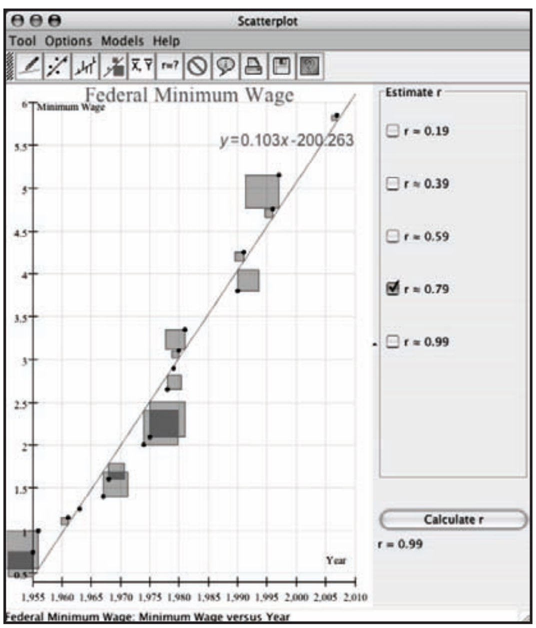
- *Algebra*—The software for work on algebra problems include an electronic spreadsheet and a computer algebra system (CAS) that produces tables and graphs of functions, manipulates algebraic expressions, and solves equations and inequalities.

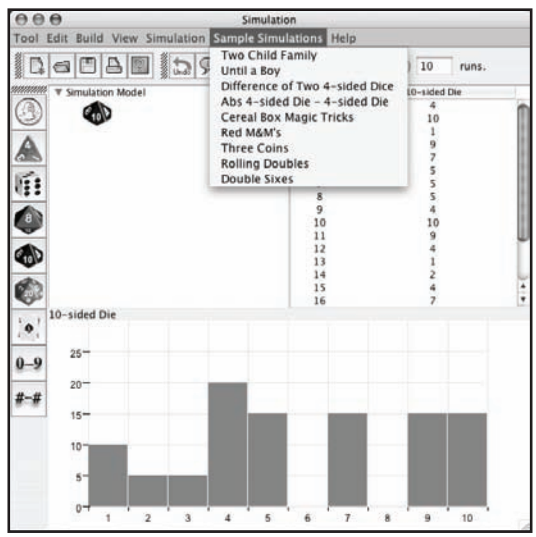


- *Geometry*—The software for work on geometry problems include an interactive drawing program for constructing, measuring, manipulating, and analyzing geometric figures in either coordinate or synthetic environments.

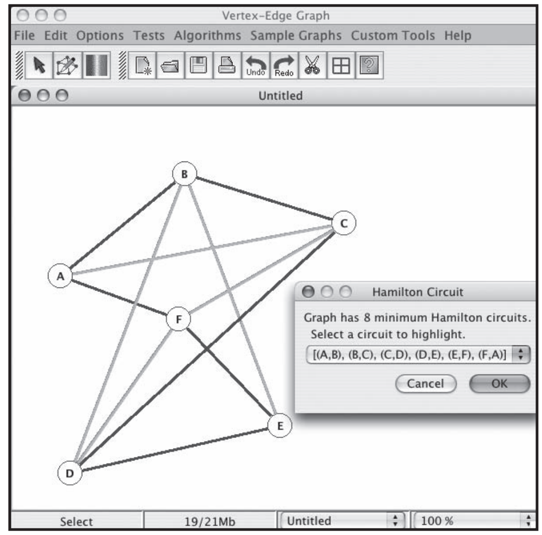


- *Statistics*—The software for work on data analysis and probability problems provides tools for graphic display and analysis of data, including finding function models for bivariate data and simulation of probability experiments. It also includes a randomization distribution and distributions of sample means, medians, and standard deviations.





- *Discrete Mathematics*—The software for work on graph theory problems provide tools for constructing, manipulating, and analyzing vertex-edge graphs and their adjacency matrices.



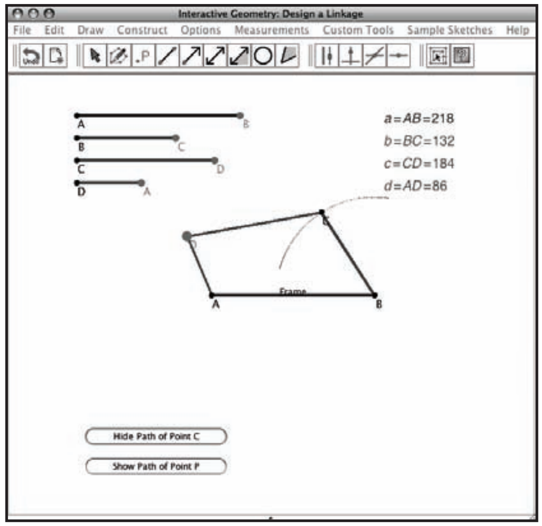
In addition to the general purpose tools provided for work on tasks in each strand of *Core-Plus Mathematics*, *CPMP-Tools* includes files that provide electronic copies of most data sets essential for work on problems in each *Core-Plus Mathematics* course. When students see an opportunity to use computer tools for work on a particular investigation, they can select the *CPMP-Tools* menu corresponding to the content involved in the problem. Then they can select the submenu items corresponding to the required mathematical operations and data sets. Each unit overview in the *Teacher's Guide* provides general information related to *CPMP-Tools* use in the unit. Technology notes at point of use alert teachers to applicable software and specific data sets included in the software.

In addition to the general purpose software, custom tools have been developed for each mathematical strand. These custom tools were developed to allow exploration and analysis of specific mathematical or statistical concepts and topics. For examples, students study geometric models of physical mechanisms, tessellations, and special shapes. A few of the custom tools are shown on the next page.

Selected Custom Tools

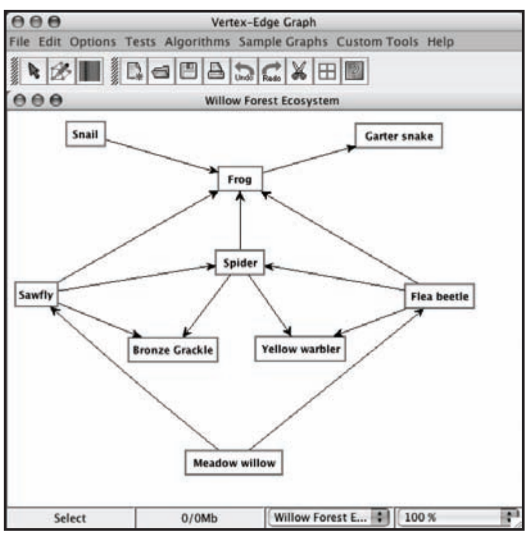
Course 1, Geometry, "Quadrilateral Linkage" Custom Tool:

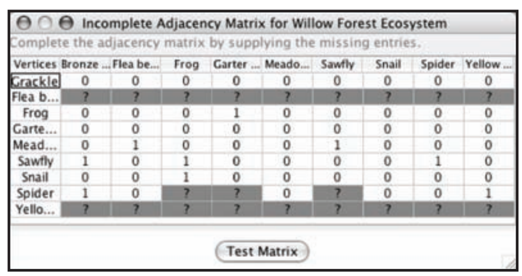
This custom tool allows students to experiment with 4-bar linkages to determine the relationship between lengths of sides of the linkage that allow one side (follower crank) to rotate partially or completely. (Grashof's Principle)



Course 2, Discrete Mathematics, "Willow Forest Ecosystem" Custom Tool:

This tool was developed to facilitate student exploration of how pollution can cause part or all of a food web in an ecosystem to become contaminated. The digraph and a partially completed adjacency matrix are built into the tool.





Incomplete Adjacency Matrix for Willow Forest Ecosystem

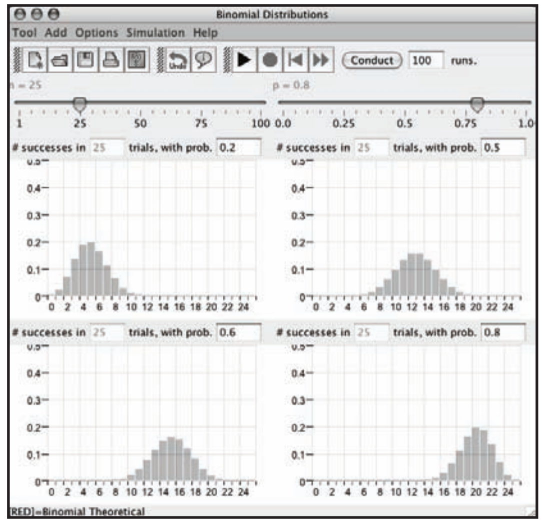
Complete the adjacency matrix by supplying the missing entries.

Vertices	Bronze ...	Flea be...	Frog	Garter ...	Meado...	Sawfly	Snail	Spider	Yellow ...
Grackle	0	0	0	0	0	0	0	0	0
Flea b...	?	?	?	?	?	?	?	?	?
Frog	0	0	0	1	0	0	0	0	0
Garte...	0	0	0	0	0	0	0	0	0
Mead...	0	1	0	0	0	1	0	0	0
Sawfly	1	0	1	0	0	0	0	1	0
Snail	0	0	1	0	0	0	0	0	0
Spider	1	0	?	?	0	?	0	0	1
Yello...	?	?	?	?	?	?	?	?	?

Test Matrix

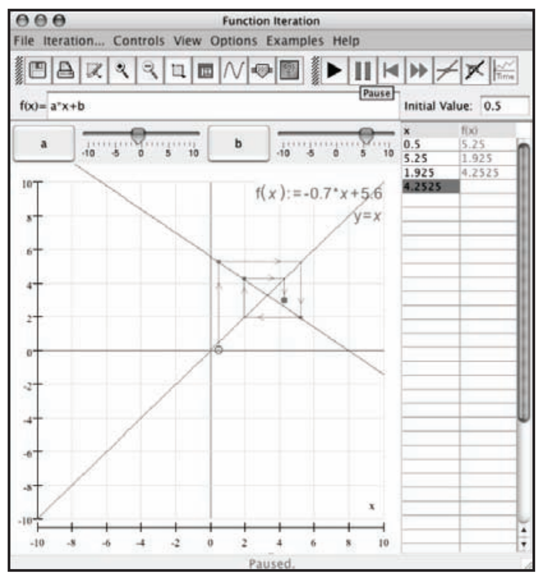
Course 3, Statistics, "Binomial Distributions" Custom Tool:

This custom tool allows students to vary the number of successes and the probability of a success on each trial to see how the shape, center, and spread of binomial distributions are affected.



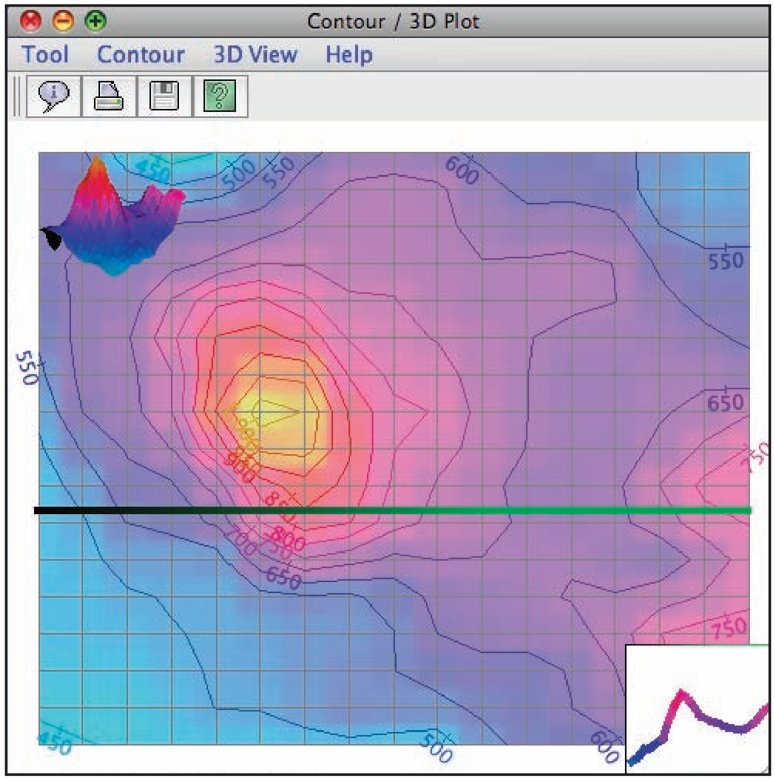
Course 3, Algebra, "Function Iteration" Custom Tool:

This custom tool illustrates the graphical iteration process for functions. Students enter a function to iterate and choose an initial value.



Course 4, Geometry, "Contour" Custom Tool:

This custom tool was developed to facilitate student connections among various representations of surfaces: data, coordinates, contour diagrams, three-dimensional shapes, and relief lines.



Common Core State Standards

Mathematical Practice #5: Use appropriate tools strategically.

CPMP-Tools public domain software as described above provides students access to powerful technology tools at school and at home. The accessibility offers flexible ways for students to develop the Mathematical Practice described below.

"Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. … They are able to use technological tools to explore and deepen their understanding of concepts." (CCSSM p. 7)

Further Reading on Technology

Alejandre, S. "The Reality of Using Technology in the Classroom." *Technology-Supported Mathematics Learning Environments Sixty-Seventh Yearbook*. NCTM, 2005.

NCTM Position Paper. "The Use of Technology in the Learning and Teaching of Mathematics." (www.nctm.org/uploadedFiles/About_NCTM/Position_Statements/Technology%20final.pdf)

Evaluation Results

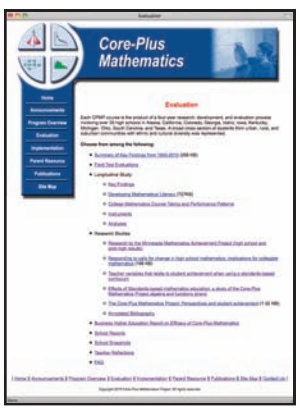
Summary of Key Research Findings (1993–2010)

The field-test evaluation of the 1st edition of *Core-Plus Mathematics* involved 36 high schools in 11 states, over 3,000 students, and over 125 teachers in rural, suburban, and urban settings. Evaluation of each of the four courses involved an iterative cycle of testing, revision, further testing, and further revision. Final curriculum materials were improved based on field-test evaluation results and face-to-face meetings with field-test teachers from the test sites. In addition to results from the field-test evaluation of the 1st edition, the summary below provides key findings of studies based on the use of the published 1st edition of *Core-Plus Mathematics.* That is followed by results from the field-test evaluation of the 2nd edition of *Core-Plus Mathematics*. Findings from use of the published 2nd edition materials are just beginning to become available. More detail on these findings and updated research can be found at: www.wmich.edu/cpmp/evaluation.html.

Formative and Summative Evaluation of the 1st Edition (1993–2002)

In comparative studies of students who studied *Core-Plus Mathematics* and comparable students who studied more conventional curricula (organized as Algebra 1, Geometry, Advanced Algebra, and Precalculus), *Core-Plus Mathematics* students:

- performed significantly better on tests of problem solving, applications, and conceptual understanding.
- performed significantly better on the SAT Mathematics Test and as well on the ACT Mathematics Test.
- elected to enroll in more high school mathematics courses.
- had more positive attitudes and perceptions about mathematics.
- at the end of Course 3, performed significantly better on measures of conceptual understanding and problem solving in applied settings, but (using field-test materials) scored significantly lower than Algebra II students on a subtest of paper-and-pencil skills.
- performed as well on tests of paper-and-pencil algebraic skills (using published 1st edition *Core Plus Mathematics* texts).
- at the end of Course 3, performed at the level of the top-scoring country, the Netherlands, on a test composed of released 1995 TIMSS Twelfth-Grade Mathematical Literacy items (using published 1st edition texts).
- at the end of Course 4, outperformed comparable students on the calculus readiness portion of a mathematics placement test at a large midwestern university. Of the 20 calculus readiness items, group means differed significantly on 12 of them, 11 in favor of CPMP students. The items were drawn from a bank of items available from the Mathematical Association of America.

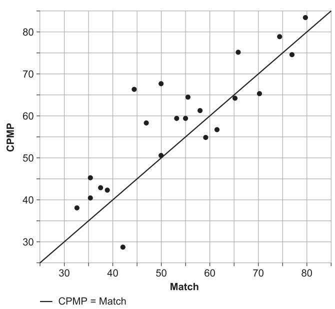


Summary of Achievement by Various School and Student Groups

CPMP students outperformed comparison students on the mathematics subtest of the nationally standardized *Iowa Tests of Educational Development (ITED) Ability to Do Quantitative Thinking* subtest.

Data was broken out and reported by various subgroups. The *adjusted effect sizes* reported below are the number of pretest standard deviations the cohort grew beyond the average growth of the nationally normed group.

- *By school type:* Rural +0.71 Urban +0.59 Suburban +0.30
- *By gender:* No statistically significant difference
- *By minority status:* African American, Hispanic, Asian American, Native Alaskan or American. Adjusted effect sizes were small but positive.
- *By high mathematical aptitude:* In spite of an expected regression to the mean effect due to very high pretest scores, the posttest growth was about double that of the ninth-grade norm group. The growth of the mean from pretest to posttest was 0.25 standard deviations higher for the CPMP cohort.



WASL Math Pass Data, Grade 10, 2004–05

A Matched-Pairs Study of Washington State 10th-Grade Assessment Scores of Students in Schools Using the 1st Edition of the *Core-Plus Mathematics* Program

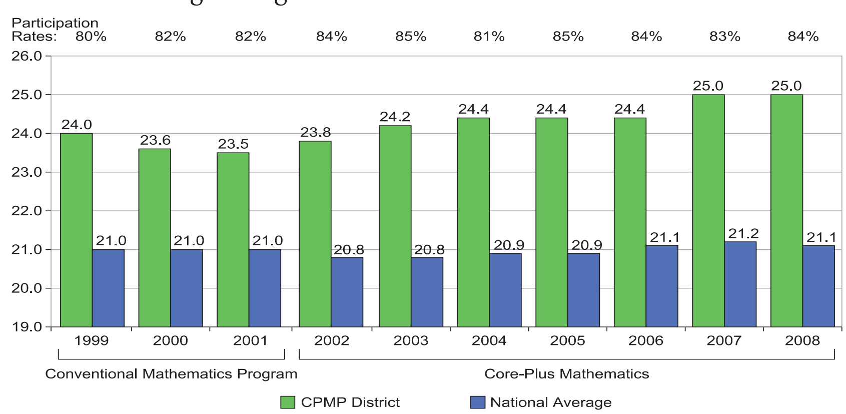
Twenty-two high schools in the state of Washington that were in at least their second year using *Core-Plus Mathematics* and 22 Washington schools using conventional high school curricula were matched on prior mathematics achievement, percent of students from low-income families, percent of underrepresented minorities, and student enrollment.

The pass rate in mathematics was higher in the schools using *Core-Plus Mathematics.* This was evident for both students from low-income families and those from other families.

College Readiness Indicators

ACT Test

Districts have monitored the ACT test performance of their students as they implemented CPMP. Below is one example showing increasing participation rates and increasing average mathematics ACT scores for a suburban district.



College Placement Test

- CPMP students performed as well as comparable students on a college placement test at a large midwestern university on basic algebra and advanced algebra subtests and performed significantly better on the calculus readiness portion of the test.

College Course Completions

- Results from a five-year longitudinal study showed that CPMP students completed first-year collegiate mathematics courses at about the same rate and with similar grades as all freshmen students with the same number of high school mathematics courses in two major research universities in two different states.
- The Minnesota Mathematics Achievement Project (MNMAP) researched the impact of curricula studied in high school (commercially developed, NSF-funded, or UCSMP) on the difficulty levels and grades of post-secondary students' mathematics courses. When taking into account student background factors, no differences across high school curricula with respect to university mathematics grades or difficulty levels across 8 semesters of college study were found.

AP Calculus and AP Statistics

Trend data supplied by CPMP districts consistently show increased enrollments in AP Calculus and AP Statistics and that their students complete AP Calculus and AP Statistics at a higher rate and with a greater percentage of high scores on the AP exams. (www.wmich.edu/cpmp/schoolreports.html)

Meta-Evaluations of Studies of 1st Edition *Core-Plus Mathematics*

To date, three meta-evaluations of the research on the efficacy of 1st edition *Core-Plus Mathematics* have been conducted, by the Strategic Ed Solutions for the Business-Higher Education Resources Forum, for the Best Evidence Encyclopedia at Johns Hopkins University, and by the What Works Clearinghouse under the U.S. Department of Education's Institute of Education Sciences.

Although each of the meta-evaluations used somewhat different criteria, the findings are all favorable to *Core-Plus Mathematics.* More detail on the meta-evaluations and web addresses for these studies is included in the Key Evaluation Findings available at: www.wmich.edu/cpmp/evaluation.html

Field-Test Evaluation of the 2nd Edition (2002–2009)

Core-Plus Mathematics 2nd edition students:

- at the end of Course 1, showed gains above the national norming group on the *Iowa Tests of Educational Development Ability to Do Quantitative Thinking* subtest.
- at the end of Course 2, outperformed multiple groups of algebra students involved in the 2004–2005 administration of the ETS *Algebra End-of-Course Assessment.*
- at the end of Course 3, outperformed the national and state-specific norming groups on the ACT preparation for college test.
- at the end of Course 4, outperformed comparison groups on basic skills, precalculus, and functions measures of college readiness.
- performed exceptionally well on the independently developed and research-based *PCA Functions Test*—a test related to understanding functions concepts. CPMP students outperformed comparable students on 21 of the 25 questions.
- outperformed *Core-Plus Mathematics* 1st edition students on measures of paper-and-pencil algebraic skills and geometry concepts and skills targeted for improvement in the revision work.

Implementing the Curriculum

Planning for Implementation

Careful consideration should be given to many issues as you study the *Core-Plus Mathematics* program and plan for an effective implementation of the program. You may wish to access resources related to improving mathematics education from organizations such as the National Council of Supervisors of Mathematics (NCSM) "Supporting Leaders in Mathematics Education: A Sourcebook of Essential Information" (ncsmonline.org/NCSMPublications/2000/sourcebook2000.html) and Mathematics Assessment Resource Service (MARS) "Resources for Leaders in Mathematics" (toolkitforchange.org).

Studying the CPMP Program

- Spend time developing goals for your mathematics program that align with various recommendations such as NCTM's *Principals and Standards for School Mathematics* (NCTM, 2000), NCTM's *Focus in High School Mathematics, the College Board Standards for College Success: Mathematics and Statistics* (2006), the American Diploma Project Standards, and your state mathematics recommendations. This work should involve representatives from all stake-holders in the district. All high school mathematics teachers should be involved at each step of the goal-setting process.
- Build understanding of your mathematics program goals and a support base from administrators, counselors, parents, board members, business/community leaders, other departments within your high school and middle school faculty.
- Form a curriculum study committee to align your district's mathematics program goals across grades K–12. One valuable resource is *Choosing a Standards-based Curriculum* from the Educational Development Center (EDC: www2.edc.org/mcc/pubs/mguide.asp).
- Research and study the *Core-Plus Mathematics* materials and instructional model as well as other programs. Consider reviewing mathematics programs with high-quality research on student achievement. Curriculum should be seen as a tool to move toward your district mathematics goals.

Advance Planning

- Assess district technology needs. A graphing calculator with statistics capabilities such as the TI-84 is required for each student. Some districts provide an at-home calculator for each student, as well as an in-class set for each mathematics teacher. The free public-domain software *CPMP-Tools* has been developed to support student learning and technology needs at home and at school. (See the *CPMP-Tools* overview on page 14.)
- Create an extended professional development plan that will provide ongoing support for high school, middle school, and special education teachers. Information on course-specific CPMP implementation workshops is available from Glencoe/McGraw-Hill and at www.wmich.edu/cpmp/ under Implementation. On-site professional development programs can be designed to meet district needs.
- Consider providing cooperative learning, technology, literacy, and alternative assessment workshop opportunities for mathematics teachers before they attend curriculum implementation workshops or begin teaching the curriculum.

- Begin adoption with Course 1 and add a course level each year. Encourage teachers to progress from Course 1 to Course 4 in stages, so they can develop an understanding of the growth of mathematical ideas across the curriculum.
- Schedule classes to allow for common planning periods for teachers teaching the same course.
- Formulate a plan to evaluate your mathematics program and the results of changes made. Plan to collect data over the long term, not just the year or two before and the year or two after the changes. (See examples of evaluations at www.wmich.edu/evaluation.html.)
- Produce a Frequently Asked Questions document containing your district's responses to local questions so that there are common responses from administrators, mathematics teachers, counselors, science teachers, and school office staff.
- Consider how district and individual teacher decisions can affect the amount of material taught each year. In particular, time should be spent aligning middle and high school mathematics programs to maximize student learning.
- Consider ways to align with your state mathematics standards and to polish skills using the practice sets and review tasks in each lesson and the practice sets available in the *Unit Resource Masters*.
- Provide opportunities for instructional leaders, particularly building principals, to understand the goals of the mathematics program and discuss ways to promote effective classroom instruction. One professional development option for teams from the school is the Lenses on Learning courses offered by the Educational Development Center. See www2.edc.org/CDT/cdt/cdt_lol1.html.

Acceleration

If your district has a history of enrolling strong eighth-grade students in an algebra course, you may wish to maintain an accelerated program using *Core-Plus Mathematics* Course 1 for select eighth-graders. These students could then enroll in AP Calculus as seniors upon completing Course 4 as juniors. Students can enroll in AP Statistics anytime after completion of Course 3.

Consider ways to schedule classes to allow for individual students to accelerate themselves. The following is a list of options that some districts implementing *Core-Plus Mathematics* have successfully used in cases in which accelerated students have not studied Course 1 as eighth-graders. Acceleration models employed by CPMP schools to allow enrollment in Advanced Placement Calculus or Statistics include the following.

Acceleration Models

	Model 1	Model 2	Model 3
8th Grade	Course 1		
9th Grade	Course 2	Courses 1–2	Courses 1–4
10th Grade	Course 3	Course 3	
11th Grade	Course 4 and AP Stat	Course 4 and AP Stat	
12th Grade	AP Calc	AP Calc	AP Calc and/or AP Stat

- Teach *Core-Plus Mathematics* Course 1 in middle school for selected eighth graders.
- Provide eighth-grade students enrolled in the grade level mathematics course additional mathematics material from *Core-Plus Mathematics* Course 1 that will allow them to enroll in Course 2 in ninth grade. This can be done during the academic year or in a summer program.
- Strong incoming ninth graders who have completed one of the NSF-funded middle school mathematics programs, or an algebra course, could be enrolled in Course 2. Prerequisite material from Course 1 can be distributed throughout the school year as needed for particular Course 2 units or taught in a summer course.
- In schools with semester block scheduling, a student could enroll in two courses in a given year.
- In schools with *alternate-day* academic-year block schedules, the schedule could be adjusted for one or more classes of a course to meet *each day* for the entire school year, thus covering two of Courses 1–4 that year.
- In schools with academic-year schedules, two mathematics classes may be scheduled back-to-back to allow study of one course in the first semester and the next course the second semester.
- A student could choose to double up on classes as a senior by enrolling in both Course 4 and AP Statistics.

Access, Equity, and Differentiation

One question frequently asked by districts adopting *Core-Plus Mathematics* is related to equity and approaches to accommodate the program for special-needs students.

Several research studies have provided evidence that introducing activities through class discussion, teaching students to explain and justify, and making real-world contexts accessible to students promote greater access and equity in mathematics classrooms. (Boaler, J. "Learning from Teaching: Exploring the Relationship Between Reform Curriculum and Equity," *Journal for Research in Mathematics Education,* 2002, Vol. 33, No. 4, 239–258, and Brown, C.A., Stein, M.K., and Forman, E. A. "Assisting Teachers and Students to Reform Their Mathematics Classroom," *Educational Studies in Mathematics,* 1996, 31–93). Practices that help promote equity are briefly discussed below.

Introducing Investigations Through Class Discussions

Group and class discussions of the aim of investigations, the meaning of contexts, the challenging points within problems, and possible problem access points to which students might turn make tasks more evenly accessible to all students.

Teaching Students to Explain and Justify their Thinking

Giving explicit attention to explaining thinking and evaluating what makes a good piece of work helps students improve their work.

Making Real-world Contexts Accessible

Considering the constraints that real situations involve and connecting these situations with issues and topics in their own lives helps students view mathematics as something that will help them interpret their world.

Other Practices that Promote Equity

Mixed-ability classes, a focus on problems solving, high expectations for all students, attention to a broad array of mathematical topics, and allowing students to restate problems in their own words also appear to help students from different racial, ethnic, and linguistic groups be more successful in mathematics.

Core-Plus Mathematics offers many opportunities for teachers to incorporate these practices into daily routines. One such built-in opportunity is the Think About This Situations (TATS) used to introduce lessons through discussions. Although no TATS questions are in the student text for individual investigations there are often suggestions in the *Teacher's Guide* for class launches of investigations. Since much of the mathematical content is based on real contexts, it is important that all students understand the contexts and draw on their own or a classmates background knowledge. Opportunities for students to explain and justify their thinking are built into all curriculum features. Look for opportunities to encourage the habit of mind of justifying their thinking, individually and in small group or class discussions.

DIFFERENTIATION Many students will already know the rule for quotients of powers. For those who do not, encourage the students to write the expressions in expanded form and then simplify.

In addition, in the *Teacher's Guide* periodically, notes provide specific ideas for differentiation. Look for **DIFFERENTIATION** margin notes and student masters.

It is important to recognize that implementing the *Core-Plus Mathematics* curriculum with classes consisting *only* of students previously unsuccessful in mathematics will create additional implementation challenges. These classes should not be expected to complete all units from a CPMP course.

Some schools provide special-needs students with a second hour of class devoted to support. This class typically follows the regular mathematics class. During this class time, students are assisted with their homework and sometimes pre-read material for the next day's class period. This support for special-needs students increases the access to the mathematics content during the regular class period.

As one special education teacher indicated:

> *"I have been teaching the CPMP in a resource setting for the past 2 years and have been teaching mathematics in the resource setting for over 8 years. I have to admit that initially I was adamant that my students (the majority have specific learning disabilities either with respect to math or reading) would be incapable of using this program. After 2 years, I have found that the opposite is true.*
>
> *My students are more engaged and achieving higher-level concepts because of CPMP. We do move at a slower pace, but they are learning the concepts of Algebra and Geometry thanks to the contextual component and the guided discovery approach. The program also provides ample opportunities to differentiate instruction. I am fortunate to work in a district where the staff received excellent training and we have access to technology. Overall, the students are doing well. I am looking forward to the training for Course 3.*
>
> *Core-Plus Mathematics is definitively a program that works well for students with learning disabilities."*

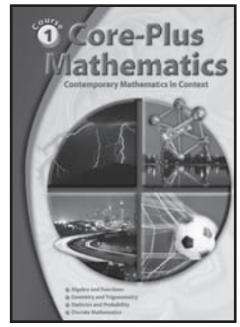
Program Components

The *Core-Plus Mathematics* four-year program consists of student materials and teacher-support materials. Familiarity with the components and their use is necessary for effectively implementing the program. The materials and their use are described in the following sections.

Student Components

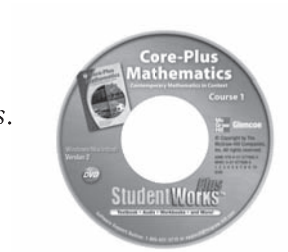
Student Texts

The student texts for each course consist of problem-based investigations through which students develop mathematical understanding and skills during class sessions and are the primary source of student homework tasks as described on page 13.



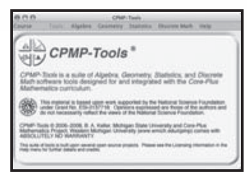
StudentWorks™ *Plus* CD

The *StudentWorks*™ *Plus* CD contains the full student text materials as well as a static version of *CPMP-Tools*.



CPMP-Tools®
Public-Domain Computer Software

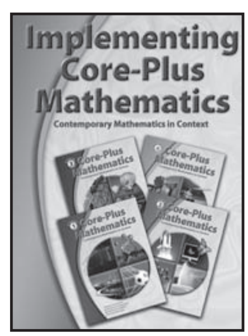
This software has been specifically designed to support student learning and problem solving in *Core-Plus Mathematics*. This software, in static versions, is located on the *StudentWorks* and *TeacherWorks* CDs. It is also available at www.wmich.edu/cpmp/CPMP-Tools/. Periodic updates of the software can also be found at that URL.



Teacher Components

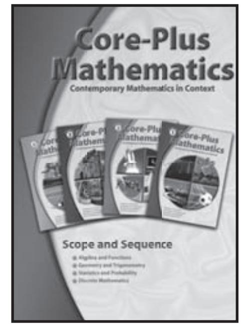
Implementing Core-Plus Mathematics

This *Implementing Core-Plus Mathematics* component is a valuable resource that teachers find useful throughout their teaching of the *Core-Plus Mathematics* program. However, it is especially appropriate to read this guide at the beginning of the implementation process. In addition to providing an overview of the curriculum, it provides helpful tips and ideas for instruction, assessment, program evaluation, and communication with parents.



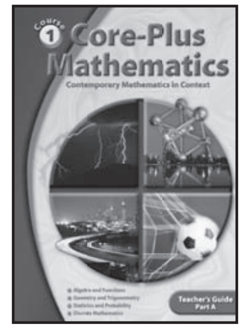
Scope and Sequence

The *Core-Plus Mathematics* curriculum includes topics found in traditional college-preparatory mathematics programs, as well as important contemporary topics from statistics, probability, and discrete mathematics. The *Scope and Sequence* booklet provides charts, organized by mathematical strand, that indicate the units in which topics appear in Courses 1–4. These charts also provide a convenient way of tracking topics in terms of initial development, growth, and connections.



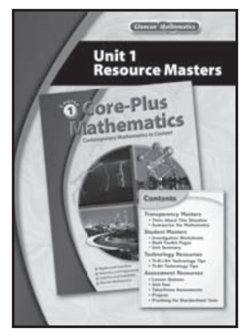
Teacher's Guides, Part A and Part B

The *Teacher's Guides* are the primary reference for the teacher containing full-size student pages with teacher material on the facing pages. For each unit, the *Teacher's Guides* contain overviews and summaries, background on the mathematical content, objectives, suggested timeline and materials needed, instructional notes and suggestions, suggested assignments for each homework (OYO) set, solutions for investigations and OYO tasks, Math Toolkit prompts, and sample class discourse scenarios. These features will be discussed in the next section of this guide, "Planning for Instruction."



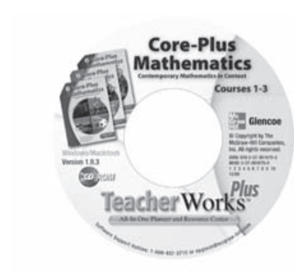
Unit Resource Masters

Each *Unit Resource Masters* book contains student activity masters for organizing work, technology tips, and practice tasks for standardized tests. Also included are masters for teachers to prepare transparencies for display on a white board to facilitate class discussions. Each unit booklet contains lesson quizzes, tests, take-home tasks, and projects.



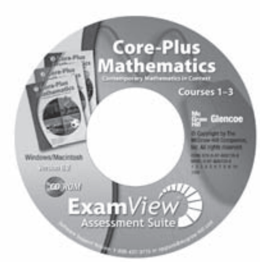
TeacherWorks™ *Plus* CD

Core-Plus Mathematics TeacherWorks™ *Plus* is easy-to-use software that combines the versatility of an Interactive Lesson Planner and an electronic *Teacher's Guide* with the convenience of the *Unit Resource Masters*, *Implementing Core-Plus Mathematics*, *Scope and Sequence*, and *CPMP-Tools* software on one CD.



ExamView Assessment CD

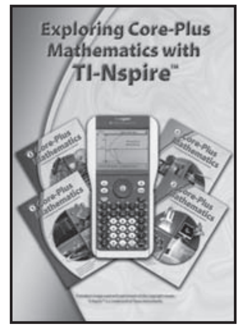
The *ExamView* Assessment CD contains the assessments from the *Unit Resource Masters*. These assessments can be customized to meet the needs of individual classes. The CD also allows teachers to develop their own assessments.



The items listed below are included in the *Unit Resource Masters* book for each unit.

- Teaching masters to facilitate TATS and STM discussions, to collect class data, to provide material for class discussion of selected problems
- Student activity masters to help organize results, record data, learn technology, summarize the main ideas of a unit, and practice for standardized tests
- Assessment masters for quizzes, tests, take-home tasks, and projects (Midterm and final assessment bank items are included in the *Unit Resource Masters* for Units 4 and 8.)

Exploring Core-Plus Mathematics with TI-Nspire™



For schools using the TI-Nspire™, a booklet is available at www.glencoe.com that provides advice on when and how to include features of this technology in teaching *Core-Plus Mathematics*. Keystroke-specific tips with expected screens are included.

Planning for Instruction

The *Core-Plus Mathematics* curriculum is not only changing what mathematics all students have the opportunity to learn, but also changing how that learning occurs and is assessed. Active learning is most effective when accompanied with active teaching. Just as the student texts are designed to actively engage students in doing mathematics, the teacher's resource materials are designed to support teachers in planning for instruction; in observing, listening, questioning, facilitating student work, and orchestrating classroom discussion; and in managing the classroom.

The *Teacher's Guides* provide suggestions, based on the experiences of 1st and 2nd edition field-test teachers, for implementing this exciting curriculum in the classroom. You may find some new ideas that can be overwhelming. The developers highly recommend that teachers who are teaching *Core-Plus Mathematics* for the first time do so at least in pairs who share a common planning period.

Each of the items listed below is included in the *Teacher's Guide* for each unit.

- Unit overview and lesson overviews
- Objectives, suggested timeline, materials needed, and suggested assignments
- Instructional notes and suggestions
- Solutions for investigations and OYO tasks
- Collaboration and Math Toolkit prompts
- Promoting Mathematical Discourse scenarios

The *Teacher's Guide* book included on the *TeachersWorks™ Plus* CD is a valuable resource for planning instruction. You will notice the small pictures in the margins of the *Teacher's Guide* that refer to the *Unit Resource Masters*. These discussion masters, activity masters, and assessment masters should be available for planning purposes and can be printed directly from the *TeacherWorks™ Plus* CD or copied from the *Unit Resource Masters* booklet.

Each unit of *Core-Plus Mathematics* includes either content that may be new to many teachers or new approaches to familiar content. Thus, a first step toward planning the teaching of a unit is to review the scope and sequence of the unit. This review provides an overall feel for the goals of the unit and how it holds together. The *Scope and Sequence* guide (available on the *TeacherWorks™ Plus* CD) shows how the specific mathematical topics fit in the complete four-year curriculum. Working through the student investigations, if possible with a colleague, provides help in thinking about and understanding mathematical ideas that may be unfamiliar.

In the *Teacher's Guides*, you will find teaching notes for each lesson, including instructional suggestions and sample student responses to investigations and OYO sets. Thinking about the range of possible responses and solutions to problems in a lesson proves to be very helpful in facilitating student work.

A summary chart at the beginning of the notes for each unit gives information about materials needed, time guidelines, and recommended assignments from the OYO sets. The Unit 1 Planning Guide for Course 1 is shown on the next page.

You will notice in the Planning Guide that the suggested time for the third lesson is approximately five 50-minute instructional periods. This may vary depending on the background and make-up of the class. Remember: *Developing deep understanding is more important than just "completing activities."*

Since acquiring the physical materials requires advance planning, specific materials for each lesson are listed in the Planning Guide. Although it is not stated in the Planning Guide, it is assumed that students have access to graphing calculators at all times for in-class work.

The developers recommend that the homework (OYO) assignment *not* be held off until the end of the lesson or the investigation just preceding the OYO set. Some teachers choose to post the OYO assignment at the beginning of a lesson along with the due date, usually a day or two following the planned completion of the lesson. Other teachers prefer to assign selected OYO tasks at appropriate times during the course of the multiday lesson and then assign the remaining tasks toward the end of the lesson. Note that all recommended assignments include provision for student choice of some tasks. This is but one of many ways in which this curriculum is designed to accommodate and support differences in students' interests and performance levels.

It is strongly recommended that student solutions to Connections tasks be discussed in class. These tasks help students organize and formalize the mathematics developed in context and connect it to other mathematics they have studied. Structuring the underlying mathematics and building connections are best accomplished by comparing and discussing student work and synthesizing key ideas within the classroom.

Unit 1 Planning Guide

Lesson Objectives	On Your Own Assignments*	Suggested Pacing	Materials
Lesson 1 ***Cause and Effect*** • Develop disposition to look for cause-and-effect relationships between variables • Review and develop skills in organizing data in tables and graphs and using words to describe patterns of change shown in those representations • Review or begin to develop knowledge about common patterns of change (linear, inverse, exponential, quadratic) and ability to use symbolic rules to represent and reason about those patterns • Use tables, graphs, and rules to solve problems of cause-and-effect change	**After Investigation 1:** A1–A3, C9, R14, R15, Rv21, Rv22 **After Investigation 2:** A4, A5, C10, Rv23–Rv26 **After Investigation 3:** A6 or A7, C11–C13, R15, R16 or R17, choose one of E18–E20, Rv27, Rv28	7 days	For each group of students: • Rubber bands and fishing weights, bags of nuts and bolts, or other weights • Meter sticks • Dice • Three different coins • Unit 1 Resource Masters
Lesson 2 ***Change Over Time*** • Develop ability to recognize recursive patterns of change • Develop ability to use calculators to iterate stages in a recursive pattern • Develop ability to write *NOW-NEXT* rules to represent recursive patterns • Develop ability to write and use spreadsheet formulas to explore recursive patterns of change (optional investigation) • Use iteration to solve problems about population and money change over time	**After Investigation 1:** See Assignment Note below. Choose one of A1–A4, A5 or A6, C10, C11, C14, C15, R19, E22 or E23, Rv26–Rv29 **After Investigation 2:** A7, A8 or A9, C12 or C13, R20 or R21, E24 or E25, Rv30, Rv31	6 days	• Access to computers with spreadsheet software, *CPMP-Tools*, or calculators with spreadsheet capabilities • ***Optional:*** *CPMP-Tools* data analysis software for C10–12, and E23 • Unit 1 Resource Masters
Lesson 3 ***Tools for Studying Patterns of Change*** • Develop skill in writing rules that express problem conditions • Review perimeter and area formulas for triangles, parallelograms, and circles, and the Pythagorean Theorem • Develop skill in producing tables and graphs for functions • Develop skill in using function tables, graphs, and computer algebra manipulations to solve problems that involve functional relationships, especially solving equations in one variable • Develop informal knowledge about connections among function rules, tables, and graphs for linear, inverse, exponential, and quadratic relations	**After Investigation 1:** Choose three of A1–A5, C13, choose two of C14–C18, E25, Rv31 **After Investigation 2:** A6 or A7, A8, A9, C19, C20, R21–R23, choose one of E26–E29, Rv32 **After Investigation 3:** A10–A12, R24, E30, Rv33–Rv36	5 days	• Access to a Computer Algebra System such as in *CPMP-Tools* • Unit 1 Resource Masters
Lesson 4 ***Looking Back*** • Review and synthesize the major objectives of the unit		2 days (including testing)	• Unit 1 Resource Masters

** When choice is indicated, it is important to leave the choice to the student.*

Note: *It is best if Connections tasks are discussed as a whole class after they have been assigned as homework.*

Orchestrating Lessons

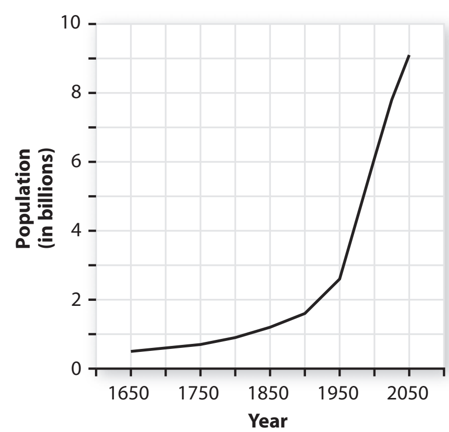
Core-Plus Mathematics materials are designed to engage students actively in a four-phase cycle of classroom activities. (See “Instructional Model” on page 12 of this guide.) The activities often require both students and teachers to assume roles quite different than those in more traditional mathematics classrooms. Becoming accustomed to these new roles usually takes time, but *Core-Plus Mathematics* teachers report that the time and effort required are worthwhile in terms of both student learning and professional fulfillment. Although realistic problem solving and investigative work by students is the heart of the curriculum, how teachers orchestrate the launching of an investigation and the sharing and summarizing of results is critical to successful implementation. Lessons should be introduced by asking students to think about a situation such as the one reproduced below, which is used to launch Lesson 2 of the *Patterns of Change* unit in Course 1.

LESSON 2 *Change Over Time*

Every 10 years, the U.S. Census Bureau counts every American citizen and permanent resident. The 2000 census reported the U.S. population to be 281 million, with growth at a rate of about 1% each year. The world population is over 6 billion and growing at a rate that will cause it to exceed 9 billion by the year 2050.

National, state, and local governments and international agencies provide many services to people across our country and around the world. To match resources to needs, it is important to have accurate population counts more often than once every 10 years. However, complete and accurate census counts are very expensive.

World Population 1650–2050



Source: www.census.gov/ipc/www/world.html

Think About This Situation

The population of the world and of individual countries, states, and cities changes over time.

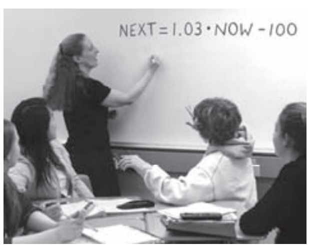
a How would you describe the pattern of change in world population from 1650 to 2050?

b What do you think are some of the major factors that influence population change of a city, a region, or a country?

c How could governments estimate year-to-year population changes without making a complete census?

Students enter the classroom with differing backgrounds, experience, and knowledge. These differences can be viewed as assets. Engaging the class in a free-flowing, give-and-take discussion of how students think about the launch situations serves to connect lessons with the informal understandings of data, shape, change, and chance that students bring to the classroom. Try to maximize the participation of students in these discussions by emphasizing that their ideas and possible approaches are valued and important and that definitive answers are not necessarily expected for all launch questions.

Once launched, a lesson may involve students working together collaboratively in small groups for a period of days, punctuated occasionally by brief, whole-class discussions of questions students have raised. In this setting, the lesson becomes driven primarily by the instructional materials themselves. Rather than orchestrating class discussion, the teacher shifts to circulating among the groups and observing, listening, and interacting with students by asking guiding or probing questions. These small-group investigations lead to (re)invention of important mathematics that makes sense to students. Sharing and agreeing as a class on the mathematical ideas that groups are developing is the purpose of the Summarize the Mathematics (STM) in the instructional materials.



As a general rule, students first should decide on responses to STM questions as a group. The sample STM shown below is the first of two in Lesson 2 of the *Patterns of Change* unit in Course 1.

Summarize the Mathematics class discussions are orchestrated somewhat differently than during the launch of a lesson. At this stage, mathematical ideas and methods still may be under development and may vary for individual groups. So, class discussion should involve groups comparing their methods and results, analyzing their work, and arriving at conclusions agreed upon by the class.

Summarize the Mathematics

In the studies of human and whale populations, you made estimates for several years based on growth trends from the past.

a What trend data and calculations were required to make these estimates:

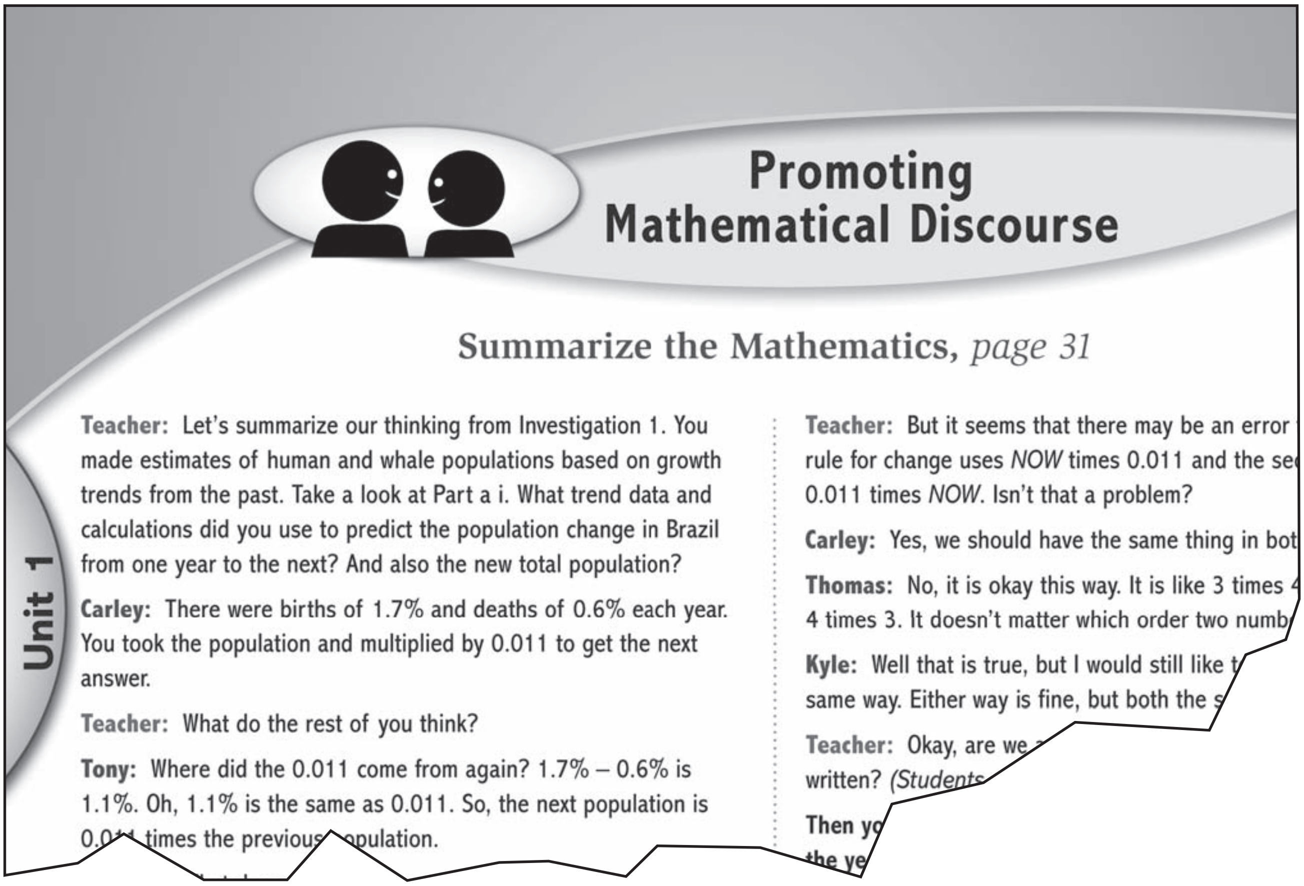
- **i.** The change in the population of Brazil from one year to the next? The new total population of that country?
- **ii.** The change in number of Alaskan bowhead whales from one year to the next? The new total whale population?

b What does a *NOW-NEXT* rule like $NEXT = 1.03 \cdot NOW - 100$ tell about patterns of change in a variable over time?

c What calculator commands can be used to make population predictions for many years in the future? How do those commands implement *NOW-NEXT* rules?

Be prepared to share your thinking with the class.

Sample discourse scenarios are available in the *Teacher's Guides*. These sample discussions, entitled Promoting Mathematical Discourse (PMD), offer possible teacher-student discourse around selected Summarize the Mathematics and Think About This Situation questions. Teachers may choose to review the PMDs with a colleague and record additional questions they might choose to ask during the class discussion. Following the class discussion, some teachers make notes on a copy of the discourse scenario to inform preparation for the following year.



Unit 1

Promoting Mathematical Discourse

Summarize the Mathematics, *page 31*

Teacher: Let's summarize our thinking from Investigation 1. You made estimates of human and whale populations based on growth trends from the past. Take a look at Part a i. What trend data and calculations did you use to predict the population change in Brazil from one year to the next? And also the new total population?

Carley: There were births of 1.7% and deaths of 0.6% each year. You took the population and multiplied by 0.011 to get the next answer.

Teacher: What do the rest of you think?

Tony: Where did the 0.011 come from again? 1.7% – 0.6% is 1.1%. Oh, 1.1% is the same as 0.011. So, the next population is 0.0[…] times the previous [p]opulation.

Teacher: But it seems that there may be an error […] rule for change uses *NOW* times 0.011 and the se[…] 0.011 times *NOW*. Isn't that a problem?

Carley: Yes, we should have the same thing in bot[…]

Thomas: No, it is okay this way. It is like 3 times 4 […] 4 times 3. It doesn't matter which order two numb[…]

Kyle: Well that is true, but I would still like t[…] same way. Either way is fine, but both the s[…]

Teacher: Okay, are we […] written? *(Students* […]

Then yo[…] **the ye**[…]

Check Your Understanding tasks immediately follow the STM. As you plan, consider whether this task should be done in class or as a homework task. If you judge that students will need your assistance or peer assistance, you might ask them to begin the task as an individual task. Similar as to what is sometimes done in Japanese classes, you might provide hints as needed for students to continue on the task. (See the TIMSS 1999 Video Study.)

For each part of the Summarize the Mathematics, different groups should be asked to share their responses and thinking before proceeding to the next part. Facilitate resolution of any differences as a step toward building agreement concerning the class's mathematical discoveries.

The investigations deepen students' understanding of mathematical ideas and extend their mathematical language in contexts. Technical terminology and symbolism are introduced as needed in the investigation. This sometimes occurs in the STM discussions. These discussions provide the agreement needed to inform student notes (called Math Toolkits).

Check Your Understanding

The 2000 United States Census reported a national population of about 281 million, with a birth rate of 1.4%, a death rate of 0.9%, and net migration of about 1.1 million people per year. The net migration of 1.1 million people is a result of about 1.3 million immigrants entering and about 0.2 million emigrants leaving each year.

a. Use the given data to estimate the U.S. population for years 2001, 2005, 2010, 2015, 2020.

b. Use the words *NOW* and *NEXT* to write a rule that shows how to use the U.S. population in one year to estimate the population in the next year.

c. Write calculator commands that automate calculations required by your rule in Part b to get the U.S. population estimates.

d. Modify the rule in Part b and the calculator procedure in Part c to estimate U.S. population for 2015 in case:

- **i.** The net migration rate increased to 1.5 million per year.
- **ii.** The net migration rate changed to −1.0 million people per year. That is, if the number of emigrants (people leaving the country) exceeded the number of immigrants (people entering the country) by 1 million per year.

Summarizing and Polishing Notes

Helping students develop skills in writing complete and concise solutions is one of the goals of this curriculum. However, always writing thorough responses can unnecessarily slow student progress through the investigations. As a guideline, we suggest that during investigations, students should make notes of their thinking and discussion of ideas rather than use complete sentences. Investigation time can be thought of as draft work or getting ideas out for discussion. For investigation problems that ask students to explain reasoning or to compare, you may want to require complete-sentence responses. Student responses to the Summarize the Mathematics and Math Toolkit entries should be more complete. If these responses are written following the class summary of important mathematical ideas, students will be able to write more thorough responses. Homework tasks from the On Your Own sets should also be thoroughly written.

Special Needs and English Language Learners

In *Core-Plus Mathematics*, students are expected to learn mathematics by reading, thinking, discussing, and writing. Teachers will always need to make some accommodations for special needs students. Based on their work with special needs students learning mathematics, the Educational Development Center (EDC) has developed the following list of strategies to guide curriculum developers interested in access for a wide range of student abilities.

Strategies that are helpful for making math more accessible for students with special needs and English language learners:

- use engaging and meaningful contexts.
- use multiple representations.
- sequence instruction to move from concrete to representational to abstract.
- offer manipulatives.
- provide examples and nonexamples.
- offer templates and graphic organizers.
- use modeling.
- use cooperative group work.
- teach metacognitive and problem-solving strategies.
- provide opportunities for students to build on their prior knowledge and experiences.
- provide opportunities for students to use their own language.
- immerse students in the language of mathematics.
- provide opportunities for guided and independent practice.
- use frequent assessments.
- provide timely and constructive feedback.
- have students create their own resources.
- use organizational systems for notebooks/binders.
- reduce amount of copying for students.
- adjust time for tasks and pacing.
- adjust amount of work.
- help students to become independent learners.

Core-Plus Mathematics field-test materials were reviewed by EDC through an accessibility lens in order to identify strengths and potential barriers for students with special needs. EDC found that many of the above strategies were already an integral part of the materials.

The full report from EDC provided many valuable suggestions that were incorporated into the published curriculum. Student materials are uncluttered and employ contextual visuals, content visuals, and graphs. Teachers will find suggestions in the *Teacher's Guide* for differentiation and masters in each unit resource booklet to help students organize their work and reduce the copying expectations.

In particular, the following features of the curriculum improve access for all students.

Focus Questions

Focus questions at the beginning of each investigation provide students with the goal(s) of the investigation. By having this identified at the beginning of the investigation, students see the underlying mathematical question(s) of the investigation.

Math Toolkit

Having a math toolkit is an excellent accessibility strategy. It is important for special needs students to create their own resources that they can use. The toolkit could be particularly helpful for students with memory difficulties. It has the added benefit of helping students to become more independent.

Technology

The use of graphing calculators and computer software is beneficial for many special needs students. Using technology allows students to make more extensive use of multiple representations without needing to construct them by hand. The availability of *CPMP-Tools* outside of class allows some students additional time to become proficient with the software and consider mathematical ideas supported by technology. Technology Tips are available in the *Unit Resource Masters* to assist students who have difficulty retaining methods.

Assessment Strategies

Ways to differentiate assessment include the following.

- Use electronic versions of the assessments and provide questions or hints for selected students.
- Provide dual-language exams where possible, or
- Offer alternatives, e.g., student could draw a diagram instead of writing an explanation.
- Provide additional time to complete assessments.
- Use many strategies suggested for assessment of Exceptional Education students such as:
    - larger font size.
    - avoid visual crowding.
    - include an organized, adequate work area.
    - scaffold the meaning of the question by including the T of a required T-chart or providing a graph grid.
    - avoiding idioms such as "best buy."
    - discretionary use of a highlighter on the ELL students' tests.

General Implementation Suggestions

As indicated in other sections of this guide, schools have found it beneficial to include special education teachers in curriculum-specific workshops. In some schools, special education teachers co-teach with mathematics teachers. To support students, some schools provide a support period following the mathematics class. During this period, students start or complete homework tasks and may preview material that will be considered at the next class period.

Resources Recommended by *Core-Plus Mathematics* Teachers

Barton, M.L. & Heidema, C. (2000) *Teaching Reading in Mathematics, 2nd Edition*, Mid-continent Research for Education and Learning (McREL).

Billmeyer, R. & Barton, M.L. (1998) *Teaching Reading in the Content Areas: If Not Me, Then Who?*, Mid-continent Research for Education and Learning (McREL).

Brenner, M.E. (1998). Development of mathematical communication in problem solving groups by language minority students. *Bilingual Research Journal* 22(2–4), 149–74.

Brenner, M., & Moschkovich, J. (2002). Everyday and academic mathematics: Implications for the classroom. Monograph Number 11 in the series published by the *Journal for Research in Mathematics Education*.

Herrell, A. & Jordan, M. *Fifty Strategies for Teaching English Language Learners*, Second Edition, Pearson Publishing.

Khisty, L.L. (1977). Making mathematics accessible to Latino students: Rethinking instructional practice. In J. Trentacosta and M. Kenney (Eds), *Multicultural and Gender Equity in the Mathematics Classroom: The Gift of Diversity, 97th Yearbook of the National Council of Teachers of Mathematics*. Reston, VA: NCTM.

Khisty, L.L. and Chval, K. (2002). Pedagogic discourse and equity in mathematics: When teachers' talk matters. *Mathematics Education Research Journal*, 14(3), 154–168.

Managing Classroom Activities

Teachers who have been teaching the *Core-Plus Mathematics* curriculum have found a variety of ways to manage classroom activities. Initially, the three biggest challenges are pacing instruction in a student-centered classroom, facilitating collaborative group work, and organizing, monitoring, and assessing student work. The following section on pacing offers a number of suggestions for meeting the first challenge. The second challenge is addressed in the section "Active Learning and Collaborative Work." The third challenge is the focus of the sections "Assessment" (pages 54–66 of this guide) and "Tips on Student Work and Writing" (pages 66–67).

Pacing Considerations

Teachers at many schools that have adopted *Core-Plus Mathematics* have indicated that they recognize that it is important to allow students time to think about, explain their reasoning on, and write mathematics. Students need to have time to develop deep understandings of important mathematics. Professional judgment is called upon daily to decide how much time to allow for student work and when to insert whole-class discussions and how much time to allow for these discussions.

District decisions such as access to technology for homework for each student, the number of minutes of mathematics classes and alignment of mathematics curricula K–12 affect the amount of material one can reasonably expect to complete in an academic year. In some cases, where middle school mathematics programs are not aligned with *Core-Plus Mathematics* or classroom hours for mathematics instruction have been reduced (such as in some block scheduling formats), completing one course level in a single year may not be an appropriate goal.

However, there are many decisions made by teachers each day that also affect the amount of material that students can complete in a year. Teachers who have been implementing the *Core-Plus Mathematics* program have found some successful strategies for addressing pacing challenges.

- In Course 1, assess students' mathematical knowledge from their middle school program to avoid repetition. Unit 1 *Patterns of Change* has been written to assist with this assessment process. Some investigations may be omitted or treated lightly depending on the background of your students.
- As a department, plan a schedule for completing units for the year. Collaborate so that all teachers of the same course continue at about the same pace.
- Know the objectives for the lesson so that you do not get sidetracked.
- Students need not write complete answers to every problem in an investigation. Solutions may be considered as records of current thinking for discussion. Complete write-ups should be made at the STM in students' Math Toolkits, and for homework tasks.
- When assigning an investigation, give time limits, for example, "You have 12 minutes to do Problems 1–5" or "Two more minutes until the STM discussion."

- Selectively facilitate whole class mini-summaries before the main STM to consolidate the learning and allow students to move efficiently through the remainder of the investigation. (This may also help bring a lagging group up to speed.)
- Occasionally, an investigation problem can be assigned for individual student consideration as homework. This problem should not be one that is so difficult that it will defeat the purpose of getting started on the item. Students should expect that they will have an opportunity to discuss their progress with others during the next class session. (This may also be a way to help bring a lagging group up to speed.)
- Check Your Understanding tasks could be assigned as homework.
- Resist the temptation to go over all the assigned OYO tasks in class. Reserve class time for the important Connections tasks.
- The curriculum builds on mathematics developed each year. Important topics will be practiced in review sets and revisited as students progress through the curriculum. Therefore, it is reasonable to expect mastery from different students at different times.

Teaching each of the four courses in the Core-Plus Mathematics curriculum will help you better understand the development of mathematical concepts and methods, student retention of mathematical ideas across courses, and students' deepening understanding of mathematics. This, in turn, will give you the confidence to make daily specific teaching decisions that affect pacing.

Active Learning and Collaborative Work

The *Core-Plus Mathematics* curriculum materials are designed to promote active, collaborative learning. Creating a classroom atmosphere conducive to effective collaborative learning requires the following: an understanding of the basic philosophy behind collaborative learning; familiarity with the factors in the composition and selection of groups; and practice in classroom techniques for managing collaborative learning. The following subsections are a summary of, and a ready reference to, these aspects of the collaborative learning model. If you choose to skim through some of this material, you may wish to read the conclusion on page 52.

Philosophy Behind Collaborative Learning

Core-Plus Mathematics deliberately incorporates collaborative learning for two reasons. First, a collaborative environment fosters students' ability to make sense of mathematics, to reason mathematically, and to develop deep mathematical understandings. Collaborative learning is an effective method for engaging all the students in the learning process, particularly students who have been underrepresented in mathematics classes. Second, practice in collaborative learning in the classroom is practice for real life; students develop and exercise many of the same skills in the classroom that they need in their lives at home, in the community, and in the workplace.

Value of Individuals

Perhaps the most fundamental belief underlying the use of collaborative learning is that every student is viewed as a valuable resource and contributor. In other words, every student participates in group work and is given the opportunity and time to voice ideas and opinions. Implementing this concept is not easy. It does not happen automatically. In order to set a tone that will promote respect for individuals and their contributions, classroom rules should be established and agreed upon by the learning community. Students should be included in the process of formulating these rules. Helpful rules might include the following.

- Refer to people by the names they prefer (provided, of course, that the names are appropriate and not offensive); there will be no name calling.
- Speak respectfully; for example, do not say, “Shut up” or “Be quiet.”
- Acknowledge that everyone’s ideas have value; listen to others, even if you disagree with them.
- Direct any criticism at ideas, not at people.
- Be willing to change your mind on the basis of mathematical or statistical reasoning.
- Include eveyone. Do not leave anyone out on purpose or by chance. If you are placed in groups with others whom you do not like, do not exclude them from the group discussion or from making decisions.
- Collaborate rather than compete. Group work is designed for cooperation, and one group or individual is not in competition for answers or for time with others in the classroom. As a last resort, agree to disagree.

You should initiate a discussion of group rules and then post them in the classroom. You should also model all of the rules correctly to show that “we” begins with “me.” Those who do not adhere to the rules must accept the consequences in accordance with classroom or school disciplinary procedures.

Importance of Social Connections

Even in classrooms in which the rules for showing respect have been clearly established, experience has shown that students still cannot talk with one another about mathematics (or social studies, or literature, or any other subject) if they do not first have positive social connections.

One way to develop this kind of common base is through team-building activities. These short activities may be used at the beginning of the year to help students get acquainted with the whole class and may be used during the year whenever new groups are formed to help groupmates know one another better.

In one such activity, called “Whip,” the members of each group give quick answers to a statement given by the teacher, such as:

“My favorite pastime is __________.”

“My favorite vacation memory is __________.”

“Something new and positive in my life is __________.”

“A hobby or sport I like is __________.”

These kinds of quick activities help students learn new and positive things about classmates with whom they may have attended classes for years but have not known well. The time taken for these quick team builders pays off later in helping students feel comfortable enough to work with the members of their group. Additional resources on collaborative learning are cited on page 52.

Need for Teaching Social Skills

Experience has also shown that social skills are critical to the successful functioning of any small group. Because there is no guarantee that students of any particular age will have the social skills necessary for effective group work, it is often necessary to teach these skills in order to build a collaborative learning environment.

These social skills are specific skills, not general goals. Examples of specific social skills that you can teach in the classroom include responding to ideas respectfully, keeping track of time, disagreeing in an agreeable way, involving everyone, and following directions. Though goals such as cooperating and listening are important, they are too general to teach and practice.

COLLABORATION SKILL
Include every group member in discussions.

PROCESSING PROMPT
I made sure
____
(person)
was included in our discussions by... .

One method of teaching social skills is to begin by selecting a specific skill and then having the class brainstorm to develop a script for practicing that skill. Next, the students practice that skill during their group work. Finally, in what is called the processing, the students discuss within their groups how well they performed the assigned social skill. Effective teaching of social skills requires practicing and processing; merely describing a specific social skill is not enough. Actual practice and processing are necessary for students to learn the skill and to increase the use of appropriate behaviors during group work and other times during class. The *Teacher's Guide* includes some suggestions for collaboration skills to practice. Look for prompts such as those at the left.

Composition, Selection, and Management of Groups

One of the premises of collaborative learning is that by developing the appropriate skills through practice, anyone in the class can learn to work in a group with anyone else. Learning to work in groups is a continuous process, however, and the process can be helped by decisions that you make with regard to the size, composition, method of selection, student reaction to, and duration of groups. Attention to dealing effectively with student absences also is important.

Group Size

Any group must have at least two students and the largest manageable groups have five. Groups of different sizes may be appropriate for different purposes. There are several factors to consider in determining group size.

One factor is available time alotted for the work. The larger the group, the more time it takes to include everyone's ideas and to reach consensus. The smaller the group, the less time it takes to complete the work and to include everyone, however, fewer ideas may be offered for consideration.

A related factor in determining group size is the level of social skills exhibited by members of the group. Larger groups require higher levels of patience, better listening skills, and the ability to accept different perspectives, while smaller groups can succeed with fewer social skills and lower levels of skills in the group.

Another factor is the willingness of students to engage in group activities. If the class contains many students who tend to hang back and avoid involvement, the best group size is two. It is hard to be left out in a group of two, and students must be more accountable when working in pairs.

Finally, if the group task involves complex problem solving or the need to generate a variety of ideas, then larger groups may be best. Students are ready for groups of four or even five if they are willing and able to take the time and if they have the skills to include everyone. One transitional alternative is to have students work in pairs initially and then have each pair share with another pair. If grouping is by 3s and there are extra persons, then one or more groups can be increased to 4.

Heterogeneity

The strongest groups contain students who are different from one another in gender, ethnicity, skills and abilities, personalities, socio-economic background, previous school experiences, and so forth. When a heterogeneous group of students work together, they bring different ideas, experiences, and points of view to the group. These differences enrich and strengthen the group discussion.

Methods of Selection

Given the above principles for forming groups, there are at least three methods of actually determining which students will be in which groups.

One way to form groups is for you to determine the groupings: You pick individual students to work with other individual students, based on considerations of group size and heterogeneity. However, you should not attempt to create the perfect group by trying to blend all of the criteria at one time; rather, you should base the groupings on the tasks at hand and on the characteristics of the class. For example, if the small groups require reading skills, individuals should be grouped so that there is a range of reading skills available in each group; or, if members of a class of ninth-grade students tend to sit and talk only with students from their own middle school, then the groupings should mix students from different feeder schools.

Another method for determining groups is random selection. While you will want to use a variety of methods during the year, one method is to simply write each student's name on a slip of paper or a Popsicle stick, put the items in a container, and then draw out the number of names corresponding to the size of the group predetermined for a particular investigation or lesson.

When a random method is used, both teacher and students must be ready for new and possibly untried combinations of students. Students often like random selection best because it seems fair, while teacher selection appears to be "fixed."

A third method that generally is not recommended is letting students select the groups themselves. When students pick their own partners, they tend to pick their friends, or people who are like themselves, not different. Some students may not be picked at all. Student selection therefore can violate the basic premises of collaborative learning.

Student selection, however, may be appropriate within limits. For example, you might say, “You may pick your groups today. Find three other students (for a total of four), include males and females, and make sure that you are in a group with students with whom you do not sit at lunch or hang out with after school. No one may be left out.” If students can follow these guidelines (or others that you choose), then student-selected groups may be used on occasion.

Student Reaction to Group Composition

No matter which method is used to select the members of each group, the students will have mixed reactions to the groupings. You should start very early to set limits on the acceptable reactions by discussing with the students what they may and may not say when they find out who is in their group. Some possible guidelines include the following.

- Groups will be changed throughout the semester and the year, but they will not be changed until any present problems are resolved.
- You must treat groupmates with respect.

Duration of Groups

Because different groups will serve different purposes, the length of time that groups stay together will vary. For example, pairs of students who sit next to each other might check homework at the beginning of the class period for three to five minutes. Two pairs might combine to create a four-person group for investigations. Work groups for projects or investigations might remain the same for each unit or even throughout a grading period. The goal is to have the students work with many different combinations of classmates during the school year.

Dealing with Student Absences

“My partner isn’t here today? Who do I meet with?” “He has the group paper, but he’s gone today!” Whatever the composition and duration of the groups, absentees can pose problems but they are manageable ones. (Some studies have even indicated that students increase their attendance in classes that use collaborative learning.) When a missing group member has materials the group needs, it is difficult for the group to continue its work. When students are absent, they miss the content and the practice provided by group work. Here are some suggestions for dealing with problems related to student absences.

- For long-term groups (two to nine weeks), use groups of four students. With groups of four, it is very likely that there will be at least two or three students in the group on a given day, so the group is still a group, and you can avoid the awkward and time-consuming task of moving a lone person to a new group.
- Do not carry over completed activities into additional class periods to provide catch-up time for students who were absent.
- Encourage each group to be responsible for helping absent members make up missed work by giving them an update by phone before class or immediately upon their return to class.

- For short-term investigations that cover two or more class periods, decide the placement of a returning student on a case-by-case basis. If a student has missed one day of a three-day investigation, the student may be placed in a group. If a student has missed two days of a three-day investigation, the student could be placed in a group for the class period and then be asked to do a shortened investigation addressing the same content, either individually or with the assistance of a classmate, following the class period.
- When a student returning to class does not have an assigned group, place the student in a group. Pick the best group for the student, that is, pick a group that is friendly, patient, and likely to include the student as a working member.
- When a group is preparing one report of their work, do not allow the report to leave the room while still in process. For any continuing activity, have group folders or a group basket ready to retain student work. The next day, the group has its work, regardless of student attendance.

Classroom Techniques for Effective Collaborative Learning

While productive and congenial collaborative learning depends to a significant degree upon student attitudes and skills and upon the composition of groups, much of the success of group learning depends upon the teacher's management of the classroom. Key areas for ensuring effective group work include:

(1) giving clear directions,

(2) establishing a materials center,

(3) setting up the work area for face-to-face interaction,

(4) establishing classroom routines,

(5) using a variety of techniques to develop positive interdependence,

(6) establishing individual accountability, and

(7) handling conflict effectively.

Giving Clear Directions

Explicit, well-organized directions are essential for effective group work. Because of the variety of reading levels and differences in learning styles that may exist within groups, it is helpful to have the directions be visible to students. Reading those directions aloud and checking for understanding before students move into groups helps auditory learners. Keeping the directions displayed during the investigation helps visual learners. If asked a question that is answered in the directions, respond by simply pointing to the display. Some experts suggest that you should always stand in the same spot in the room when giving directions.

Clear directions for problems in *Core-Plus Mathematics* are often provided in the student text; however, if you create or revise directions, care should be taken that those directions are clear. Time spent providing clear directions pays off later.

Establishing a Materials Center

Another way to help maximize efficiency in a collaborative learning environment is to establish a materials center, a single location where designated students can get all supplies and handouts for their groups and store their group work. You should place the supplies there before class. Students should get in the routine of getting all materials from, and returning them to, the materials center. This approach saves time and encourages both you and the students to get and remain organized.

Setting Up for Face-to-Face Interaction

The physical location of students in the room affects how they respond to the content of the lesson, to the teacher, and to one another. For group work, students need to be "eye-to-eye" and "knee-to-knee." In other words, students in the same group should face their groupmates and face away from the teacher and other groups. A minimum of furniture is best. Less table surface is needed when students share materials than when all students have their own materials. (See the discussion on "limited" and "jigsawed" materials on page 48 of this guide.) With shared materials, three or four chairs can be clustered around a single desk or at the end of a rectangular table. In any group, students should be close together so that everyone can see easily and can hear without talking loudly.

Establishing Classroom Routines

Establishing routines for getting into groups, moving furniture, and having the class become quiet all help to facilitate cooperative learning. The Dishon/Wilson O'Leary model recommends the use of the "3 Rs" to teach a routine: *reveal*, *rehearse*, and *reinforce* (Dishon and Wilson O'Leary, 1994). A "Quiet Signal" is one important routine for a collaborative classroom.

1. *Reveal:* Name the routine, explain why it is needed, tell how it works, and demonstrate the behavior.

    "The routine you will learn today is called the Quiet Signal. It is necessary to have a Quiet Signal because you will be busy working in groups and at some point I will need your attention. I would like to do this in a quiet way so that you can quickly finish your sentence and give me your attention. I will simply raise my hand. If you are not talking and see my hand, continue listening to whomever is speaking and then raise your hand. If you are speaking, finish your sentence and then raise your hand to acknowledge the signal." At this point, you raise a hand.

2. *Rehearse:* Students practice the procedure while you observe. The students should practice several times under different conditions.

    "Now I would like to have you practice this routine. In a moment, I want you to turn to your neighbor and tell about something you like to do on the weekend. Then listen while your neighbor does the same. You will have two minutes." Move quietly around the room listening while students talk. At the end of two minutes, stand quietly at the front of the room and raise one hand. Students' hands go up slowly at first and then more quickly as more students see the signal. "Thank you. That worked quite well. Remember to finish your sentence before you raise your hand. This time, let's practice with several people walking around the room, some people talking quietly, and the rest working at your desks." This time, let students' movement continue for 30 seconds or so and then raise one hand. "I noticed people seeing my hand and raising one of theirs even when they were out of their seats. That was just fine!"

3. *Reinforce:* It is important to practice, give feedback, review, and reteach if necessary until students use the routine consistently in appropriate ways. Also, on some occasions after using the routine, be sure to ask students for their perception of how it is working.

   "I want to ask you about how the Quiet Signal is working. Did the person who was speaking when the Quiet Signal was noticed finish his or her sentence?" "What can you do so that the person talking finishes the sentence and doesn't start another one?" Students offer possibilities. The discussion continues and another brief practice session is conducted.

One caution: Be sure that you are using the routine as it was taught. Teachers who have problems with the Quiet Signal often are talking while they have their hand up or are walking around reminding people to be quiet. Teach the routine by standing quietly, not looking directly at any student, and then *waiting* until it is quiet. Taking the time to teach the routine will be beneficial to you and your students and will facilitate successful collaborative group work.

Developing Positive Interdependence

Positive interdependence, the extrinsic conditions that motivate students to work together, is at the heart of collaborative learning. There are various types of tools for building positive interdependence:

- focusing on group processes
- sharing materials
- sharing roles

At least one of these tools must be used during any collaborative learning lesson, although more can be included.

Focusing on Group Processes

The most obvious tools for ensuring that students work together in their groups have to do with focusing on group processes. For example, the procedures described for giving directions clearly focus on the group and reinforce the principle of group structure.

The practice of answering only group questions is a powerful tool for developing positive interdependence. When students are working in a group, you should establish the general practice of answering only those questions that have been decided upon by the group. If a student has a question about the group's work, the group should be consulted. If no one has a response that helps, or if there are various responses and an agreement cannot be reached, this group of students may ask you. When the group has a question, all members must raise a hand. This signals to you that everyone agrees that help from you is needed. It should be understood by all that, when you arrive at the group, *anyone* in the group may be called on, not just the person who asked the original question.

If the person called on does not know the question, you may say, "I will come back when you have a group question." If the person called on knows the question, you may ask probing questions to assess student understanding or prompt thinking, give a hint to help the group determine an appropriate answer, refer them to another group, or answer the question.

Before beginning group work, it would be helpful for students to understand the reasons for responding only to group questions. The belief behind the practice of the group question is that the group is a student's main resource. It is important for students to realize that they can think and solve problems within their groups. The teacher's job is to facilitate group work, not to participate as a member of any one group.

Establishing a group goal is another way to help students achieve positive interdependence within their groups. One such goal might be to work through an investigation or to prepare for a class discussion at a STM.

Still another way to increase the likelihood that students will think, talk, and work together in a group is to have the group create a single group product. The one product, which must be specifically described in the directions for the problem, is worked on by everyone in the group. Everyone must give ideas or opinions; everyone must write, draw, or use technology tools when agreed to by the group. This product is not created in an assembly line, like an add-a-line poem; rather, it is shared, but not divided, equally among the group members.

Sharing Materials

Another tool for developing positive interdependence is sharing materials in one of two ways. With "limited" materials, the group is limited to one set of materials. If the problems require a ruler, marker, scale, and technology, there is only one of each item in each group. These materials then rotate among the members of the group, with each student taking a turn measuring, writing, weighing, or calculating, at the direction of the group.

With "jigsawed" materials, the different items needed for the activity are distributed among the group members. In other words, everyone in the group has something that the other group members need. If measurements are required, one person may hold the item to be measured while others do the measurement and check each other's measurements by remeasuring. Or each student may have a different piece of information, and the group needs all the pieces of information to respond to the problem at hand.

Sharing materials in these ways helps ensure that the quiet, shy, or inactive student is included in the task, and the sharing also helps reduce the influence of dominating students who want to use only their own ideas. Because individual students who have their own materials have less reason to interact with other group members, they are more likely simply to socialize in their talking or not talk at all. Sharing materials thus encourages students to engage in conversation about the work, to explain, predict, negotiate, and reach consensus.

Sharing Roles

Sharing roles is also a type of tool to develop interdependence within groups, and there are several approaches to the assignment of roles. In what Dishon and Wilson O'Leary call "specifically assigned rotated roles," each student in the group has a particular function for a particular group session. This method of assignment can be seen in the references to the roles of Experimenter, Recorder, and Quality Controller in Unit 1 of Course 1. Depending upon the circumstances and the task at hand, of course, there can be different combinations of roles. For example, when students work in groups of three or four, the roles might be selected from Reader, Recorder, Quality Controller, Coordinator, and Reporter. With this type of role sharing, you may rotate the job assignments each class period or before the beginning of each new investigation.

In another type of role sharing, everyone is responsible for taking a turn at each job during group work time. For example, each student takes a turn being the measurer or being the writer. Then when it is time to report on group work, each student does part of the reporting. All group members are thus responsible for contributing ideas and information, as well as for checking the accuracy of their work.

Still another way to deal with roles is not to assign them at all. Instead, after the collaboration is completed, ask students to reflect on the tasks that had to be done and how these tasks were handled.

Establishing Individual Accountability

While the use of the techniques described above for development of positive interdependence is at the heart of collaborative learning, establishing individual accountability is also a very important component of collaborative learning.

One way to start developing accountability is to require individual student signatures on group products. At the end of work time, each student signs the group product with full name, not scrawled initials. A signature means, "I helped and I agree." This declaration of ownership and participation helps formalize the importance of being involved and being responsible for what the group discusses, decides, creates, agrees upon, and learns. Such visible ownership of the group product also decreases the feeling of anonymity that can accompany group work and begins the accountability process. In addition, the signature is a tool to use if a student says later, "Well, I didn't agree with what my group did." In such a case, the teacher responds, "Isn't this your signature? When you sign, you become responsible."

In some instances, of course, groups cannot reach consensus. Students may then designate and sign the parts to which they do agree, and indicate where and how they disagree.

Another fundamental way to ensure individual accountability is by holding every student responsible for being able to describe any aspect of the group activity. For example, you may decide not to select students for the role of reporter until after the investigation has been completed. In this way, all students must be ready to report on their group's findings by describing the group product, by explaining why it looks the way it does, by giving reasons for the group response, and so on.

When designating who will be the reporters, you may choose one of three different selection methods. One method is random selection within each group, with one person selected at random to report from each group. Another method of selecting reporters involves the idea of shared roles: You may randomly select a group or groups to report and have everyone in the group present part of the report.

No matter how the reporters are selected, to be fair and to motivate the students to pay attention to the task at hand, the method of selection, but not the identity of the reporters, should be announced before the investigation begins.

Opportunities for individual accountability are an integral part of each lesson in *Core-Plus Mathematics*. For example, at the STM, some students are called upon to report for their groups; random selection of reporters works well here. Following each STM, another opportunity for individual accountability is provided by the Check Your Understanding (CYU) task. After developing the mathematical ideas in groups and then sharing, refining, and summarizing them in a class discussion, students individually complete the CYU task based on their understanding of the mathematics. Their work on this task can be evaluated and recorded in a gradebook. This kind of accountability should also be decided upon and announced before work begins.

When students work in groups, it can be difficult for the teacher to know what an individual student participated in or learned. By including individual accountability in every investigation, the teacher alerts the students to the fact that they will be held accountable, and thus increases the likelihood that students will pay attention to, and participate in, group work.

Handling Conflict Effectively

A final area in which effective classroom management contributes to the collaborative learning process is in the handling of counterproductive conflict. The key to preventing such conflict lies in adherence to the rules for promoting respect. Even when rules for acceptable behavior have been discussed and posted, conflict can arise during group work. The amount of potential conflict is related to students' social skills. Students with a high degree of social skills will discuss, negotiate, and reach consensus with minimal conflict. When presented with new or different ideas, students with few social skills may use group work as a time to argue. The goal is to encourage healthy debate and to eliminate negative conflict.

To reduce further the likelihood of negative conflict, teachers may set the stage for the investigation by initiating a discussion to check for understanding or by conducting a brainstorming session (without evaluation of the ideas contributed). The Think About This Situation that begins each lesson serves this purpose.

Another way of preparing students to handle conflict is to have them finish and then discuss, either as a class or as groups, statements such as, "When we had a disagreement, it was helpful to... ." Role playing can also help students understand the differences between healthy debate and negative conflict and help them practice handling conflict.

When a group seems to be at an impasse, you should help the group resolve a conflict rather than simply break up the group. In resolving conflict, as in other aspects of collaborative learning, the teacher functions as guide or coach. Giving help does not mean entering into the conflict or solving the problem for the students. You must help the students to become their own problem solvers.

Conflict can be healthy and can stimulate thinking if teachers help students learn how to question each other with a positive tone; however, conflict resolution requires complex skills and will take time for students to practice and learn.

Conclusion

An atmosphere in which students feel free to express their thoughts without derision from classmates, are encouraged to think deeply about mathematics, and are provided the opportunity to grow intellectually is truly a challenge to create. The culture created within the classroom is crucial to the success of the *Core-Plus Mathematics* program. It is important to inculcate in students a sense of inquiry and responsibility for their own learning. Without this commitment, active, collaborative learning by students cannot be effective. The percentage of ninth-grade students who have already attained these skills may be small. The work ethic and social skills of ninth graders is not always ideal in many cases, but teachers indicate that, with persistence, this changes. In some cases, students returning to school as tenth graders seem to have miraculously matured over the summer and are actually eager to work collaboratively with their classmates.

In order for students to work collaboratively, they must be able to understand the value of working together. Some students seem satisfied with the rationale that this is important in the business world. Others may need to understand that the struggle with verbalizing their thinking, listening to others' thinking, questioning themselves and other group members, and coming to an agreement increases their understanding and retention of the mathematics and contributes to forming important thinking skills or habits of mind.

This level of understanding of the importance of collaborative groups is particularly important for students who are inclined to quickly and superficially cover the mathematics and then move on. They often resist thinking deeply about the mathematics and fail to see the value in a multiple-perspective approach. Once these students understand that collaborative work broadens and deepens their own mathematical understanding, they can exert a positive influence on their classmates.

Issues involved in helping students to work collaboratively will become less pressing as both you and your students gain experience in this type of learning. You may find it helpful to refer back to this guide and discuss effective collaborative group strategies with your colleagues a few weeks into the semester.

Further Reading

Artzt, A.F., and Newman, C.M. (1997). *"How to use cooperative learning in the mathematics class"*. Reston, VA: The Council.

Baxter, Stephen. "Assessing Groups." *The Mathematics Teacher* 85, no. 8 (1992): 637–639.

Bellanca, J., & Fogarty, R. (1991). *Blueprints for thinking in the cooperative classroom (Second ed.)*. Palatine, Illinois: Skylight Publishing, Inc.

Cohen, Elizabeth G. *Designing Groupwork*, 2nd ed. New York: Teachers College Press, 1994.

Cohen, Elizabeth G. "Complex Instruction: Equity in Cooperative Learning Classrooms." *Theory into Practice* 38, no. 2 (Spring, 1999): 80–86.

Davidson, Neil (editor). *Cooperative Learning in Mathematics: A Handbook for Teachers*. Menlo Park, CA: Addison Wesley, 1990.

Davidson, Neil and Toni Worsham (editors). *Enhancing Thinking Through Cooperative Learning*. New York, NY: Teachers College Press, 1992.

Dishon, Dee, and Pat Wilson O'Leary. *A Guidebook for Cooperative Learning: A Technique for Creating More Effective Schools*. Holmes Beach, FL: Learning Publications, Inc., 1994.

Johnson, David, and Roger Johnson. *Cooperation in the Classroom*. Minneapolis: Interaction Book Company, 1988.

Johnson, David, Roger Johnson, and Edythe Johnson Holubec. *The New Circles of Learning: Cooperation in the Classroom and School*. Alexandria, VA: ASCD, 1994.

Kagan, Spencer. "The Structural Approach to Cooperative Learning." *Educational Leadership* 47, no. 4 (1989): 12–15.

Reynolds, B. E., Hagelgans, N.L., Schwingendorf, K.E., Vidakovic, D., Dubinsky, E., Shahin, M., et al (Eds.). (1995). *A practical guide to cooperative learning in collegiate mathematics.* Mathematical Association of America.

Rogers, E. C., Reynolds, B.E., Davidson, N.A., & Thomas, A.D. (Eds.). (2001). *Cooperative learning in undergraduate mathematics: Issues that matter & strategies that work.* Mathematical Association of America.

Schultz, James L. "Cooperative Learning: Refining the Process." *Educational Leadership* 47, no. 4 (1989): 43–45.

Sharan, Yael and Sharan, Shlomo (1992). *Expanding Cooperative Learning Through Group Investigation.* New York, New York: Teachers College Press.

Assessment

Throughout the *Core-Plus Mathematics* curriculum, the term "assessment" is meant to include all instances of gathering information about students' levels of understanding of and their disposition toward mathematics for purposes of making decisions about instruction. The dimensions of student performance that are assessed in this curriculum (see chart below) are consistent with the assessment recommendations of the National Council of Teachers of Mathematics *Assessment Standards for School Mathematics* (NCTM, 1995). They are much broader than those of a typical testing program.

Assessment Dimensions		
Process	**Content**	**Attitude**
Problem Solving Reasoning Communication Connections	Concepts Applications Mathematical Representations Procedures	Beliefs Perseverance Confidence Enthusiasm

Sources of Assessment Information

Several kinds of assessment are available to teachers using *Core-Plus Mathematics*. Some of these sources reside within the student text itself, some of them are student-generated, and some are supplementary materials designed specifically for assessment. Understanding the nature of these sources is a prerequisite for selecting assessment tools, establishing guidelines on how to score assessments, making judgments about what students know and are able to do, and assigning grades.

Curriculum Sources

Two features of the curriculum, questioning and observation by the teacher, provide fundamental and particularly useful ways of gathering assessment information.

Questions and Questioning The student texts use questions to facilitate student understanding of new concepts, of how these concepts fit with earlier ideas and with one another, and of how they can be applied in problem situations. Whether students are working individually or in groups, the teacher is given a window to watch how the students think about and apply mathematics as they attempt to answer the questions posed by the curriculum materials. In fact, by observing how students respond to the curriculum-embedded questions, the teacher can assess student performance across all process, content, and attitude dimensions described in the chart above.

Student responses to questions such as "What if. . . ?" and "Why?", along with their explanations and justifications provide insights into how students reason about and communicate mathematics. How well students see connections among mathematical ideas and their applications can be assessed with questions such as "How is this like [an earlier idea] and how is it different?" A question such as "What do you predict will happen?" may provide insights not only into students' understanding of the relevant content but also into their dispositions toward mathematics. These are just some of the types of questions that can provide good assessment information to help teachers make appropriate instructional decisions. Although such questions are commonplace in the instructional materials, similar questions can be framed by teachers in order to further probe student understanding. While observing individual students as they respond to such questions, teachers often take notes or complete checklists to help judge the growth of individual students or to provide detailed information for grade reports.

Specific features in the student material that focus on different ways students respond to questions are the Summarize the Mathematics, Check Your Understanding, and On Your Own homework sets. The Summarize the Mathematics sections are intended to bring students together, usually after they have been working in small groups, so they may share and discuss the progress each group has made during a sequence of related problems. The questions in the Summarize the Mathematics (STM) are focused on the mathematical concepts and procedures developed in the investigation. They should help the teacher and student identify and formalize the key ideas of the investigation. Each STM is intended to be a whole-class discussion, so it should provide an opportunity for teachers to informally assess the levels of understanding that the various groups of students have reached.

Following each Summarize the Mathematics, the Check Your Understanding tasks are meant to be completed by students working individually. Student responses to these tasks provide an opportunity for teachers to assess the level of understanding of each student.

The tasks in the On Your Own homework sets serve many purposes, including post-investigation assessment. Each type of task in the On Your Own homework sets has a different instructional purpose. *Applications tasks* provide opportunities for students to demonstrate how well they understand and can use the ideas they learned in the investigations of the lesson. Work on *Connections tasks* demonstrates how well the students understand links between mathematical topics they studied in the lesson and their ability to connect those topics with other mathematics that they know. *Reflections tasks* provide insight into students' mathematical thinking and strategic competence. *Extensions tasks* reveal how well students are able to extend the present content beyond the level addressed in the investigations. The *Review tasks* allow for pre-assessment of students' understanding of ideas or procedures needed in the upcoming lessons and also provide information on how well students are retaining previously learned mathematics. The performance of students or groups of students on each of these types of tasks provides the teacher with further information to help assess each student's evolving ability to use, connect, and extend the mathematics of the lesson.

Finally, an opportunity for group self-assessment is provided in the last element of each unit, the Looking Back lesson. These tasks help students pull together and demonstrate what they have learned in the unit and at the same time provide helpful review and confidence-building for students.

Additionally, in each of Courses 1–3, there are Practicing for Standardized Test masters (see page 13) and in each Course 4 *Unit Resource Masters*, there are two Preparing for Undergraduate Mathematics Placement (PUMP) exercise sets. Each set provides practice and assessment of skills and reasoning techniques commonly assessed on college mathematics placements tests.

Observation The other fundamental method of assessment that is built into *Core-Plus Mathematics* is observation by the teacher. Teachers, of course, have always learned a great deal by observing individual students as they do mathematics. Such observation continues to be a valuable source of assessment information. Because many problems are completed by students working in small groups, teachers using these materials have many opportunities to observe the performance of students as they work with others.

While students are working on tasks from the investigations, you might listen for evidence of student progress in understanding and applying the mathematical content of the lessons. Before students begin their work, you should identify several tasks that will help you determine the progress that they have made in understanding the key mathematical ideas or procedures of the investigation. Then, as you observe each group, you can focus on their work on those items. Assessing student thinking by listening and questioning students as they work through an investigation will also allow you to identify and address misconceptions as they arise.

In addition to developing content knowledge, *Core-Plus Mathematics* provides students an opportunity to develop skill in working with others. In today's society, working well with others in problem-solving groups is important. To help develop these skills, teachers should observe and assess behaviors of group members such as those listed below and provide students feedback.

- Communicating mathematical ideas to other members of the group
- Dividing a task fairly among group members
- Agreeing on a structure for completing a task
- Taking time to ensure that all group members understand
- Recording results regularly and clearly
- Soliciting and using, in appropriate ways, the suggestions and ideas of all group members
- Fairly representing a group consensus
- Pushing the group to think deeply about the mathematics under investigation
- Reporting the group's progress to the whole class in a complete and interesting way

Student-Generated Sources

Other possible sources of assessment information are writings and materials produced by students in the form of mathematics toolkits, journals, and portfolios. (See p. 65, Tips on Student Work and Writing.)

Mathematics Toolkits Each student should create a Math Toolkit that organizes important class-generated ideas and selected Checkpoint responses as they complete investigations. Constructing a Math Toolkit prompts are provided in the *Teacher's Guide* to assist in identifying key concepts and methods as they are developed by students.

Unit Summaries A summary template intended to help students organize and record the main ideas learned in the unit is provided in each *Unit Resource Masters*. The synthesis of ideas that occurs during completion of the Looking Back lesson and the final unit Summarize the Mathematics discussion should provide the background for student completion of the unit summary.

Journals Student journals are notebooks in which students are encouraged to write (briefly but frequently), their personal reflections concerning the class, the mathematics they are learning, and their progress. These journals are an excellent way for the teacher to gain insights into how individual students are feeling about the class, what they do and do not understand, and what some of their particular learning difficulties are. This information can be very useful for planning instruction that will meet the needs of individual students in the class. For many students, the journal is a non-threatening way to communicate with the teacher about matters that may be too difficult or too time-consuming to talk about directly. Journals also encourage students to assess their own understanding of, and feelings about, the mathematics they are studying.

One effective approach to the use of journals is for the teacher to provide prompts (questions or statements) to which the students give a written response in their journals. Such prompts may be given one or two times a week, and the writing may be done during the last few minutes of class or at home. The best journal prompts are open-ended and encourage students to reflect on the mathematics they have been doing and learning or to reflect on the learning process. Some possible prompts are given below.

- The key idea of the lesson today was . . .
- What questions were still unanswered at the end of class today?
- Find something that you learned today that is connected to something that you already knew. Write about how the two things are connected.
- How do you feel about sharing your work with the class?
- My three personal goals in math this term are . . .
- When I study for a test, I . . .
- Describe one thing you did well today and one thing upon which you could improve.
- The problem-solving techniques that I used today were . . .
- Describe a way in which you helped a classmate today or a way in which a classmate helped you.
- When you get stuck on a problem, what are some things you try in order to get unstuck?
- Identify a task that was difficult (easy) for you today. Describe why it was difficult (easy).
- In what ways did technology help you to better understand the big ideas in today's lesson?

The teacher should collect, read, and respond to each journal regularly. Teacher responses to the journal entries should be nonjudgmental statements of interest or questions seeking clarification. Many teachers stagger their journal collections, reviewing perhaps one-fourth of them each week. Teachers are encouraged to have their students keep journals and write in them regularly.

Portfolios A portfolio is a collection of a student's work accumulated over time. Typically, portfolios provide a tool for assessing one or more of the following outcomes: student thinking, growth over time, mathematical connections, a student's views of herself or himself as a mathematician, and the problem-solving process as employed by the student. An excellent, practical source about the use of portfolios in mathematics is the NCTM publication, *Mathematics Assessment: A Practical Handbook for Grades 9–12* (1999).

The *Core-Plus Mathematics* assessment program provides many items that would be appropriate for a student's portfolio, including reports of individual and group projects, Math Toolkits or journal entries, teacher-completed observation checklists, and end-of-unit assessments, especially the take-home tasks and projects. One way students can develop a portfolio is to collect all of their written work in a folder, sometimes called a "working portfolio." Then, at least once each semester, students can go through their working portfolios and choose items that they think best represent their growth during that time period. After writing a paragraph or two explaining why each piece of work was chosen, each student can place the chosen items with the written rationales into a new folder that becomes the actual portfolio.

Supplementary Assessment Materials

The *Core-Plus Mathematics Unit Resource Masters* are an additional source of assessment information.

Each *Unit Resource Masters* includes lesson quizzes and unit assessments in the form of tests, take-home tasks, and projects. There are also banks of questions and projects from which you can form end of semester exams following the Unit 4 and Unit 8 assessment masters. Calculators are assumed in most cases and are intended to be available to students. Teacher discretion should be used regarding student access to their textbook and Math Toolkit for assessments. In general, if the goals to be assessed are problem solving and reasoning, while memory of facts and procedural skill are of less interest, resources may be allowed. However, if automaticity of procedures or unaided recall are being assessed, it is appropriate to prohibit resource materials.

The *ExamView Pro* software can be used to modify the curriculum provided assessment items or to create formal assessments using a combination of curriculum supplied items and ones written by the teacher.

Lesson Quizzes Two forms of a quiz covering the main ideas of each lesson are provided. These quizzes are comprised of problems meant to determine if students have developed understanding of the important concepts and procedures of each lesson. The two forms of each quiz are not necessarily equivalent, although they assess essentially the same mathematical ideas. Since many rich opportunities for assessing students are embedded in the curriculum itself, you may choose not to use a quiz at the end of every lesson.

Unit Tests Two forms of tests are provided for each unit and are intended to be completed in a 50-minute class period. The two forms of each test are not necessarily equivalent, although they assess essentially the same mathematical ideas. Teachers should preview the two versions carefully to be sure that the unit assessment aligns with the learning goals emphasized.

Take-Home Assessments Take-home assessment tasks are included for each unit. The students or the teacher should choose one or, at most, two of these tasks. These assessments, some of which are best done by students working in pairs or small groups, provide students with the opportunity to organize the information from the completed unit, to work with another student or group of students, to engage in in-depth problem solving, to grapple with new and more complex situations related to the mathematics of the unit, and to avoid the time pressure often generated by in-class exams. These problems may also require more extensive use of technology than is often available in the regular classroom during testing situations. You may wish to use these more in-depth problems as a replacement for a portion of an in-class end-of-unit exam.

Projects Assessment traditionally has been based on evaluating work that students have completed in a very short time period and under restricted conditions. Some assessment, however, should involve work done over a longer time period and with the aid of resources. Thus, assessment projects are included in unit assessments. These projects, which are intended to be completed by small groups of students, provide an opportunity for students to conduct an investigation that extends and applies the main ideas from the unit and to write a summary of their findings. Many of these might also allow for students to present their work in a variety of ways. You may have students who would rather prepare and present their work orally or visually using computers and/or video equipment. In this way, the projects can provide an opportunity for students to use their creativity while demonstrating their understanding of mathematics.

Midterm and Final Assessments A bank of assessment tasks, from which to construct midterm and final exams that fit your particular class needs and emphases, are provided in the Unit 4 and Unit 8 *Unit Resource Masters*. In addition to problems similar in form to those on the quizzes and tests, these assessment banks include several multiple-choice problems for each unit.

Extended assessment projects are also included with the end-of-year assessments. These projects are investigations that make use of many of the main ideas encountered in the curriculum. They require use of material from more than one unit. The projects are intended to be completed by small groups of students working over a period of time. You may wish to have different groups work on different projects and then give presentations or create posters of their work.

Scoring of Assessments

High expectations of the quality of students' written work will encourage students to reach their potential. You will notice the solutions will sometimes indicate that "responses may vary." This is used when more than one correct response is acceptable. Students should always have reasonable explanations for their responses. The quality of the responses will vary and some responses may simply be incorrect. Some teachers use quality student responses as model responses for the class throughout the course. Continued emphasis on reasonable and clear explanations helps students develop their thinking and communication skills throughout their mathematics studies.

Assigning scores to open-ended assessments and to observations of students' performance requires more subjective judgment by the teacher than does grading short-answer or multiple-choice tests. It is therefore not possible to provide a complete set of explicit guidelines for scoring open-ended assessment items and written or oral reports. However, the following general guidelines may be helpful. (Adapted from *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions,* the National Council of Teachers of Mathematics, 1991.) Teachers can use this general framework to develop guidelines for specific assessment items.

Open-Ended Items

When scoring student work on open-ended assessment tasks, the goal is to reward in a fair and consistent way the kinds of thinking and understanding that the task is meant to measure. Preparing to score open-ended assessment tasks is best done using a three-step process. First, teachers should have a *general rubric*, or scoring scheme, with several response levels in mind. As the name suggests, the general rubric is the foundation for scoring across a wide range of types of open-ended tasks. The following general rubric can be used for most assessment tasks provided with the *Core-Plus Mathematics* materials.

To score a particular task in a reasonably unambiguous, fair way, the teacher will need a *specific rubric* which is the result of rewriting the definitions of the scoring levels in the general rubric so that they explicitly account for the details of the assessment task at hand and apply to the range of actual student responses to that task. (As an illustration, consider the sample task on page 60.) This rubric should take into consideration the ways in which students construct their responses to meet the expectations of that task. The specific rubric is the second step in the three-step process.

Finally, since verbal definitions for score levels can never be entirely unambiguous, the teacher will need to identify examples of student work to anchor the specific rubric. These examples are often referred to as *anchor items*. The anchor items for each scoring level should be chosen to illustrate the range of responses that are typical of students who score at that level. On the next three pages, the complete process for scoring is described using a sample open-ended task and examples of actual student work for *Core-Plus Mathematics*.

General Scoring Rubric

For most open-ended mathematics tasks, credit can be assigned in decreasing amounts according to the following levels of performance. This general rubric has five scoring levels corresponding respectively to 0 to 4 points, but three or four levels may work better for some tasks. Adjustments to the rubric for the number of levels can be made in obvious ways. Not all parts of the description of each level will be relevant to every open-ended task.

4 points	Contains complete response with clear, coherent, and unambiguous explanation; includes clear and simple diagram, if appropriate; communicates effectively to identified audience; shows understanding of question's mathematical ideas and processes; identifies all important elements of question; includes examples and counterexamples; gives strong supporting arguments
3 points	Contains good solid response with some, but not all, of the characteristics above; explains less completely; may include minor error of execution but not of understanding
2 points	Contains complete response, but explanation is muddled; presents incomplete arguments; includes diagrams that are inappropriate or unclear, or fails to provide a diagram when it would be appropriate; indicates some understanding of mathematical ideas, but in an unclear way; shows clear evidence of understanding some important ideas while also making one or more fundamental, specific errors
1 point	Omits parts of question and response; has major errors; uses inappropriate strategies
0 points	No response; frivolous or irrelevant response

Sample Assessment Task The length s, in inches, of a spring is a function of the weight w, in pounds, that is attached to the spring. For a particular spring, the modeling equation is

$$s = 20 + 5w.$$

a. Explain what the 20 in the equation tells you about the spring.

b. Solve the following equation and show your work: $36 = 20 + 5w$.

c. Explain what the solution in Part b tells you about the spring.

d. How long is the spring if you place a weight of 1.2 pounds on it?

The process of developing a specific rubric for this task involves an initial decision concerning whether to score the entire task as a unit or to score each part separately. In this case, it could be reasoned that the parts are sufficiently different to make scoring each part the better decision. Following that decision, consider Part a by first asking what a top-level (4-point) response should contain. After specifying the characteristics of a solution required for 4 points, next ask what a minimal solution must contain; that is, what must the response include for a score of 1 rather than 0. Identifying both the "best" and "least" acceptable solution can usually be done without reference to student work, although some minor changes in these initial definitions may need to be made later to account for some unanticipated student responses.

Distinctions between scores of 4 and 3, 3 and 2, and 2 and 1 are more difficult. It is helpful to make an initial attempt at defining scores of 3 and 2, using only an analysis of the item and what are likely responses, but then to move quickly to examining a set of student work. Actual student responses help refine definitions, forcing decisions about what should count in each scoring category. Refinements can be made to account for unexpected student responses, while keeping in mind the general rubric and the ultimate goal: to reward in a fair and consistent way the kinds of thinking and understanding that the task is meant to measure.

By repeating the process described in the previous paragraph for Parts b–d, a specific rubric similar to the following could be constructed. Recall that a score of 0 is given for either no response or for a completely irrelevant response.

Specific Scoring Rubric

Part	4 points	3 points	2 points	1 point
a	Initial length in inches of spring *or* length in inches when no weight is attached	Correct answer is implied, but answer is vague and may not contain units of measure, e.g., "What the spring started at."	Confusion between weight and stretch; does not state that it is a stretch length, *or* incorrect response, but containing word synonymous with weight or stretch, e.g., "How long the spring is."	Incorrect response that shows some relevance to a spring, length, weight, stretch, or 20
b	Correct answer (3.2 or 16/5) with correct algebraic work and explanation of how tables or graphs were used	Correct answer, but no work shown or wrong steps in the solution process *or* correct solution process, but arithmetic error leads to incorrect answer	Incorrect numerical answer with flawed, or partially correct, solution process	Incorrect numerical answer with no work or inappropriate work or incomplete work that seems to be partially correct, but no answer is given
c	Correct answer similar to: "When weight of 3.2 lb is attached to the spring, its length is 36 in."	Right idea, but missing labels make distinction between weight and length unclear *or* relationship between length and weight is mentioned, but answer is vague	Mentions at least one of 3.2 and 36 and at least one of pounds and inches (or length and weight), but does not connect them or switches them	An attempt that mentions 36 or 3.2, but not both or an attempt that is in some way relevant to a spring, weight, or length
d	Writes the equation $s = 20 + 5(1.2)$ correctly and gets the answer 26 pounds, or indicates an answer of 26 pounds and describes how tables or graphs were used	Correct answer, 26 pounds, but no work or an equation that is incorrect *or* a correct equation with an arithmetic error leading to an incorrect answer	Incorrect answer with correct equation, but work has one or more algebraic errors (not just arithmetic)	Incorrect numerical answer with wrong equation, inappropriate work, or no work

Anchor Items

The following responses demonstrate the range of responses to Part a at each score level.

Score	Student Responses to Part a
4	(1) "That the spring is 20 inches long before adding weight to it." (2) "The spring is 20 inches long to start with."
3	(1) "What the spring started at." (2) "20 tells me that the spring has to be at least 20 inches long."
2	(1) "It tells me the length in inches." (2) "It explains that there are 20 inches in the spring."
1	(1) "That you can have anything you want be equal to 20." (2) "The spring is 20 more than five times the weight." (3) "20 inches"

Note that a score of 4 requires explicit mention of 20 inches being the length of the spring at the start or with no weight attached. The responses that were scored 3 suggest that the students may have had the correct idea, but they failed to express it explicitly. Most responses in the 2-point category simply said the spring is 20 inches long, an incomplete response, but at least it identifies 20 with the length of the spring. The responses assigned a score of 1 failed to relate 20 to the length of the spring and, in some cases, gave a clearly incorrect description of the role of 20.

The responses in the chart below are meant to clarify the rubric for Part b.

Scores for Student Responses

Scores for Part b

4	$36 = \quad 20 + 5w$ $-20 \qquad -20$ $\frac{16}{5} = \frac{5w}{5}$ $w = 3.2$	$36 - 20 = 16$ $\begin{array}{r} 3.2 \\ 5\overline{)16} \end{array}$ $w = 3.2$	Go to Y= menu and enter $20 + 5x$. Use the table. Press the down arrow until you reach 36 in the Y1 column. Look across from 36 to get $w = 3.2$.
3	$36 - 20 = 20 - 20 + 5w$ $\frac{16}{5} = \frac{5w}{5}$ $w = 3.1$	$w = 3.2$	
2	$20 + 36 = 20 - 20 + 5w$ $56 = 5w$ $\frac{56}{5}$ $w = 11.2$	$36 - 20 = 20 - 20 + 5w$ $16 - 5 = 5 - 5w$ $w = 11$	
1	$20 + 36 = 20 - 20 + 5w$ $56 = 5w$ $\frac{56}{5}$	$\frac{5}{5w} = 5$ $25 + 5 + 36$ $w = 66$	

The three main categories of responses that are assigned a score of 4 are illustrated by the examples in the table on page 54. In the first sample response, the standard paper-and-pencil approach to solving a linear equation is clearly applied correctly. In the second sample response, the student showed the computations, but not what was done. While the second response is a less complete solution than the first response, it was judged as including the correct answer and the correct work and so assigned a score of 4. If the use of a graphing calculator is acceptable for this item, then the third sample response should also receive a score of 4.

In the examples receiving a score of 3, an arithmetic error in an otherwise correct solution is shown in the first sample response, and the correct answer only with no work is given in the second response.

For a score of 2, the student in the first example erroneously added 20 to the left side, but then correctly subtracted 20 from the right side and finally correctly divided by 5. Similarly, in the second example that was scored 2, the student correctly subtracted 20 from both sides, but then got off track by subtracting 5 (thinking that $5 - 5w = w$).

The first example that was scored 1 is very similar to the first example that was scored 2. The work is partially correct in both cases, but the student who received the score of 1 did not indicate a value for w. The second example that was scored 1 has a numerical answer, but no part of the work seems to be on target.

No sample work is provided for Parts c and d. You and a colleague may wish to examine the rubric for Parts c and d carefully in light of the foregoing examples of responses to Parts a and b and generate examples of student work for each part that would receive any score from 1 to 4.

Reports and Presentations

Four or five levels of holistic scoring may be used for reports and presentations. At each level the teacher must be able to identify the key characteristics of a quality report, such as a particular table, graph, diagram, or function. The teacher also will need to judge the quality of the student's reasoning, problem solving, and communication skills as displayed in the report or presentation. Holistic scoring by the teacher or by classmates could also be used to assess individual or group presentation of work.

An alternative to the holistic approach involves making separate judgments of the student's reasoning, problem solving, communication skills, and perhaps other categories that may be appropriate for a given report. Then, weighted in a manner that the teacher deems appropriate, these scores may be combined to determine a single grade for the report. Whether scoring in different categories is better than holistic scoring is debatable, but breaking down the grade into components probably makes the overall grade more understandable to the student and easier for the teacher to defend.

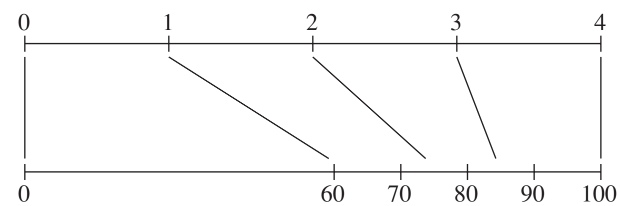
Specific ideas about evaluating extended projects are given in the teacher notes that accompany the extended project assignments.

Assigning Grades

Since the *Core-Plus Mathematics* approach and materials provide a wide variety of assessment information, the teacher will be in a good position to assign appropriate grades. With such a wide choice of assessment opportunities, a word of caution is appropriate: *It is easy to overassess students, and care must be taken to avoid doing so.* A quiz need not be given after every lesson nor an in-class exam after every unit. The developers believe it is best to vary assessment methods from lesson to lesson, and from unit to unit. If information on what students understand and are able to do is available from their homework and in-class work, it may not be necessary to take the time for a formal quiz after each lesson. Similarly, information from project work may replace an in-class exam. In some schools, teachers of each course work together to prepare and grade common assessments.

Deciding exactly how to weigh the various kinds of assessment information is a decision that you will need to make and communicate clearly to your students. Commonly, teachers assign a percentage of the final grade to different types of assessment such as in-class participation (dispositions), quizzes, exams, homework, take-home tasks, and projects. Sometimes teachers make more holistic judgments.

Some flexibility in grading is required. Recall that a score of 1 on a 0–4 scoring rubric indicates some understanding and that a score of 2 indicates a considerable amount of progress on challenging problem solving and reasoning assessment tasks. Even though 1/4 is 25% and 2/4 is 50%, which typically might be equated with failing grades, the level of understanding actually may be worthy of a grade of D or C. In short, the usual 90% for A, 80% for B, and so forth, may not work as a grading scale. It is possible to use a slightly different approach in which you directly translate a rubric score to the usual scale. Since 0%–59% is the interval of failure on this scale, there are really five intervals: 0–59, 60–69, 70–79, 80–89, and 90–100. In this approach, the teacher maps the five intervals of a rubric score directly into these intervals, using the following scheme or some variation on it:



With this translation, the teacher, students, and their parents simply (and quite legitimately) think of the rubric scores as equivalent to the corresponding scores on the percent scale.

Whatever the method for determining grades, the developers recommend that you take advantage of the multiple sources of assessment information available, and do not base grades merely on quizzes and unit exams. The following is typical of the way the pilot- and field-test teachers weighed factors when assigning grades. Of course, individual teachers may decide on their own breakdowns.

Unit Tests	25%
Quizzes	25%
Homework	20%
Group work/participation	15%
Written/oral reports	10%
Notebooks/journals	5%

One grading issue in using *Core-Plus Mathematics* relates to group work. Every student working in a group has a responsibility to help the group operate smoothly and to facilitate the learning of other group members. These outcomes are very important in this curriculum, and assessing and encouraging them for each student is no less important. For grading group assignments which result in a single report, all group members receive the same grade. So, how can individual grades be used to promote successful group work? One approach that seems to work well is to give extra credit to all group members when all members of a group do very well on a quiz or exam. Generally, this extra credit should be difficult, but not impossible, to earn.

Another grading issue arises from the fact that this curriculum is designed for *all* students. It intends to challenge the strongest students, while being accessible to all. Fair grading of such a wide range of students is not easy. Schools may choose to have two grading scales, "regular" or "core" and "honors" or "Core-Plus." The latter category typically requires consistently high-quality work on the core topics, plus work of similar quality on additional Extensions tasks in every unit. A grade in the Core-Plus category could be assigned more weight than a corresponding grade in the core category in the computation of a student's grade point average. For example, as many schools do with Advanced Placement classes, an A in the Core-Plus category could count as 5 quality points (or grade points), whereas an A in the core category might count as 4.

Tips on Student Work and Writing

Most teachers require students to maintain a three-ring binder containing their written work from investigations, Check Your Understanding, and On Your Own homework sets. Written assessments and projects that have been returned to students may be organized in the three-ring binder or held in a student portfolio that remains in the classroom at all times.

Writing Complete Responses

Helping students develop skills in writing complete and concise solutions is one of the goals of this curriculum. However, always writing thorough responses can unnecessarily slow student progress through the investigations. As a guideline, we suggest that during investigations, students should make notes of their thinking and discussion of ideas rather than use complete sentences. Investigation time can be thought of as draft work or getting ideas out for discussion. For investigation problems that ask students to explain reasoning or to compare, you may want to require complete-sentence responses. Student responses to the Summarize the Mathematics and Math Toolkit entries should be more complete. If these responses are written following the class summary of important mathematical ideas, students will be able to write more thorough responses. Homework tasks from the On Your Own sets should also be thoroughly written.

Paperwork Management

In order to avoid being overwhelmed with paperwork and to avoid over-assessing students, teachers use a variety of sampling techniques. By selecting from the options on the next page, individual teachers can develop their own scheme for paperwork management.

1. Check only one paper per group for each investigation.
2. Periodically check for completion of homework by a quick survey while students work on an investigation.
3. Collect papers from the entire class, check the papers for completeness, and then examine one or two questions or parts of questions for correctness, complete sentence responses, and thoroughness.
4. Check different tasks for different students. (By assessing different items, teachers have a broader view of what students know and are able to do. Some teachers require students who wish to earn an A to do one or two Extensions tasks for every lesson.)
5. Check for understanding by using a *homework quiz*. (Students copy their already-written responses to selected homework tasks such as Applications 1b or Connections 4f, or they simply circle their responses and hand in these items.)
6. Collect notebooks and grade for completion at the end of each unit.
7. Collect papers randomly from about one-third of the class for each assignment. Grade responses to all or selected questions.
8. Have students exchange papers in class as they go over the Connections tasks from the assignment. Students can address strengths and weaknesses of the responses and, with teacher guidance, assign a check, check-plus, or check-minus.
9. If students are required to have all investigations and homework completed before taking a written assessment, it is helpful to keep a file card for each student that lists tasks yet to be completed.

Math Toolkit

Some teachers also have students include a section (often containing colored paper) in the three-ring binder for the student-constructed Math Toolkit. Other teachers require a separate spiral notebook, a booklet made from grid paper folded in half with a colored cover, or note cards on a ring. One advantage of a separate location for the Math Toolkit is that students have long-term access to their compilations of class-generated definitions, examples, theorems, and mathematical concepts that they have developed throughout the year. If the Math Toolkit is kept in the three-ring binder, it should be maintained throughout the year, even though other sections of the notebook may be emptied when starting each new unit. Students should also continue constructing their Math Toolkits during all four courses of *Core-Plus Mathematics*. The toolkits give students a valuable reference, as well as a long-term record of their mathematical growth.

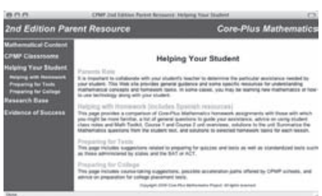
It is important to help students develop good work habits. Students' previous mathematics programs may not have emphasized writing. Students studying *Core-Plus Mathematics* for the first time will need to develop the ability to write clear, concise, mathematical explanations and justifications. Throughout the four courses, students will need to be encouraged to express their ideas completely and clearly. It is very important to have high expectations and hold students to high standards.

Communicating with Parents

General Suggestions

Parents are understandably concerned about the implementation of any major curriculum or instructional changes in programs that affect their children. Their understanding and support will play an important role in the successful implementation of the *Core-Plus Mathematics* program. This understanding and support can also be a big factor in their children's success with mathematics. For many parents (who attended high school in the United States) the sequence of algebra, geometry, advanced algebra, and precalculus courses was the gateway to college. The *Core-Plus Mathematics* curriculum provides an alternative, contemporary, *Standards*-based route to both college and careers in the technology-based workplace of the 21st century. Its organization is similar to that of high-performing countries on international assessments.

It is important that parents are provided information about *Core-Plus Mathematics* well in advance of its implementation and periodically while it is in use. Schools participating in the testing and evaluation of the curriculum organized "Math Nights" where parents were provided a rationale for the proposed changes in curriculum and instructional practices, including expectations of colleges, small businesses, and industry; were guided through the mathematics of the curriculum; and were given opportunities to raise questions or concerns. Opportunities for parents to review the textbooks permit them to see that the texts include important ideas and methods of algebra, geometry, and functions—and much more: statistics, probability, trigonometry, and discrete mathematics. Parents also see in the texts the kinds of mathematics that they might possibly use in their jobs. Parents can be referred to the Parent Resource at www.wmich.edu/cpmp/parentresource2/ for assistance in understanding the curriculum and in helping their children. Follow-up "Math Nights" can feature upcoming content, technology assistance, and student presentations.



As noted in the next section on college admissions, students studying *Core-Plus Mathematics* have been admitted to highly selective national colleges and universities. This information and information from your local colleges is helpful in relieving concerns of some parents. It is also important to note that increasing numbers of colleges and universities are offering "reform" calculus programs that share many of the features of this curriculum: emphasis on developing conceptual understanding and presenting problems in context, use of technology tools such as graphing calculators or computers, collaborative group work, student projects, and writing in and about mathematics.

Parent Letters

As an initial step toward communication, you might consider sending periodic information letters to parents. A sample letter for Course 1 follows.

[school letterhead]

[date]

Dear Parent or Guardian:

We would like to share with you information about our new mathematics program. The program was developed by the Core-Plus Mathematics Project through a grant from the National Science Foundation. The text, *Core-Plus Mathematics,* is based on national standards for curriculum and teaching developed by the National Council of Teachers of Mathematics and endorsed by 15 mathematical sciences organizations. The text also reflects the needs of business and industry today who are calling for 21st century workers who can think and reason about quantitative situations, who are innovative, who can communicate effectively, and who can work together in teams. In each course, students study algebra, geometry, statistics, and discrete mathematics.

The first two units of Course 1 are devoted to the study of important and broadly useful topics in algebra and statistics.

> In Unit 1, *Patterns of Change*, students extend their understanding and skill in algebra in three ways. They learn how to recognize relationships among independent and dependent variables in problems and experiments and describe patterns in quantitative variables that change over time. They learn how to read and construct data tables and graphs that display relationships among variables. They begin developing symbol sense—the ability to connect important patterns of change to linear, exponential, quadratic, and inverse variation rules. After completing this unit, your student should be able to solve problems like 1–4 on pages 70–72 of the text.
>
> In Unit 2, *Patterns in Data*, students learn to organize and analyze data using various graphical displays (histogram, dot plot, box plot, and stem-and-leaf plot) and to summarize data using measures of center (mean, median, mode) and measures of variability (range, interquartile range, percentiles, and standard deviation). For the kinds of problems your student should be able to solve after completing the unit, see Tasks 1–3 on pages 145 and 146 of the textbook.

Your student should be developing a Math Toolkit which summarizes concepts, facts, skills, and methods which he or she is learning. You may wish to review his or her Math Toolkit from time to time or consult it as you are providing assistance on homework.

We believe that through the CPMP materials your student will develop a new excitement about mathematics and will grow confident in his or her ability to think mathematically. You can contribute to this growth by supporting completion of work that is assigned as homework. A parent resource is available at www.wmich.edu/cpmp/parentresource2/.

If you have any questions about our mathematics program, please do not hesitate to contact us.

Sincerely yours,

[Local principal, mathematics department chair, teacher]

Throughout the school year, you may wish to periodically send additional letters to parents. The following pages include background information for Course 1 Units 3–8 and other ideas for material to include in parent letters.

Units 3–8 Content Overviews

In Unit 3, *Linear Functions*, students learn how to recognize situations in which key variables change at a constant rate. They learn how to express and interpret those patterns of change in data tables, slopes and intercepts of straight-line graphs, and equations in the form $y = a + bx$. They learn techniques for solving linear equations and inequalities that arise in science and business problems. For the kinds of problems your student should be able to solve after completing this unit, see Tasks 1–6 on pages 232–236 of the textbook.

In Unit 4, *Vertex-Edge Graphs*, students learn basic concepts of graph theory. They use vertex-edge graphs to solve problems related to many different types of networks, including communication, computer, transportation, and distribution networks. Problems involve find the optimal (best) route and avoiding conflict among objects. Upon completion of this unit, your student should be able to solve problems like 1, 2, and 3 on pages 286–288.

In Unit 5, *Exponential Functions*, students learn how to construct and use data tables, graphs, and equations in the form $y = a(b^x)$ to describe and solve problems about exponential relationships such as population growth, investment of money, and decay of medicines and radioactive materials. Upon completing this unit, your student should be able to solve problems like 3, 4, and 5 on pages 357 and 358.

In Unit 6, *Patterns in Shape*, students learn to describe, classify, and visualize two-dimensional and three-dimensional shapes. They consider the Pythagorean Theorem and triangle congruence relationships. They also use experimentation and reasoning to investigate properties of polygons and polyhedra. For the kind of problem your student should be able to solve after completing this unit, see Tasks 3, 4, 7, and 8 on pages 457–459.

In Unit 7, *Quadratic Functions*, students learn to identify quadratic patterns in tables, graphs, and problem conditions. They write quadratic functions to represent situations, combine terms and factor expressions, and solve quadratic equations. The concepts and skills developed in this unit will be practiced in Unit 8 and units in later courses. Upon completing this unit, your student should be able to solve problems such as 3, 4, and 5 on page 528.

In Unit 8, *Patterns in Chance*, students learn to construct probability distributions and use the Addition Rule to solve problems involving chance. They design and carry out simulations to estimate answers to questions about probability. See Tasks 1–5 on pages 586–588 for a sample of the kind of problem your student should be able to solve after completing this unit.

Other Possible Topics for Parent Letters

One important topic to address is how parents can help with homework. In addition to providing the typical advice about providing a quiet location and holding expectations that homework will be completed, you could provide some or all of the questions below to help parents guide their children.

General Questions to Ask Your Student–Sample Text

To support learning, it is best to not do homework problems for your student, even if this is accompanied by an explanation. Instead, we suggest you ask questions like the following. Questions such as these work to enhance learning and success, whether the homework question is very basic, or very complex. This is a good place to start all help sessions. (Spanish versions of these questions are available from the CPMP Parent Resource Web site or your student's teacher.)

- What have you been doing in class that relates to this problem? Can you explain the main ideas to me so I can think about the problem with you?
- Do you have some examples in your notes or toolkit that would help us think about this problem? Did your class do a Summarize the Mathematics recently that would be relevant?
- Explain to me what you know right now and why that is not enough to do this problem.
- Explain these vocabulary words to me. Are there other words that you don't understand?
- What have you tried? Explain the steps to me. Can you explain this another way?
- Is there a way to organize this, with a sketch or a diagram or a graph or a table, that might help us get a handle on the problem?
- If you cannot complete the problem, can you make a simpler problem that you can complete?
- Write down a question that tells your teacher where you got lost.
- Is there someone else on your team that you can call to ask if they are also having difficulty?
- Now that you have a solution, does it make sense? Can you check your solution? Have you answered clearly and completely? Convince me.

Practicing for Standardized Tests–Sample Text

In addition to the practice embedded in problems that students do in class and in Connections, Reflections, and Review homework tasks, we periodically provide practice in the format used by most standardized tests such as state tests and the ACT and SAT tests. Attached to this letter is one such practice set. Although some of the items may look unfamiliar to students, they should have the background knowledge and problem solving abilities to complete these items. (Attach a Practicing for Standardized Test set. See Course 1 *Unit 1 Resource Masters* pages 73 and 74.)

College Admissions

Since most college preparatory curricula in the United States historically have been sequences of Algebra 1, Geometry, Advanced Algebra, and Precalculus, implementation of *Core-Plus Mathematics* may raise questions related to college or university admissions for students who have successfully completed three or four years of this program.

Some people are not aware that integrated high school mathematics programs are the norm in countries other than the United States. College admissions departments accepting students from overseas commonly receive transcripts indicating students have studied mathematics courses in high school without any designation of courses such as a Geometry or Advanced Algebra. (Community college admissions departments may see fewer applications from outside of the U.S.)

In addition, integrated high school mathematics courses such as *Core-Plus Mathematics* and other NSF-funded high school curricula have been published since the late 1990s. Many higher education admissions offices are aware of these programs. Graduates who have studied *Core-Plus Mathematics* have been accepted into a wide variety of higher education institutions. The first graduates from *Core-Plus Mathematics* began enrolling in colleges in the year 2000. Schools have not seen any students refused admittance to a college or university due to their study of *Core-Plus Mathematics*. A comprehensive list of these colleges is far too long to include in this resource, but a list of selective national colleges where students have been admitted prior to 2008 is included on page 72.

Titles of Courses

In cases where university mathematics departments have reviewed course descriptions of *Core-Plus Mathematics*, these courses have been approved as meeting admissions requirements of the institutions. Typically, Courses 1–3 are considered the equivalent of the college preparatory Algebra-Geometry-Advanced Algebra sequence. High school counselors can submit copies of the course descriptions found on pages 7–11 of this guide or summary paragraphs based on those course descriptions. Such course descriptions with accompanying transcripts showing Integrated Mathematics I, II, III, and IV or Mathematics 1, 2, 3, and 4 have proved sufficient for admissions purposes. Some schools have used course titles such as Integrated Algebra/Geometry or Integrated Algebra/Geometry/Statistics.

NCAA Clearinghouse

The NCAA clearinghouse for student athlete eligibility regularly examines courses based on short course descriptions of the mathematical content rather than textbook titles or course titles. Schools have included the unit descriptions on pages 7–11 of this book when submitting courses to NCAA for review. (These descriptions can also be sent to post-secondary admissions departments.) NCAA considers CPMP Course 1 to satisfy Level I requirements for student eligibility. Courses 2 and 3 satisfy Level II requirements.

Selective National Universities

Sometimes the question arises about whether or not students who studied *Core-Plus Mathematics* have gained admission into exclusive or selective universities. Teachers of *Core-Plus Mathematics* have reported that their students have been accepted into almost all of the "most selective national universities" as defined by *U.S. News and World Report* in 2008.

In 2008, *U.S. News and World Report* ranked 38 national universities as the "most selective" for admissions. These universities offer a full range of undergraduate majors, Master's and Ph.D. programs, and emphasize faculty research. According to *U.S. News and World Report*, there are three factors contributing to their selective score for national universities: test scores of enrollees on the Critical Reading and Math portions of the SAT or Composite ACT score (50% of the selectivity score); the proportion of enrolled freshmen who graduated in the top 10% of their high school classes (40% of the selectivity score) and the acceptance rate, or the ratio of students admitted to applicants (10% of the selectivity score).

The 38 "most selective national universities" as ranked by *U.S. News and World Report* in 2008 are listed below. The developers of *Core-Plus Mathematics* are aware of students having been accepted into at least 34 of these 38 universities.

Selective National Universities That Have Accepted CPMP Students

Boston College	Tufts University
Brandeis University	University of California—Berkeley
Brown University	University of California—Los Angeles
Carnegie Mellon University	University of California—San Diego
Columbia University	University of California—Santa Barbara
Cornell University	University of Chicago
Dartmouth College	University of Michigan—Ann Arbor
Duke University	University of North Carolina—Chapel Hill
Emory University	University of Notre Dame
Georgetown University	University of Pennsylvania
Harvard	University of Rochester
Massachusetts Institute of Technology	University of Southern California
New York University	University of Virginia
Northwestern University	Vanderbilt University
Princeton	Wake Forest University
Rice University	Washington University in St. Louis
Stanford	Yale

As of the year 2008, the developers of *Core-Plus Mathematics* have not received information from schools indicating that any graduates have applied to the other four selective national universities listed below.

California Institute of Technology	College of William and Mary
Johns Hopkins University	Lehigh University

Visit us at www.glencoe.com

The McGraw·Hill Companies

ISBN: 978-0-07-893587-9
MHID: 0-07-893587-3

9 780078 935879 90000

www.glencoe.com

Macmillan/McGraw-Hill
Glencoe