Achieve, Inc. – High School Integrated Model Course Sequence
Alignment to *Core-Plus Mathematics* ©2008-2009

This document lists the Achieve Secondary Mathematics Benchmarks and provides the unit or units from *Core-Plus Mathematics* where each standard is addressed. For more information on *Core-Plus Mathematics* see www.wmich.edu/cpmp. For more information on the Achieve benchmarks, see www.achieve.org/node/969.

Core-Plus Mathematics Courses Aligned to Achieve Model for Integrated Mathematics Course 1

A. Proportion, Scale, and Similarity Rates, ratios and proportions are a major focus of a middle school curriculum. This course builds on that knowledge, extending proportions and scaling to arithmetic and geometric applications. The section following this one will extend these concepts to proportional functions in algebra.	
A1 Extend and apply understanding about rates and ratios, estimation, and measurement to derived measures, including weighted averages, using appropriate units and unit analysis to express and check solutions.	
a. Create and interpret scale drawings as a tool for solving problems.	Opportunity to Address: Course 1 Unit 6
b. Use unit analysis to clarify appropriate units in calculations.	Not Addressed
c. Identify applications that can be expressed using derived measures or weighted averages; use and identify potential misuses of derived measures or weighted averages.	Course 1 Unit 2 Course 2 Unit 8 Opportunity to address misuses
A2 Use ratios and proportional reasoning to apply a scale factor to a geometric object, a drawing, a three-dimensional space, or a model and analyze the effect.	
a. Extend the concept of scale factor to relate the length, area, and volume of other figures and objects.	Course 1 Unit 6 Course 2 Unit 3
A3 Identify and use relationships among volumes of common solids.	
a. Identify and apply the 3:2:1 relationship between the volumes of circular cylinders, hemispheres, and cones of the same height and circular base.	Opportunity to Address: Course 1 Unit 6 pp. 447-448
b. Recognize that the volume of a pyramid is one-third the volume of a prism of the same base area and height and use this to solve problems involving such measurements.	Course 1 Unit 6
A4 Analyze, interpret, and represent origin-centered dilations and relate them to scaling and similarity.	
a. Interpret and represent origin-centered dilations of objects on the coordinate plane.	Course 2 Unit 3
b. Explain why the image under an origin-centered dilation is similar to the original figure.	Course 2 Unit 3
c. Show that an origin-centered dilation maps a line to a line with the same slope, that dilations map parallel lines to parallel lines (lines passing through the origin remain unchanged and are parallel to themselves), and that a dilation maps a figure into a similar figure.	Course 2 Unit 3

A5 Identify and apply conditions that are sufficient to guarantee similarity of triangles.	
a. Identify two triangles as similar if the ratios of the lengths of corresponding sides are equal (SSS criterion), if the ratios of the lengths of two pairs of corresponding sides and the measures of the corresponding angles between them are equal (SAS criterion), or if two pairs of corresponding angles are congruent (AA criterion).	Course 3 Unit 3
b. Apply the SSS, SAS, and AA criteria to verify whether or not two triangles are similar.	Course 3 Unit 3
c. Apply the SSS, SAS, and AA criteria to construct a triangle similar to a given triangle using straightedge and compass or geometric software.	Course 3 Unit 3
d. Identify the constant of proportionality and determine the measures of corresponding sides and angles for similar triangles.	Course 2 Unit 3 Course 3 Unit 3
e. Use similar triangles to demonstrate that the rate of change (slope) associated with any two points on a line is a constant.	Opportunity to Address: Course 2 Unit 7
f. Recognize, use, and explain why a line drawn inside a triangle parallel to one side forms a smaller triangle similar to the original one.	Course 3 Unit 3
A6 Identify congruence as a special case of similarity; determine and apply conditions that guarantee congruence of triangles.	
a. Determine whether two plane figures are congruent by showing whether they coincide when superimposed by means of a sequence of rigid motions (translation, reflection, or rotation).	Course 3 Unit 3
b. Identify two triangles as congruent if the lengths of corresponding sides are equal (SSS criterion), if the lengths of two pairs of corresponding sides and the measures of the corresponding angles between them are equal (SAS criterion), or if two pairs of corresponding angles are congruent and the lengths of the corresponding sides between them are equal (ASA criterion).	Course 3 Unit 3
c. Apply the SSS, SAS, and ASA criteria to verify whether or not two triangles are congruent.	Course 3 Unit 3
d. Apply the definition and characteristics of congruence to make constructions, solve problems, and verify basic properties of angles and triangles.	Course 3 Unit 3
A7 Extend the concepts of similarity and congruence to other polygons in the plane.	
a. Identify two polygons as similar if have the same number of sides and angles, if corresponding angles have the same measure, and if corresponding sides are proportional; identify two polygons as congruent if they are similar and their constant of proportionality equals one.	Course 3 Unit 3
b. Determine whether or not two polygons are similar.	Course 3 Unit 3
c. Use examples to show that analogues of the SSS, SAS, and AA criteria for similarity of triangles do not work for polygons with more than three sides.	Course 3 Unit 3

B. Proportional Functions Linear patterns of growth are a focus of the middle school curriculum. Description, analysis, and interpretation of lines should continue to be reinforced and extended, as students work in this course with functions that express direct proportions. The reciprocal functions introduced here should be linked back to student experience with proportions and with the simple exponential patterns of growth studied in middle school. These functions are strongly linked to the concepts of scaling and similarity addressed earlier in this course. Also included here is a first look at how changes in parameters affect the graph of a function.	
B1 Recognize, graph, and use direct proportional relationships.	
a. Analyze the graph of a direct proportional relationship, $f(x) = kx$ and identify its key characteristics.	Course 1 Unit 3 Course 2 Unit 1
b. Compare and contrast the graphs of x = k, y = k and y = kx, where k is a constant.	Course 1 Unit 3
c. Recognize and provide a logical argument that if $f(x)$ is a linear function, $g(x) = f(x) - f(0)$ represents a direct proportional relationship.	Opportunity to Address: Course 3 Unit 5
d. Recognize quantities that are directly proportional and express their relationship symbolically.	Course 2 Unit 1
B2 Recognize, graph, and use reciprocal relationships.	
a. Analyze the graph of reciprocal relationships, $f(x) = k/x$ and identify its key characteristics.	Course 2 Unit 1
b. Recognize quantities that are inversely proportional and express their relationship symbolically.	Course 2 Unit 1
B3 Distinguish among and apply linear, direct proportional, and reciprocal relationships; identify and distinguish among applications that can be expressed using these relationships.	
a. Identify whether a table, graph, formula, or context suggests a linear, direct proportional, or reciprocal relationship.	Course 2 Unit 1
b. Create graphs of linear, direct proportional, and reciprocal functions by hand and using technology.	Course 2 Unit 1
c. Distinguish practical situations that can be represented by linear, directly proportional, or inversely proportional relationships; analyze and use the characteristics of these relationships to answer questions about the situation.	Course 2 Unit 1
B4 Create, interpret, and apply mathematical models to solve problems arising from contextual situations that involve linear relationships.	
a. Distinguish relevant from irrelevant information, identify missing information, and find what is needed or make appropriate estimates.	Course 1 Unit 3 Course 2 Unit 1
b. Apply problem solving heuristics to practical problems: Represent and analyze the situation using symbols, graphs, tables, or diagrams; assess special cases; consider analogous situations; evaluate progress; check the reasonableness of results; and devise independent ways of verifying results.	Throughout – In the algebra strand see: Course 1 Unit 1, Unit 3, Unit 5, and Unit 7, Course 2 Unit 1, Unit 2, Unit 5
B5 Explain and illustrate the effect of varying the parameters *m* and *b* in the family of linear functions and varying the parameter *k* in the families of directly proportional and reciprocal functions.	Course 1 Unit 3 Course 2 Unit 1

C. Fundamentals of Logic This relatively short unit formalizes the vocabulary and methods of reasoning that form the foundation for logical arguments in mathematics. Examples should be taken from numeric and algebraic branches of mathematics as well as from everyday reasoning and argument. While this unit emphasizes the application of reasoning in a broad spectrum of contexts, the following unit will mainly apply logical thinking to geometric contexts.	
C1 Use mathematical notation, terminology, syntax, and logic; use and interpret he vocabulary of logic to describe statements and the relationship between statements.	
a. Identify and give examples of definitions, conjectures, theorems, proofs, and counterexamples.	Course 2 Unit 3 Course 3 Unit 1 and Unit 3
b. Describe logical statements using such terms as assumption, hypothesis, conclusion, converse, and contrapositive.	Course 3 Unit 1 and Unit 3
C2 Make, test, and confirm or refute conjectures using a variety of methods.	
a. Distinguish between inductive and deductive reasoning; explain and illustrate the importance of generalization in mathematics.	Course 3 Unit 1
b. Construct simple logical arguments and proofs; determine simple counterexamples.	Course 3 Unit 1 and Unit 3 Course 2 Unit 3
c. Demonstrate through example or explanation how indirect reasoning can be used to establish a claim.	Course 3 Unit 1
d. Recognize syllogisms, tautologies, and circular reasoning and use them to assess the validity of an argument.	Not Addressed
e. Recognize and avoid flawed reasoning; recognize flaws or gaps in the reasoning used to support an argument.	Course 3 Unit 1
C3 Analyze and apply algorithms for searching, for sorting, and for solving optimization problems.	
a. Identify and apply algorithms for searching, such as sequential and binary.	Not addressed
b. Describe and compare simple algorithms for sorting, such as bubble sort, quick sort, and bin sort.	Not addressed
c. Know and apply simple optimization algorithms.	Course 1 Unit 4 Course 2 Unit 6
D. Geometric Relationships, Proof, and Constructions Once students have gained experience with logic in multiple venues, geometry—partially because of its physical aspects—provides an excellent context in which to hone reasoning skills. This section identifies coordinate transformations as one example of generalization in mathematics. It applies generalization as well as inductive and deductive reasoning to establish similarity theorems (introduced earlier) and geometric constructions. This topic also offers the opportunity to reinforce the theorems about angles and triangles encountered in middle school.	
D1 Interpret, represent, and verify geometric relationships.	
a. Use the Pythagorean theorem to determine slant height, surface area, and volume for pyramids and cones; justify the process through diagrams and logical reasoning.	Course 1 Unit 6
b. Present and analyze geometric proofs using paragraphs or two-column or flow-chart formats.	Course 3 Unit 1, Unit 3
c. Use coordinates and algebraic techniques to interpret, represent, and verify geometric relationships in the plane.	Course 2 Unit 3, Unit 7
D2 Analyze, execute, explain, and apply simple geometric constructions.	
a. Perform and explain simple straightedge and compass constructions.	Course 1 Unit 6 Course 3 Unit 1, Unit 3
b. Apply properties of lines and angles to perform and justify basic geometric constructions.	Course 3 Unit 1, Unit 3

c. Use geometric computer or calculator packages to create and test conjectures about geometric properties or relationships.	CPMP-Tools Computer Software Course 2 Unit 3 Course 3 Unit 3, Unit 6
D3 Show how similarity of right triangles allows the trigonometric functions sine, cosine, and tangent to be properly defined as ratios of sides.	
a. Know the definitions of sine, cosine, and tangent as ratios of sides in a right triangle and use trigonometry to calculate the length of sides, measure of angles, and area of a triangle.	Course 2 Unit 7
b. Derive, interpret and use the identity $\sin^2\_ + \cos^2\_ = 1$ for angles _ between 0° and 90°.	Course 4 Unit 4 Opportunity to Address: Course 2 Unit 7 p. 478 Course 3 Unit 1 p. 68
E. **Linear Equations, Inequalities and Systems** Considering what happens when two or more conditions exist is the theme that ties together the ideas found in the final two sections of the course. Understanding the language and meaning of mathematical terms lays the foundation for the solution of systems of equations and inequalities. Linear systems provide another opportunity to reinforce the basics of linear functions and offer a myriad of opportunities for contextual problem solving.	
E1 Know the concepts of sets, elements, empty set, relations (e.g., belong to), and subsets, and use them to represent relationships among objects and sets of objects.	
a. Recognize and use different methods to define sets (lists, defining property).	Course 4 Unit 10 Opportunity to Address: Course 3 Unit 1
b. Perform operations on sets: union, intersection, complement.	Course 4 Unit 10 Opportunity to Address: Course 3 Unit 1
c. Create and interpret Venn diagrams to solve problems.	Course 1 Unit 8 Opportunity to Address: Course 3 Unit 1 and review tasks
d. Identify whether a given set is finite or infinite; give examples of both finite and infinite sets.	Opportunity to Address: Course 1 Unit 2 Course 3 Unit 1
E2 Use and interpret relational conjunctions (“and,” “or,” “not”), terms of causation (“if… then”), and equivalence (“if and only if”).	
a. Distinguish between the common uses of such terms in everyday language and their use in mathematics.	Course 3 Unit 1, Unit 2, Unit 3
b. Relate and apply these operations to situations involving sets.	Course 4 Unit 10
E3 Solve equations and inequalities involving the absolute value of a linear expression in one variable.	
a. Use conjunctions and disjunctions to express equations and inequalities involving absolute value as compound sentences that do not involve absolute value.	Opportunity to Address: Course 3 Unit 2
b. Graph the solution of a single-variable inequality involving the absolute value of a linear expression as an open or closed interval on the number line or as a union of two of them.	Course 3 Unit 2
E4 Solve and graph the solution of a linear inequality in two variables.	
a. Know what it means to be a solution of a linear inequality in two variables, represent solutions algebraically and graphically, and provide examples of ordered pairs that lie in the solution set.	Course 2 Unit 1 Course 3 Unit 2
b. Graph a linear inequality in two variables and explain why the graph is always a half-plane (open or closed).	Course 2 Unit Course 3 Unit 2

E5 Solve systems of two or more linear inequalities in two variables and graph the solution set.	Course 2 Unit 1 Course 3 Unit 2
E6 Solve systems of linear equations in two and three variables using algebraic procedures; describe the possible arrangements of the graphs of three linear equations in three variables and relate these to the number of solutions of the corresponding system of equations.	Course 2 Unit 1 Course 3 Unit 2
E7 Recognize and solve problems that can be modeled using a linear inequality or a system of linear equations or inequalities; interpret the solution(s) in terms of the context of the problem.	Course 2 Unit 1 Course 3 Unit 2
F. Counting and Computing Probability for Compound Events The final topic addressed in this integrated course extends the compound thinking developed earlier from algebraic contexts to those involving discrete events. Counting the number of ways a series of events can occur and applying prior knowledge of probability encourages students to see linkages across mathematical content areas. As with linear equations, inequalities, and systems, these topics have important contextual applications.	
F1 Represent and calculate probabilities associated with compound events.	
a. Distinguish between dependent and independent events.	Course 2 Unit 8
b. Use Venn diagrams to summarize information about compound events.	Course 1 Unit 8
c. Represent bivariate categorical data in a two-way frequency table; show how such a table can be used effectively to calculate and study relationships among probabilities for two events.	Course 2 Unit 8
d. Recognize probability problems that can be represented by geometric diagrams, on the number line, or in the coordinate plane; represent such situations geometrically and apply geometric properties of length or area to calculate the probabilities.	Course 1 Unit 8 Course 2 Unit 8
e. Use probability to interpret odds and risks and recognize common misconceptions.	Course 3 Unit 4
F2 Construct and interpret discrete graphs and charts to represent contextual situations.	
a. Construct and interpret network graphs and use them to diagram social and organizational networks.	Course 1 Unit 4 Course 2 Unit 6
b. Construct and interpret decision trees to represent the possible outcomes of independent events.	Course 2 Unit 8
c. Construct and interpret flow charts.	Not Addressed
F3 Determine the number of ways events can occur using permutations, combinations, and other systematic counting methods.	
a. Know and apply organized counting techniques such as the Fundamental Counting Principle.	Course 4 Unit 8
b. Distinguish between counting situations that do not permit replacement and situations that do permit replacement.	Course 4 Unit 8
c. Distinguish between situations where order matters and situations where it does not; select and apply appropriate means of computing the number of possible arrangements of the items in each case.	Course 4 Unit 8
d. Interpret and simplify expressions involving factorial notation; use factorial notation to express permutations and combinations.	Course 4 Unit 8

Core-Plus Mathematics Courses Aligned to Achieve Model for Integrated Mathematics Course 2

A. Reasoning from Data Integrated Mathematics Course 1 ended with a look at probability and its applications, and this course begins by extending those concepts to probability distributions and the information they convey that leads to rational, reasoned decision-making. Since issues of precision and number comprehension often affect decisions, a short section on those topics is included as well.	
A1 Describe key characteristics of a distribution.	
a. Identify and distinguish between discrete and continuous probability distributions.	Course 1 Unit 8 Course 2 Unit 8
b. Calculate and use the mean and standard deviation to describe the characteristics of a distribution.	Course 1 Unit 2 Course 3 Unit 4
c. Reason from empirical distributions of data to make assumptions about their underlying theoretical distributions.	Course 1 Unit 2, Unit 8 Course 3 Unit 4 Course 4 Unit 9
A2 Know and use the chief characteristics of the normal distribution.	
a. Identify examples that demonstrate that the mean and standard deviation of a normal distribution can vary independently of each other (e.g., that two normal distributions with the same mean can have different standard deviations).	Course 1 Unit 2 Course 3 Unit 4
b. Identify common examples that fit the normal distribution (height, weight) and examples that do not (salaries, housing prices, size of cities) and explain the distinguishing characteristics of each.	Course 1 Unit 2 Course 3 Unit 4
A3 Apply probability to make and communicate informed decisions.	
a. Apply probability to practical situations.	Course 1 Unit 8 Course 2 Unit 8
b. Calculate the expected value of a random variable having a discrete probability distribution and interpret the results.	Course 2 Unit 8
A4 Interpret and apply numbers used in practical situations.	
a. Interpret and compare extreme numbers.	Opportunity to Address: Course 1 Unit 5
b. Determine a reasonable degree of precision in a given situation.	Not Addressed
c. Assess the amount of error resulting from estimation and determine whether the error is within acceptable tolerance limits.	Not Addressed
d. Choose appropriate techniques and tools to measure quantities in order to achieve specified degrees of precision, accuracy, and error (or tolerance) of measurements.	Not Addressed
e. Apply significant figures, orders of magnitude, and scientific notation when making calculations or estimations.	Course 1 Unit 5 (scientific notation) Opportunity to Address: Course 2 Unit 7

B. Applying Exponents Building on the understanding of whole number exponents, students in Integrated Mathematics Course 2 will develop an understanding of the impact of a negative exponent and generalize the properties of exponents to all rational exponents. Application of the laws of exponents to numerical and algebraic monomials and their use in operations with binomials forms a foundation for important algebraic skills. Basic factoring and multiplication enable algebraic expressions to be written in various forms that provide insight and clarify information. The binomial theorem is an example of the multiplication of a binomial. Its links to the binomial distribution and to probability studied in the previous unit provide an effective bridge from the study of reasoning with data and distributions of data to the study of algebraic expressions, equations, and functions.	
B1 Interpret negative integer and rational exponents; use them to rewrite numeric expressions in alternative forms.	
a. Convert between expressions involving negative exponents and those involving only positive ones; apply the properties as necessary.	Course 1 Unit 5
b. Convert between expressions involving rational exponents and those involving roots and integral powers; apply the properties of exponents as necessary.	Course 1 Unit 5
B2 Apply the properties of exponents to transform variable expressions involving integer exponents.	
a. Know and apply the laws of exponents for integer exponents.	Course 1 Unit 5
b. Factor out common factors in expressions involving integer exponents.	Course 1 Unit 7 Course 3 Unit 5
B3 Make regular fluent use of basic algebraic identities such as $(a + b)^2 = a^2 + 2ab + b^2$; $(a - b)^2 = a^2 - 2ab + b^2$; and $(a + b)(a - b) = a^2 - b^2$.	
a. Use the distributive law to derive each of these formulas.	Course 1 Unit 7 (reviewed and used in Courses 2 and 3)
b. Use geometric representations to illustrate these formulas.	Course 1 Unit 7
B4 Know and use the binomial expansion theorem.	
a. Relate the expansion of $(a + b)^n$ to the possible outcomes of a binomial experiment and the n^{th} row of Pascal's triangle.	Course 4 Unit 8
B5 Convert between forms of numerical expressions involving roots and perform operations on numbers expressed in radical form.	Course 1 Unit 5
B6 Solve linear and simple nonlinear equations involving several variables for one variable in terms of the others; use fractional exponents and roots as needed to express the solution.	Course 2 Unit 1
C. Quadratic Functions and Equations with Real Zeros/Roots The study of quadratic functions and equations builds on the work with algebraic identities and forms begun in the last unit. Early work with quadratic functions and equations should focus on those with real zeros/roots.	
C1 Identify quadratic functions expressed in multiple forms; identify the specific information each form clarifies.	
a. Express a quadratic function as a polynomial, $f(x) = ax^2 + bx + c$, where a, b and c are constants with $a \neq 0$, and identify its graph as a parabola that opens up when a > 0 and down when $a < 0$; relate c to where the graph of the function crosses the y-axis.	Course 1 Unit 7 Course 2 Unit 5 Course 3 Unit 5
b. Express a quadratic function in factored form, $f(x) = (x - r)(x - s)$, when r and s are integers; relate the factors to the solutions of the equation $(x - r)(x - s) = 0$ ($x = r$ and $x = s$) and to the points ((r, 0) and (s, 0)) where the graph of the function crosses the x-axis.	Course 1 Unit 7 Course 2 Unit 5 Course 3 Unit 5

C2 Transform quadratic functions and relate their symbolic and graphical forms.	
a. Write a quadratic function in polynomial or standard form, $f(x) = ax^2 + bx + c$, to identify the y-intercept of the function's parabolic graph or the x-coordinate of its vertex, $x = -\frac{b}{2a}$	Course 1 Unit 7 Course 2 Unit 5
b. Write a quadratic function in factored form, $f(x) = a(x - r)\ (x - s)$, to identify the zeros of the function.	Course 1 Unit 7 Course 2 Unit 5 Course 3 Unit 5
c. Write a quadratic function in vertex form, $f(x) = a(x - h)^2 + k$, to identify the vertex and axis of symmetry of the function's parabolic graph.	Course 3 Unit 5
d. Describe the effect that changes in the leading coefficient or constant term of $f(x) = ax^2 + bx + c$ have on the shape, position, and characteristics of the graph of $f(x)$.	Course 1 Unit 7 Course 2 Unit 5
e. Determine domain and range, intercepts, axis of symmetry, and maximum or minimum.	Course 1 Unit 7 Course 2 Unit 5
C3 Solve and graph quadratic equations having real solutions using a variety of methods.	
a. Solve quadratic equations having real solutions by factoring, by completing the square, and by using the quadratic formula.	Course 1 Unit 7 Course 2 Unit 5 Course 3 Unit 5
b. Estimate the real zeros of a quadratic function from its graph; identify quadratic functions that do not have real zeros by the behavior of their graphs.	Course 1 Unit 7 Course 2 Unit 5 Course 3 Unit 5
c. Use a calculator to approximate the roots of a quadratic equation and as an aid in graphing.	Course 1 Unit 7 Course 3 Unit 5
D. Quadratic Functions and Equations with Complex Zeros/Roots This unit begins with the definition of complex numbers. Extension of the real number system to the complex number system permits solution of all quadratic equations. Students should be comfortable using a variety of solutions methods for quadratic equations and in identifying and interpreting their graphs. These techniques should then be applied to solving and graphing quadratic inequalities and transforming quadratic expressions and equations, including those that are not functions, to extract information.	
D1 Know that if *a* and *b* are real numbers, expressions of the form *a* + *bi* are called complex numbers and explain why every real number is a complex number.	
a. Explain why every real number is a complex number.	Course 3 Unit 5
b. Express the square root of a negative number in the form bi, where b is real.	Course 3 Unit 5
c. Identify complex conjugates.	Course 3 Unit 5
D2 Solve and graph quadratic equations having complex roots and find those roots.	
a. Use the quadratic formula to solve any quadratic equation and write it as a product of linear factors.	Course 1 Unit 7 Course 2 Unit 5 Course 3 Unit 5
b. Use the discriminant $D = b^2 - 4ac$ to determine the nature of the roots of the equation $ax^2 + bx + c = 0$.	Course 3 Unit 5
c. Know that complex solutions of quadratic equations with real coefficients occur in conjugate pairs and show that multiplying factors related to conjugate pairs results in a quadratic equation having real coefficients.	Course 3 Unit 5

D3 Recognize and solve practical problems that can be expressed using simple quadratic equations; interpret their solutions in terms of the context of the situation.	
a. Create, interpret, and apply mathematical models to solve problems arising from contextual situations that involve quadratic relationships; distinguish relevant from irrelevant information, identify missing information, and find what is needed or make appropriate estimates and apply problem solving heuristics.	Course 1 Unit 7 Course 2 Unit 1, Unit 5 Course 3 Unit 2, Unit 5
b. Select and explain a method of solution (e.g., exact vs. approximate) that is effective and appropriate to a given problem.	Throughout-See Course 1 Unit 7 Course 2 Unit 7
D4 Solve and graph quadratic inequalities in one or two variables.	Course 2 Unit 1 Course 3 Unit 2
D5 Manipulate quadratic equations to extract information.	Course 1 Unit 7 Course 2 Unit 5 Course 3 Unit 2, Unit 5
E. Power and Polynomial Functions and Expressions Power and polynomial functions are natural extensions of the work done in this course with quadratic functions. The majority of work in this unit involves recognizing power and polynomial functions, identifying some of their characteristics, and applying them to contextual situations. Manipulation of polynomial and rational expressions completes the unit.	
E1 Analyze power functions and identify their key characteristics.	
a. Recognize that the inverse proportional function $f(x) = k/x$ ($f(x) = kx^n$ for $n = -1$) and the direct proportional function $f(x) = kx$ ($f(x) = kx^n$ for $n = 1$) are special cases of power functions.	Course 2 Unit 1
b. Distinguish between *odd* and *even* power functions.	Course 2 Unit 1
E2 Transform the algebraic expression of power functions using properties of exponents and roots.	
a. Explain and illustrate the effect that a change in a parameter has on a power function (a change in a or n for $f(x) = ax^n$).	Course 2 Unit 1
E3 Analyze polynomial functions and identify their key characteristics.	
a. Know that polynomial functions of degree n have the general form $f(x) = ax^n + bx^{n-1} + \ldots + px^2 + qx + r$ for n an integer, $n \geq 0$ and $a \neq 0$.	Course 3 Unit 5
b. Know that a power function with an exponent that is a positive integer is a particular type of polynomial function, a *monomial* function, whose graph contains the origin.	Course 3 Unit 5
c. Distinguish among polynomial functions of low degree, i.e., constant functions, linear functions, quadratic functions, or cubic functions.	Course 3 Unit 5
d. Explain why every polynomial function of odd degree has at least one zero; identify any assumptions that contribute to your argument.	Course 3 Unit 5
e. Communicate understanding of the concept of the multiplicity of a root of a polynomial equation and its relationship to the graph of the related polynomial function.	Course 3 Unit 5
E4 Use key characteristics to identify the graphs of simple polynomial functions.	
a. Decide if a given graph or table of values suggests a simple polynomial function.	Course 3 Unit 5 Course 4 Unit 3
b. Distinguish between the graphs of simple polynomial functions.	Course 3 Unit 5 Course 4 Unit 3
c. Where possible, determine the domain, range, intercepts and end behavior of polynomial functions.	Course 3 Unit 5 Course 4 Unit 3

E5 Recognize and solve problems that can be modeled using power or polynomial functions; interpret the solution(s) in terms of the context of the problem.	
a. Use power or polynomial functions to represent quantities arising from numeric or geometric contexts such as length, area, and volume.	Course 2 Unit 1 Course 3 Unit 5
b. Solve simple polynomial equations and use technology to approximate solutions for more complex polynomial equations.	Course 3 Unit 5
E6 Perform operations on polynomial expressions.	
a. Add, subtract, multiply, and factor polynomials.	Course 3 Unit 5 Course 4 Unit 3
b. Divide one polynomial by a lower-degree polynomial.	Course 3 Unit 5 Course 4 Unit 3
E7 Use factoring to reduce rational expressions that consist of the quotient of two simple polynomials.	Course 3 Unit 5 Course 4 Unit 3
E8 Perform operations on simple rational expressions.	
a. Add, subtract, multiply, and divide rational expressions having monomial or binomial denominators.	Course 3 Unit 5 Course 4 Unit 3
b. Rewrite complex fractions composed of simple rational expressions as a simple fraction in lowest terms.	Course 3 Unit 5 Course 4 Unit 3

Core-Plus Mathematics Courses Aligned to Achieve Model for Integrated Mathematics Course 3

A. Reasoning and Proof Extending the fundamentals of mathematical reasoning introduced in Integrated Mathematics Course 1, students will formalize their understanding of mathematical logic and proof. In this course, reasoning is applied to numeric as well as geometric properties.	
A1 Use geometric examples to illustrate the relationships among undefined terms, axioms/postulates, definitions, theorems, and various methods of reasoning.	
a. Analyze and illustrate the effect of changing a definition or an assumption.	Course 3 Unit 1, Unit 3
b. Analyze the consequences of using alternative definitions; apply this especially to definitions of geometric objects.	Course 3 Unit 1, Unit 3
c. Demonstrate the effect that changing an assumption has on the validity of a conclusion.	Not addressed
A2 Present and analyze direct and indirect proofs using paragraphs or two column or flow-chart formats.	Course 3 Unit 1 and Unit 3
A3 Establish simple facts about rational and irrational numbers using logical arguments and examples.	Not addressed
A4 Given a degree of precision, determine a rational approximation for an irrational number.	Not Addressed
B. Geometric Reasoning and Proof Work with circles provides opportunities to prove and apply important and more complex geometric theorems than those encountered in previous courses in the integrated course sequence. Geometric reasoning is extended to three dimensions, assisting students in developing better spatial sense and analysis skills. An optional section applying possible changes to the parallel postulate of Euclidean geometry may be used to introduce students to a very practical example of non-Euclidean space.	
B1 Know and apply the definitions and properties of a circle and the radius, diameter, chord, tangent, secant, and circumference of a circle.	Course 2 Unit 3 Course 3 Unit 6
B2 Recognize, verify and apply statements about the properties of a circle.	
a. Recognize and apply the fact that a tangent to a circle is perpendicular to the radius at the point of tangency.	Course 3 Unit 6
b. Recognize, verify, and apply the relationships between central angles, inscribed angles, and circumscribed angles and the arcs they define.	Course 3 Unit 6
c. Recognize, verify, and apply the relationships between inscribed and circumscribed angles of a circle and the arcs and segments they define.	Course 3 Unit 6
B3 Determine the length of line segments and arcs, the magnitude of angles, and the area of shapes that they define in complex geometric drawings.	Course 2 Unit 7 Course 3 Unit 7
B4 Interpret and use locus definitions to generate two- and three-dimensional geometric objects.	Course 4 Unit 6
B5 Analyze cross-sections of basic three-dimensional objects and identify the resulting shapes.	Course 1 Unit 6 Course 4 Unit 6
a. Describe all possible results of the intersection of a plane with a cube, prism, pyramid, or sphere.	Course 1 Unit 6 Course 4 Unit 6
B6 Describe the characteristics of the three-dimensional object traced out when a one- or two-dimensional figure is rotated about an axis.	Course 4 Unit 6

B7 Analyze all possible relationships among two or three planes in space and identify their intersections.	
a. Identify a physical situation that illustrates two distinct parallel planes; identify a physical situation that illustrates two planes that intersect in a line.	Course 4 Unit 6
b. Demonstrate that three distinct planes may be parallel; two of them may be parallel to each other and intersect with the third, resulting in two parallel lines; or none may be parallel, in which case the three planes intersect in a single point, a single line, or by pairs in three parallel lines.	Course 4 Unit 6
B8 Recognize that there are geometries other than Euclidean geometry, in which the parallel postulate is not true.	Course 3 Unit 1
B9 Analyze and interpret geometry on a sphere. [OPTIONAL ENRICHMENT UNIT]	
a. Identify the parallel postulate as key in Euclidean geometry and analyze the effect of changes to that postulate	Course 3 Unit 1
b. Know and apply the definition of a great circle.	Course 3 Unit 1
c. Use latitude, longitude, and great circles to solve problems relating to position, distance, and displacement on the earth's surface.	Not Addressed
d. Interpret various two-dimensional representations for the surface of a sphere (e.g., two-dimensional maps of the Earth), called projections, and explain their characteristics.	Course 4 Unit 6
e. Describe geometry on a sphere as an example of a non-Euclidean geometry in which any two lines intersect.	Course 3 Unit 1
C. Iteration and Its Applications Recursive thinking is an important mathematical idea that naturally connects to the study of sequences and series. Sequences and series is included here as an optional topic and may be omitted if time or other constraints make its inclusion difficult.	
C1 Analyze, interpret, and describe relationships represented iteratively and recursively including those produced using a spreadsheet.	Course 1 Unit 3, Unit 5
C2 Generate and describe sequences having specific characteristics; use calculators and spreadsheets effectively to extend sequences beyond a relatively small number of terms.	
a. Generate and describe the factorial function or the Fibonacci sequence recursively.	Course 3 Unit 1, Unit 7
b. Generate and describe *arithmetic* sequences recursively; identify arithmetic sequences expressed recursively.	Course 3 Unit 7
c. Generate and describe *geometric* sequences recursively; identify geometric sequences expressed recursively.	Course 3 Unit 7
C3 Represent, derive, and apply sequences and series. [OPTIONAL ENRICHMENT UNIT]	
a. Know and use subscript notation to represent the general term of a sequence and summation notation to represent partial sums of a sequence.	Course 3 Unit 7
b. Derive and apply the formulas for the general term of arithmetic and geometric sequences.	Course 3 Unit 7
c. Derive and apply formulas to calculate sums of finite arithmetic and geometric series.	Course 3 Unit 7
d. Derive and apply formulas to calculate sums of infinite geometric series whose common ratio r is in the interval $(-1, 1)$.	Course 3 Unit 7
e. Model, analyze, and solve problems using sequences and series.	Course 3 Unit 7

D. Piecewise-Linear and Exponential Functions Linear, proportional, reciprocal, quadratic, power, and polynomial functions have been studied in previous courses. This course rounds out the function toolkit with the introduction of piecewise-linear and exponential functions and their applications.	
D1 Identify key characteristics of absolute value, step, and other piecewise-linear functions and graph them.	
a. Interpret the algebraic representation of a piecewise-linear function; graph it over the appropriate domain.	Course 1 Unit 1(step p. 16) Course 3 Unit 8 (ceiling and floor functions p. 553) Opportunity to Address: Course 1 Unit 3
b. Write an algebraic representation for a given piecewise-linear function.	Course 1 Unit 1 (step p. 16) Opportunity to Address: Course 1 Unit 3
c. Determine vertex, slope of each branch, intercepts, and end behavior of an absolute value graph.	Course 3 Unit 2
d. Recognize and solve problems that can be modeled using absolute value, step, and other piecewise-linear functions.	Course 1 Unit 1 (step p.16) Course 3 Unit 2 (absolute value) Course 3 Unit 8 (ceiling and floor functions p. 553) Opportunity to Address: Course 1 Unit 3
D2 Graph and analyze exponential functions and identify their key characteristics.	
a. Describe key characteristics of the graphs of exponential functions and relate these to the coefficients in the general form $f(x) = ab^x + c$ for $b > 0$, $b \neq 1$.	Course 1 Unit 5 Course 3 Unit 7
b. Explain and illustrate the effect that a change in a parameter has on an exponential function (a change in a, b, or c for $f(x) = ab^x + c$).	Course 1 Unit 5 Course 3 Unit 7
D3 Demonstrate the effect of compound interest, decay, or growth using iteration.	
a. Identify the diminishing effect of increasing the number of times per year that interest is compounded and relate this to the notion of instantaneous compounding.	Course 1 Unit 5 Course 3 Unit 7
D4 Determine the composition of simple functions, including any necessary restrictions on the domain. [OPTIONAL ENRICHMENT UNIT]	
a. Know the relationship among the identity function, composition of functions, and the inverse of a function, along with implications for the domain.	Course 4 Unit 1 Course 3 Unit 8
D5 Determine and identify key characteristics of inverse functions. [OPTIONAL ENRICHMENT UNIT]	
a. Analyze characteristics of inverse functions.	Course 3 Unit 8
b. Identify the conditions under which the inverse of a function is a function.	Course 3 Unit 8
c. Determine whether two given functions are inverses of each other.	Course 3 Unit 8
d. Explain why the graph of a function and its inverse are reflections of one another over the line $y = x$.	Course 3 Unit 8
e. Determine the inverse of linear and simple non-linear functions, including any necessary restrictions on the domain.	Course 3 Unit 8
f. Determine the inverse of a simple polynomial or simple rational function.	Course 3 Unit 8

D6 Identify characteristics of logarithmic functions; apply logarithmic functions. **[OPTIONAL ENRICHMENT UNIT]**	
a. Identify a logarithmic function as the inverse of an exponential function.	Course 3 Unit 8 Course 4 Unit 5
b. Know and use the definition of logarithm of a number and its relation to exponents.	Course 3 Unit 8 Course 4 Unit 5
c. Prove basic properties of logarithms using properties of exponents (or the inverse exponential function).	Course 3 Unit 8 Course 4 Unit 5
d. Use properties of logarithms to manipulate logarithmic expressions in order to extract information.	Course 3 Unit 8 Course 4 Unit 5
e. Use logarithms to express and solve equations and problems.	Course 3 Unit 8 Course 4 Unit 5
f. Solve logarithmic equations; use logarithms to solve exponential equations.	Course 3 Unit 8 Course 4 Unit 5
E. Characteristics and Transformations of Function and Equation Families Students are expected to refresh their knowledge of all function relationships and deepen their understanding by distinguishing among them and identifying the result when simple coordinate transformations are applied. Building on prior experience with linear, simple polynomial, power, and exponential equations, students will solve rational and radical equations.	
E1 Distinguish among the graphs of linear, exponential, power, polynomial, or rational functions by their key characteristics.	
a. Decide whether a given exponential or power function is suggested by the graph, table of values, or underlying context of a problem.	Course 1 Unit 5, Unit 7 Course 2 Unit 1 Course 3 Unit 5, Unit 7
b. Distinguish between the graphs of exponential growth functions and those representing exponential decay.	Course 1 Unit 5 Course 3 Unit 7, Unit 8
c. Distinguish among the graphs of power functions having positive integral exponents, negative integral exponents, and exponents that are positive unit fractions ($f(x) = x^{\frac{1}{n}} = \sqrt[n]{x}$, $n > 0$, n an integer).	Course 2 Unit 1
d. Identify and explain the symmetry of an even or odd power function.	Course 2 Unit 1 Course 4 Unit 1
e. Where possible, determine the domain, range, intercepts, asymptotes, and end behavior of linear, exponential, power, polynomial, or rational functions.	Course 1 Unit 3, Unit 5, Unit 7 Course 2 Unit 1, Unit 5 Course 3 Unit 2, Unit 5 Course 4 Unit 1
E2 Distinguish among linear, exponential, polynomial, rational, and power expressions; equations; and functions by their symbolic form.	
a. Identify linear, exponential, polynomial, rational, or power expressions, equations, or functions by their general form and the position of the variable.	Course 1 Unit 3, Unit 5, Unit 7 Course 2 Unit 1, Unit 5 Course 3 Unit 2, Unit 5 Course 4 Unit 1
b. Distinguish among power expressions, equations, and functions by the type of exponent.	Course 1 Unit 5 Course 2 Unit 1
E3 Solve simple rational and radical equations in one variable.	
a. Use algebraic, numerical, graphical, and/or technological means to solve radical and rational equations.	Course 3 Unit 2, Unit 5 Course 4 Unit 3

b. Know which operations on an equation produce an equation with the same solutions and which may produce an equation with fewer or more solutions (lost or extraneous roots) and adjust solution methods accordingly.	Course 3 Unit 2, Unit 5
E4 Recognize and solve problems that can be modeled using exponential or power functions; interpret the solution(s) in terms of the context of the problem.	
a. Use exponential functions to represent *growth* functions, such as $f(x) = an^x$ ($a > 0$ and $n > 1$), and *decay* functions, such as $f(x) = an^{-x}$ ($a > 0$ and $n > 1$).	Course 1 Unit 5 Course 3 Unit 7, Unit 8 Course 4 Unit 1, Unit 5
b. Use power functions to represent quantities arising from geometric contexts such as length, area, and volume.	Course 1 Unit 7 Course 2 Unit 1
c. Use the laws of exponents to determine exact solutions for problems involving exponential or power functions where possible; otherwise approximate the solutions graphically or numerically.	Course 1 Unit 5 Course 2 Unit 1, Unit 5 Course 3 Unit 8
E5 Explain, illustrate, and identify the effect of simple coordinate transformations on the graph of a function.	
a. Interpret the graph of $y = f(x - a)$ as the graph of $y = f(x)$ shifted $\lvert a \rvert$ units to the right ($a > 0$) or the left ($a < 0$).	Course 4 Unit 1
b. Interpret the graph of $y = f(x) + a$ as the graph of $y = f(x)$ shifted $\lvert a \rvert$ units up ($a > 0$) or down ($a < 0$).	Course 4 Unit 1 See also: Course 1 Unit 5, Unit 7 Course 3 Unit 7
c. Interpret the graph of $y = f(ax)$ as the graph of $y = f(x)$ expanded horizontally by a factor of $\frac{1}{\lvert a \rvert}$ if $0 < \lvert a \rvert < 1$ or compressed horizontally by a factor $\lvert a \rvert$ if $\lvert a \rvert > 1$ and reflected over the y-axis if $a < 0$.	Course 4 Unit 1
d. Interpret the graph of $y = af(x)$ as the graph of $y = f(x)$ compressed vertically by a factor of $\frac{1}{\lvert a \rvert}$ if $0 < \lvert a \rvert < 1$ or expanded vertically by a factor of $\lvert a \rvert$ if $\lvert a \rvert > 1$ and reflected over the x-axis if $a < 0$.	Course 4 Unit 1 See also: Course 1 Unit 5, Unit 7 Course 3 Unit 7
e. Relate the algebraic properties of a function to the geometric properties of its graph.	Course 1 Unit 3, Unit 5, Unit 7 Course 2 Unit 1, Unit 5 Course 3 Unit 2, Unit 5, Unit 6, Unit 7 Course 4 Unit 1
F. Mathematical Modeling with Data Now that students have amassed experience with various function prototypes and with the effect of transformations on them, they would benefit from engaging in a project collecting and analyzing data. They will need to understand the differences among the major types of statistical studies. For the purposes of applying what they have learned about functions, a project that generates bivariate data would be most effective. As time permits, an optional section on transformation of data may be included to provide students with an introduction to how statisticians generally develop models for real data.	
F1 Describe the nature and purpose of sample surveys, experiments, and observational studies, relating each to the types of research questions they are best suited to address.	
a. Identify specific research questions that can be addressed by different techniques for collecting data.	Course 3 Unit 1, Unit 4
b. Critique various methods of data collection used in real-world problems, such as a clinical trial in medicine, an opinion poll, or a report on the effect of smoking on health.	Course 3 Unit 1

c. Explain why observational studies generally do not lead to good estimates of population characteristics or cause-and-effect conclusions regarding treatments.	Course 3 Unit 1
F2 Plan and conduct sample surveys, observational studies, or experiments.	
a. Recognize and explain the rationale for using randomness in research designs; distinguish between random sampling from a population in sample surveys and random assignment of treatments to experimental units in an experiment.	Course 3 Unit 1
b. Use simulations to analyze and interpret key concepts of statistical inference.	Course 1 Unit 8
F3 Determine, interpret, and compare linear models for data that exhibit a linear trend.	
a. Identify and evaluate methods of determining the goodness of fit of a linear model.	Course 1 Unit 3 Course 2 Unit 4
b. Use a computer or a graphing calculator to determine a linear regression equation (least-squares line) as a model for data that suggest a linear trend.	Course 1 Unit 3 Course 2 Unit 4
c. Use and interpret a residual plot or correlation coefficient to evaluate the goodness of fit of a regression line.	Course 2 Unit 4
d. Note the effect of outliers on the position and slope of the regression line; interpret the slope and *y*-intercept of the regression line in the context of the relationship being modeled.	Course 2 Unit 4
F4 Apply transformations to data that exhibit curvature to analyze the underlying pattern of growth and its characteristics. [OPTIONAL ENRICHMENT UNIT]	
a. Analyze and compare key characteristics of different families of functions; identify prototypical functions as potential models for given data.	Course 4 Unit 1, Unit 5
b. Apply transformations of data for the purpose of "linearizing" a scatter plot that exhibits curvature.	Course 4 Unit 5
c. Interpret the results of specific transformations in terms of what they indicate about the trend of the original data.	Course 4 Unit 5
d. Estimate the rate of exponential growth or decay by fitting a regression model to appropriate data transformed by logarithms.	Course 4 Unit 5
e. Estimate the exponent in a power model by fitting a regression model to appropriate data transformed by logarithms.	Course 4 Unit 5
f. Analyze how linear transformations of data affect measures of center and spread, the slope of a regression line, and the correlation coefficient.	Course 1 Unit 2 Course 2 Unit 4
g. Use transformation techniques to select, interpret, and apply mathematical functions to summarize and model data; include models involving the functions and relationships found in all three model integrated courses.	Course 4 Unit 1, Unit 5