

1-1 Study Guide and Intervention***A Plan for Problem Solving*****Four-Step Problem-Solving Plan**

When solving problems, it is helpful to have an organized plan to solve the problem. The following four steps can be used to solve any math problem.

- 1. Explore** – Get a general understanding of the problem. What information is given?
- 2. Plan** – Select a strategy to solve the problem and estimate the answer.
- 3. Solve** – Carry out your plan to solve the problem.
- 4. Check** – Determine the reasonableness of your answer compared to your estimate.

Example 1 Use the four-step plan to solve the problem.

RECREATION A canoe rental store along the Illinois River in Oklahoma has 30 canoes that it rents on a daily basis during the summer season. If canoes rent for \$15 per day, how much money can the store collect for canoe rentals during the month of July?

Explore You know that they rent 30 canoes per day for \$15 each. You need to determine the total amount of money that can be collected during the month of July.

Plan First, find the total amount of money that can be collected each day by finding the product of 30 and 15. Next, multiply the previous result by 31, the number of days in July. You can estimate this result by 30. $30 \times 15 \times 30 = 13,500$

Solve Since $30 \times \$15 = \450 , the canoe rental store can collect \$450 in rental fees each day. This means the total amount of money that could be collected during the month of July is $\$450 \times 31$ or \$13,950.

Check Is your answer reasonable? The answer is close to the estimate of \$13,500.

Exercises

Use the four-step plan to solve each problem.

- 1. MONEY** Colin works for his dad during summer vacation. His dad pays him \$5.20 per hour and he works 20 hours per week. How much will Colin earn during his 8-week summer vacation?
- 2. BOOKS** A student assistant in the school library is asked to shelve 33 books. If he puts away 9 books the first hour and then 6 books each hour after that, how long will it take him to shelve all 33 books?
- 3. SHOPPING** Alicia bought a \$48 sweater on sale for \$25 and a \$36 purse on sale for \$22. How much did Alicia save?
- 4. MAIL** It cost Ramon \$3.73 to mail a package to his grandmother. The post office charged \$2.38 for the first pound and 45 cents for each additional pound. How much did the package weigh?

1-2 Study Guide and Intervention***Powers and Exponents***

$$\begin{array}{c}
 \text{Exponent} \\
 \swarrow \\
 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81 \\
 \uparrow \quad \underbrace{\hspace{2cm}} \\
 \text{Base} \quad \text{common factors}
 \end{array}$$

The **exponent** tells you how many times the **base** is used as a factor.

Example 1 Write 6^3 as a product of the same factor.

The base is 6. The exponent 3 means that 6 is used as a factor 3 times.

$$6^3 = 6 \cdot 6 \cdot 6$$

Example 2 Evaluate 5^4 .

$$\begin{aligned}
 5^4 &= 5 \cdot 5 \cdot 5 \cdot 5 \\
 &= 625
 \end{aligned}$$

Example 3 Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ in exponential form.

The base is 4. It is used as a factor 5 times, so the exponent is 5.

$$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5$$

Exercises

Write each power as a product of the same factor.

1. 7^3

2. 2^7

3. 9^2

4. 15^4

Evaluate each expression.

5. 3^5

6. 7^3

7. 8^4

8. 5^3

Write each product in exponential form.

9. $2 \cdot 2 \cdot 2 \cdot 2$

10. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

11. $10 \cdot 10 \cdot 10$

12. $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

13. $12 \cdot 12 \cdot 12$

14. $5 \cdot 5 \cdot 5 \cdot 5$

15. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

16. $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

1-3 Study Guide and Intervention**Squares and Square Roots**

The product of a number and itself is the **square** of the number. Numbers like 4, 25, and 2.25 are called **perfect squares** because they are squares of rational numbers. The factors multiplied to form perfect squares are called **square roots**. Both $5 \cdot 5$ and $(-5)(-5)$ equal 25. So, 25 has two square roots, 5 and -5 . A **radical sign**, $\sqrt{\quad}$, is the symbol used to indicate the *positive* square root of a number. So, $\sqrt{25} = 5$.

Examples

- 1 Find the square of 5.**

$$5 \cdot 5 = 25$$

- 2 Find the square of 16.**

$$16 \quad x^2 \quad \text{ENTER} \quad 256$$

- 3 Find $\sqrt{49}$.**

$$7 \cdot 7 = 49, \text{ so } \sqrt{49} = 7.$$

- 4 Find $\sqrt{169}$.**

$$\text{2nd} \quad [\sqrt{\quad}] \quad 169 \quad \text{ENTER} \quad 13$$

$$\text{So, } \sqrt{169} = 13.$$

Example 5 A square tile has an area of 144 square inches. What are the dimensions of the tile?

$$\text{2nd} \quad [\sqrt{\quad}] \quad 144 \quad \text{ENTER} \quad 12 \quad \text{Find the square root of 144.}$$

So, the tile measures 12 inches by 12 inches.

Exercises

Find the square of each number.

1. 2

2. 9

3. 14

4. 15

5. 21

6. 45

Find each square root.

7. $\sqrt{16}$

8. $\sqrt{36}$

9. $\sqrt{256}$

10. $\sqrt{1,024}$

11. $\sqrt{361}$

12. $\sqrt{484}$

1-4 Study Guide and Intervention***Order of Operations***

Use the **order of operations** to evaluate numerical expressions.

1. Do all operations within grouping symbols first.
2. Evaluate all powers before other operations.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Example 1 Evaluate $(10 - 2) - 4 \cdot 2$.

$$\begin{aligned} (10 - 2) - 4 \cdot 2 &= 8 - 4 \cdot 2 && \text{Subtract first since } 10 - 2 \text{ is in parentheses.} \\ &= 8 - 8 && \text{Multiply 4 and 2.} \\ &= 0 && \text{Subtract 8 from 8.} \end{aligned}$$

Example 2 Evaluate $8 + (1 + 5)^2 \div 4$.

$$\begin{aligned} 8 + (1 + 5)^2 \div 4 &= 8 + 6^2 \div 4 && \text{First, add 1 and 5 inside the parentheses.} \\ &= 8 + 36 \div 4 && \text{Find the value of } 6^2. \\ &= 8 + 9 && \text{Divide 36 by 4.} \\ &= 17 && \text{Add 8 and 9.} \end{aligned}$$

Exercises

Evaluate each expression.

1. $(1 + 7) \times 3$

2. $28 - 4 \cdot 7$

3. $5 + 4 \cdot 3$

4. $(40 \div 5) - 7 + 2$

5. $35 \div 7(2)$

6. 3×10^3

7. $45 \div 5 + 36 \div 4$

8. $42 \div 6 \times 2 - 9$

9. $2 \times 8 - 3^2 + 2$

10. $5 \times 2^2 + 32 \div 8$

11. $3 \times 6 - (9 - 8)^3$

12. 3.5×10^2

1-5 Study Guide and Intervention***Problem Solving Investigation: Guess and Check***

When solving problems, one strategy that is helpful to use is guess and check. Based on the information in the problem, you can make a guess of the solution. Then use computations to check if your guess is correct. You can repeat this process until you find the correct solution.

You can use guess and check, along with the following four-step problem solving plan to solve a problem.

- Explore** • Read and get a general understanding of the problem.
- Plan** • Make a plan to solve the problem and estimate the solution.
- Solve** • Use your plan to solve the problem.
- Check** • Check the reasonableness of your solution.

Example

VETERINARY SCIENCE Dr. Miller saw 40 birds and cats in one day. All together the pets he saw had 110 legs. How many of each type of animal did Dr. Miller see in one day?

Explore You know that Dr. Miller saw 40 birds and cats total. You also know that there were 110 legs in all. You need to find out how many of each type of animal he saw in one day.

Plan Make a guess and check it. Adjust the guess until you get the correct answer.

Solve

Number of birds	Number of cats	Total number of feet
20	20	$2(20) + 4(20) = 120$
30	10	$2(30) + 4(10) = 100$
25	15	$2(25) + 4(15) = 110$

Check 25 birds have 50 feet. 15 cats have 60 feet. Since $50 + 60$ is 110, the answer is correct.

Exercise

GEOMETRY In a math class of 26 students, each girl drew a triangle and each boy drew a square. If there were 89 sides in all, how many girls and how many boys were in the class?

1-6 Study Guide and Intervention**Algebra: Variables and Expressions**

To evaluate an algebraic expression you replace each variable with its numerical value, then use the order of operations to simplify.

Example 1 Evaluate $6x - 7$ if $x = 8$.

$$\begin{aligned} 6x - 7 &= 6(8) - 7 && \text{Replace } x \text{ with } 8. \\ &= 48 - 7 && \text{Use the order of operations.} \\ &= 41 && \text{Subtract 7 from 48.} \end{aligned}$$

Example 2 Evaluate $5m - 3n$ if $m = 6$ and $n = 5$.

$$\begin{aligned} 5m - 3n &= 5(6) - 3(5) && \text{Replace } m \text{ with } 6 \text{ and } n \text{ with } 5. \\ &= 30 - 15 && \text{Use the order of operations.} \\ &= 15 && \text{Subtract 15 from 30.} \end{aligned}$$

Example 3 Evaluate $\frac{ab}{3}$ if $a = 7$ and $b = 6$.

$$\begin{aligned} \frac{ab}{3} &= \frac{(7)(6)}{3} && \text{Replace } a \text{ with } 7 \text{ and } b \text{ with } 6. \\ &= \frac{42}{3} && \text{The fraction bar is like a grouping symbol.} \\ &= 14 && \text{Divide.} \end{aligned}$$

Example 4 Evaluate $x^3 + 4$ if $x = 3$.

$$\begin{aligned} x^3 + 4 &= 3^3 + 4 && \text{Replace } x \text{ with } 3. \\ &= 27 + 4 && \text{Use the order of operations.} \\ &= 31 && \text{Add 27 and 4.} \end{aligned}$$

Exercises

Evaluate each expression if $a = 4$, $b = 2$, and $c = 7$.

1. $3ac$

2. $5b^3$

3. abc

4. $5 + 6c$

5. $\frac{ab}{8}$

6. $2a - 3b$

7. $\frac{b^4}{4}$

8. $c - a$

9. $20 - bc$

10. $2bc$

11. $ac - 3b$

12. $6a^2$

13. $7c$

14. $6a - b$

15. $ab - c$

1-7 Study Guide and Intervention**Algebra: Equations**

- An **equation** is a sentence in mathematics that contains an equals sign, =.
- The **solution** of an equation is the value that when substituted for the variable makes the equation true.

Example 1 Solve $23 + y = 29$ mentally.

$23 + y = 29$ Write the equation.
 $23 + 6 = 29$ You know that $23 + 6$ is 29.
 $29 = 29$ Simplify.
 The solution is 6.

Example 2

TRAVEL On their annual family vacation, the Whites travel 790 miles in two days. If on the first day they travel 490 miles, how many miles must they drive on the second day to reach their destination?

The total distance to travel in two days is 790 miles.

Let m represent the distance to travel on day two.

$$m + 490 = 790$$

$m + 490 = 790$ Write the equation.

$300 + 490 = 790$ Replace m with 300 to make the equation true.

$790 = 790$ Simplify.

The number 300 is the solution. The distance the Whites must travel on day two is 300 miles.

Exercises

Solve each equation mentally.

1. $k + 7 = 15$

2. $g - 8 = 20$

3. $6y = 24$

4. $\frac{a}{3} = 9$

5. $\frac{x}{6} = 9$

6. $8 + r = 24$

7. $12 \cdot 8 = h$

8. $n \div 11 = 8$

9. $48 \div 12 = x$

10. $h - 12 = 24$

11. $19 + y = 28$

12. $9f = 90$

Define a variable. Then write and solve an equation.

13. **MONEY** Aaron wants to buy a video game. The game costs \$15.50. He has \$10.00 saved from his weekly allowance. How much money does he need to borrow from his mother in order to buy the video game?

1-8 Study Guide and Intervention**Algebra: Properties**

Property	Arithmetic	Algebra
Distributive Property	$5(3 + 4) = 5(3) + 5(4)$	$a(b + c) = a(b) + a(c)$
Commutative Property of Addition	$5 + 3 = 3 + 5$	$a + b = b + a$
Commutative Property of Multiplication	$5 \times 3 = 3 \times 5$	$a \times b = b \times a$
Associative Property of Addition	$(2 + 3) + 4 = 2 + (3 + 4)$	$(a + b) + c = a + (b + c)$
Associative Property of Multiplication	$(4 \times 5) \times 6 = 4 \times (5 \times 6)$	$(a \times b) \times c = a \times (b \times c)$
Identity Property of Addition	$5 + 0 = 5$	$a + 0 = a$
Identity Property of Multiplication	$5 \times 1 = 5$	$a \times 1 = a$

Example 1 Use the Distributive Property to write $6(4 + 3)$ as an equivalent expression. Then evaluate the expression.

$$\begin{aligned} 6(4 + 3) &= 6 \cdot 4 + 6 \cdot 3 && \text{Apply the Distributive Property.} \\ &= 24 + 18 && \text{Multiply.} \\ &= 42 && \text{Add.} \end{aligned}$$

Example 2 Name the property shown by each statement.

$$\begin{aligned} 5 \times 4 &= 4 \times 5 && \text{Commutative Property of Multiplication} \\ 12 + 0 &= 12 && \text{Identity Property of Addition} \\ 7 + (6 + 3) &= (7 + 6) + 3 && \text{Associative Property of Addition} \end{aligned}$$

Exercises

Use the Distributive Property to write each expression as an equivalent expression. Then evaluate the expression.

- $5(7 + 2)$
- $4(9 + 1)$
- $2(6 + 7)$

Name the property shown by each statement.

- $9 \times 1 = 9$
- $(7 + 8) + 2 = 7 + (8 + 2)$
- $15 + 12 = 12 + 15$
- $(9 \times 5) \times 2 = 9 \times (5 \times 2)$
- $7 \times 3 = 3 \times 7$
- $6(3 + 2) = 6(3) + 6(2)$
- $1 \times 20 = 20$
- $3 = 0 + 3$

1-9 Study Guide and Intervention**Algebra: Arithmetic Sequences**

An **arithmetic sequence** is a list in which each term is found by adding the same number to the previous term. 1, 3, 5, 7, 9, ...

$$\begin{array}{ccccccc} 1, & 3, & 5, & 7, & 9, & \dots & \\ \uparrow & \uparrow & \uparrow & \uparrow & & & \\ +2 & +2 & +2 & +2 & & & \end{array}$$

Example 1 Describe the relationship between terms in the arithmetic sequence 17, 23, 29, 35, ... Then write the next three terms in the sequence.

$$\begin{array}{ccccccc} 17, & 23, & 29, & 35, & \dots & & \\ \uparrow & \uparrow & \uparrow & & & & \\ +6 & +6 & +6 & & & & \end{array} \quad \begin{array}{l} \text{Each term is found by adding 6 to the previous term.} \\ 35 + 6 = 41 \qquad 41 + 6 = 47 \qquad 47 + 6 = 53 \end{array}$$

The next three terms are 41, 47, and 53.

Example 2

MONEY Brian's parents have decided to start giving him a monthly allowance for one year. Each month they will increase his allowance by \$10. Suppose this pattern continues. What algebraic expression can be used to find the number of pastries offered after any given number of months? How much money will Brian receive for allowance for 10th month?

Make a table to display the sequence.

Position	Operation	Value of Term
1	$1 \cdot 10$	10
2	$2 \cdot 10$	20
3	$3 \cdot 10$	30
n	$n \cdot 10$	$10n$

Each term is 20 times its position number. So, the expression is $10n$.
How much money will Brian earn after 10 months?

$10n$ Write the expression.

$10(10) = 100$ Replace n with 10

So, for the 10th month Brian will receive \$100.

Exercises

Describe the relationship between terms in the arithmetic sequences.
Write the next three terms in the sequence.

- 2, 4, 6, 8, ...
- 4, 7, 10, 13, ...
- 0.3, 0.6, 0.9, 1.2, ...
- 200, 212, 224, 236, ...
- 1.5, 2.0, 2.5, 3.0, ...
- 12, 19, 26, 33, ...

- 7. SALES** Mama's bakery just opened and is currently selling only two types of pastry. Each month, Mama's bakery will add two more types of pastry to their menu. Suppose this pattern continues. What algebraic expression can be used to find the number of pastries offered after any given number of months? How many pastries will be offered in one year?

1-10 Study Guide and Intervention**Algebra: Equations and Functions**

The solution of an equation with two variables consists of two numbers, one for each variable that makes the equation true. When a relationship assigns exactly one output value for each input value, it is called a function. Function tables help to organize input numbers, output numbers, and function rules.

Example 1 Complete a function table for $y = 5x$. Then state the domain and range.

Choose four values for x . Substitute the values for x into the expression. Then evaluate to find the y value.

x	$5x$	y
0	$5(0)$	0
1	$5(1)$	5
2	$5(2)$	10
3	$5(3)$	15

The domain is $\{0, 1, 2, 3\}$. The range is $\{0, 5, 10, 15\}$.

Exercises

Complete the following function tables. Then state the domain and range.

1. $y = x + 4$

x	$x + 4$	y
0		
1		
2		
3		

2. $y = 10x$

x	$10x$	y
1		
2		
3		
4		

3. $y = x - 1$

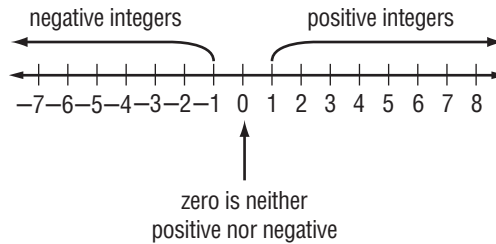
x	$x - 1$	y
2		
3		
4		
5		

4. $y = 3x$

x	$3x$	y
10		
11		
12		
13		

2-1 Study Guide and Intervention***Integers and Absolute Value***

Integers less than zero are **negative integers**. Integers greater than zero are **positive integers**.



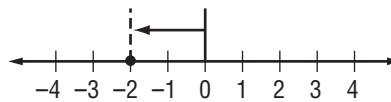
The **absolute value** of an integer is the distance the number is from zero on a number line. Two vertical bars are used to represent absolute value. The symbol for absolute value of 3 is $|3|$.

Example 1 Write an integer that represents 160 feet below sea level.

Because it represents *below* sea level, the integer is -160 .

Example 2 Evaluate $|-2|$.

On the number line, the graph of -2 is 2 units away from 0. So, $|-2| = 2$.

**Exercises**

Write an integer for each situation.

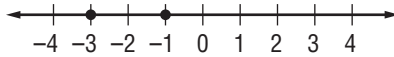
- | | |
|---------------------------------|-------------------|
| 1. 12°C above 0 | 2. a loss of \$24 |
| 3. a gain of 20 pounds | 4. falling 6 feet |

Evaluate each expression.

- | | |
|------------|-------------|
| 5. $ 12 $ | 6. $ -150 $ |
| 7. $ -8 $ | 8. $ 75 $ |
| 9. $ -19 $ | 10. $ 84 $ |

2-2**Study Guide and Intervention****Comparing and Ordering Integers**

When two numbers are graphed on a number line, the number to the left is always less than ($<$) the number to the right. The number to the right is always greater than ($>$) the number to the left.

Model**Words**

-3 is less than -1 . -1 is greater than -3 .

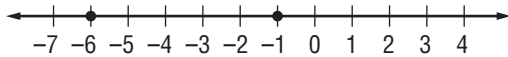
Symbols

$-3 < -1$ $-1 > -3$

The symbol points to the lesser number.

Example 1 Replace the \bullet with $<$ or $>$ to make $-1 \bullet -6$ a true sentence.

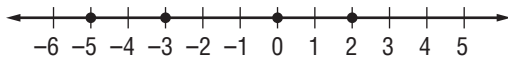
Graph each integer on a number line.



Since -1 is to the right of -6 , $-1 > -6$.

Example 2 Order the integers $2, -3, 0, -5$ from least to greatest.

To order the integers, graph them on a number line.



Order the integers by reading from left to right: $-5, -3, 0, 2$.

Exercises

1. Replace the \bullet with $<$ or $>$ to make $-5 \bullet -10$ a true sentence.
2. Order $-1, 5, -3,$ and 2 from least to greatest.
3. Order $0, -4, -2,$ and 7 from greatest to least.
4. Order $-3, |-2|, 4, 0,$ and -5 from greatest to least.

2-3 Study Guide and Intervention

The Coordinate Plane

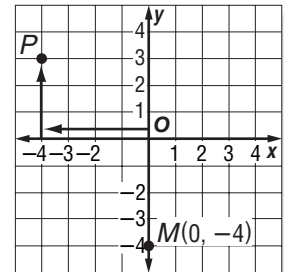
The **coordinate plane** is used to locate points. The horizontal number line is the **x-axis**. The vertical number line is the **y-axis**. Their intersection is the **origin**.

Points are located using **ordered pairs**. The first number in an ordered pair is the **x-coordinate**; the second number is the **y-coordinate**.

The coordinate plane is separated into four sections called **quadrants**.

Example 1 Name the ordered pair for point P. Then identify the quadrant in which P lies.

- Start at the origin.
 - Move 4 units left along the x-axis.
 - Move 3 units up on the y-axis.
- The ordered pair for point P is $(-4, 3)$.
P is in the upper left quadrant or quadrant II.



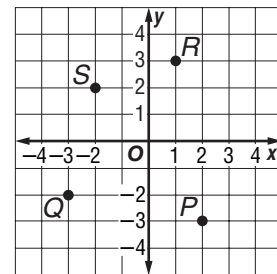
Example 2 Graph and label the point $M(0, -4)$.

- Start at the origin.
- Move 0 units along the x-axis.
- Move 4 units down on the y-axis.
- Draw a dot and label it $M(0, -4)$.

Exercises

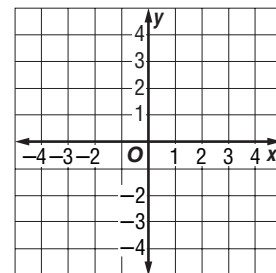
Name the ordered pair for each point graphed at the right. Then identify the quadrant in which each point lies.

- | | |
|------|------|
| 1. P | 2. Q |
| 3. R | 4. S |



Graph and label each point on the coordinate plane.

- | | |
|---------------|----------------|
| 5. $A(-1, 1)$ | 6. $B(0, -3)$ |
| 7. $C(3, 2)$ | 8. $D(-3, -1)$ |
| 9. $E(1, -2)$ | 10. $F(1, 3)$ |



2-4 Study Guide and Intervention***Adding Integers***

For integers with the same sign:

- the sum of two positive integers is positive.
- the sum of two negative integers is negative.

For integers with different signs, subtract their absolute values. The sum is:

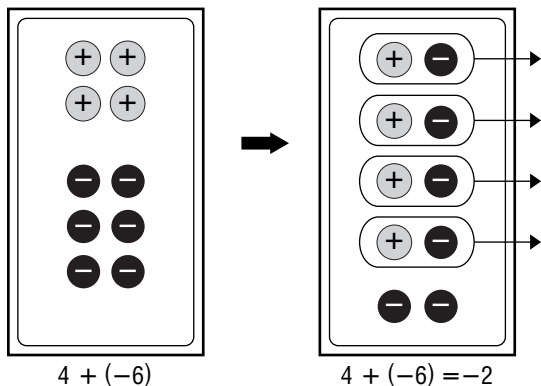
- positive if the positive integer has the greater absolute value.
- negative if the negative integer has the greater absolute value.

To add integers, it is helpful to use counters or a number line.

Example Find $4 + (-6)$.

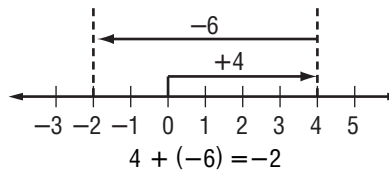
Method 1 Use counters.

Combine a set of 4 positive counters and a set of 6 negative counters on a mat.



Method 2 Use a number line.

- Start at 0.
- Move 4 units right.
- Then move 6 units left.

**Exercises**

Add.

1. $-5 + (-2)$

2. $8 + 1$

3. $-7 + 10$

4. $16 + (-11)$

5. $-22 + (-7)$

6. $-50 + 50$

7. $-10 + (-10)$

8. $100 + (-25)$

9. $-35 + -20$

Evaluate each expression if $a = 8$, $b = -8$, and $c = 4$.

10. $a + 15$

11. $b + (-9)$

12. $a + b$

13. $b + c$

14. $-10 + c$

15. $12 + b$

2-5**Study Guide and Intervention*****Subtracting Integers***

To subtract an integer, add its opposite.

Example 1 Find $6 - 9$.

$$\begin{aligned} 6 - 9 &= 6 + (-9) \\ &= -3 \end{aligned}$$

To subtract 9, add -9 .
Simplify.

Example 2 Find $-10 - (-12)$.

$$\begin{aligned} -10 - (-12) &= -10 + 12 \\ &= 2 \end{aligned}$$

To subtract -12 , add 12.
Simplify.

Example 3 Evaluate $a - b$ if $a = -3$ and $b = 7$.

$$\begin{aligned} a - b &= -3 - 7 \\ &= -3 + (-7) \\ &= -10 \end{aligned}$$

Replace a with -3 and b with 7.
To subtract 7, add -7 .
Simplify.

Exercises**Subtract.**

1. $7 - 9$

2. $20 - (-6)$

3. $-10 - 4$

4. $0 - 12$

5. $-7 - 8$

6. $13 - 18$

7. $-20 - (-5)$

8. $-8 - (-6)$

9. $25 - (-14)$

10. $-75 - 50$

11. $15 - 65$

12. $19 - (-10)$

Evaluate each expression if $m = -2$, $n = 10$, and $p = 5$.

13. $m - 6$

14. $9 - n$

15. $p - (-8)$

16. $p - m$

17. $m - n$

18. $-25 - p$

2-6**Study Guide and Intervention*****Problem-Solving Investigation: Look for a Pattern***

Looking for a pattern is one strategy that can help you when solving problems. You can use the four-step problem-solving plan along with looking for a pattern to solve problems.

Explore	• Determine what information is given in the problem and what you need to find.
Plan	• Select a strategy including a possible estimate.
Solve	• Solve the problem by carrying out your plan.
Check	• Examine your answer to see if it seems reasonable.

Example

MEMBERSHIP The local tennis club started the year with 675 members. In one month they had 690 members. After two months they had 705 members. After three months they had 720 members. When the tennis club reaches 750 members they will close their enrollment. How many months will it take the club to reach their maximum enrollment if they continue adding new members at the same rate?

Explore The club began with 590 members and is adding new members every month. It needs to find out when it reaches its maximum enrollment of 750 members.

Plan Look for a pattern or rule that increases the membership each month. Then use the rule to extend the pattern to find the solution.

Solve After the initial 575 members, 15 new members joined each month. Extend the pattern to find the solution.

$$\begin{array}{cccccc} 675, & 690, & 705, & 720, & 735, & 750 \\ +15 & +15 & +15 & +15 & +15 & \end{array}$$

They will have reached their maximum enrollment in 5 months.

Check They increased by $5 \cdot 15$ or 75 members in 5 months which when added to the original 675 members is $675 + 75 = 750$. So, 5 months is a reasonable answer.

Exercises

- PRODUCE** A farmer has 42 apples sitting on his front porch. The next day there are only 36 apples left on the porch. After 2 days there are only 30 apples left on the porch and in 3 days 24 apples remain on the porch. After how many days will there be no more apples on the porch if the same amount continue to disappear each day?
- TELEPHONE** A local phone company charges a standard rate of \$3 per call. After one minute the charge is \$4.50. In two minutes the charge is \$6.00. If Susan only has \$10.00, how long can her phone conversation be if the charges per minute stay constant?

2-7**Study Guide and Intervention*****Multiplying Integers***

The product of two integers with **different** signs is **negative**.

The product of two integers with the **same** sign is **positive**.

Example 1 Multiply $5(-2)$.

$5(-2) = -10$ The integers have different signs. The product is negative.

Example 2 Multiply $-3(7)$.

$-3(7) = -21$ The integers have different signs. The product is negative.

Example 3 Multiply $-6(-9)$.

$-6(-9) = 54$ The integers have the same sign. The product is positive.

Example 4 Multiply $(-7)^2$.

$(-7)^2 = (-7)(-7)$ There are 2 factors of -7 .
 $= 49$ The product is positive.

Example 5 Simplify $-2(6c)$.

$-2(6c) = (-2 \cdot 6)c$ Associative Property of Multiplication.
 $= -12c$ Simplify.

Example 6 Simplify $2(5x)$.

$2(5x) = (2 \cdot 5)x$ Associative Property of Multiplication.
 $= 10x$ Simplify.

Example**Multiply.**

- | | | |
|------------|---------------|-------------|
| 1. $-5(8)$ | 2. $-3(-7)$ | 3. $10(-8)$ |
| 4. $-8(3)$ | 5. $-12(-12)$ | 6. $(-8)^2$ |

ALGEBRA Simplify each expression.

- | | | |
|-------------|---------------|--------------|
| 7. $-5(7a)$ | 8. $3(-2x)$ | 9. $4(6f)$ |
| 10. $7(6b)$ | 11. $-6(-3y)$ | 12. $7(-8g)$ |

ALGEBRA Evaluate each expression if $a = -3$, $b = -4$, and $c = 5$.

- | | | |
|------------|-------------|-----------|
| 13. $-2a$ | 14. $9b$ | 15. ab |
| 16. $-3ac$ | 17. $-2c^2$ | 18. abc |

2-8**Study Guide and Intervention*****Dividing Integers***

The quotient of two integers with different signs is negative.

The quotient of two integers with the same sign is positive.

Example 1 Divide $30 \div (-5)$.

$30 \div (-5)$ The integers have different signs.

$30 \div (-5) = -6$ The quotient is negative.

Example 2 Divide $-100 \div (-5)$.

$-100 \div (-5)$ The integers have the same sign.

$-100 \div (-5) = 20$ The quotient is positive.

Exercises**Divide.**

1. $-12 \div 4$

2. $-14 \div (-7)$

3. $\frac{18}{-2}$

4. $-6 \div (-3)$

5. $-10 \div 10$

6. $\frac{-80}{-20}$

7. $350 \div (-25)$

8. $-420 \div (-3)$

9. $\frac{540}{45}$

10. $\frac{-256}{16}$

ALGEBRA Evaluate each expression if $d = -24$, $e = -4$, and $f = 8$.

11. $12 \div e$

12. $40 \div f$

13. $d \div 6$

14. $d \div e$

15. $f \div e$

16. $e^2 \div f$

17. $\frac{-d}{e}$

18. $ef \div 2$

19. $\frac{f^2}{e^2}$

20. $\frac{de}{f}$

3-1 Study Guide and Intervention***Estimating with Decimals***

- When estimating a product, round to the nearest whole number by looking at the digit in the ones place and rounding down when the digit is 4 or less and up when the digit is 5 or higher.
- When estimating a quotient, find numbers that are compatible to each other in order to make it easier to divide mentally.

Example 1 Estimate a Product**Estimate 9.6×4.2 .**

$$\begin{array}{r} 9.6 \rightarrow 10 \\ \times 4.2 \rightarrow \times 4 \\ \hline 40 \end{array}$$

Round 9.6 to 10.
Round 4.2 to 4.

The product is *about* 40.**Example 2 Estimate a Quotient****Estimate $15.1 \div 5.7$.**

$$5.7 \overline{)15.1} \rightarrow 5 \overline{)15}^3$$

Round 5.7 to 5 since 15 and 5 are compatible numbers.

The quotient is *about* 3.**Exercises****Estimate the following products.**

1. 7.4×2.7

2. 9.2×8.8

3. 22.1×9.9

4. 1.1×7.5

5. $\begin{array}{r} 35.1 \\ \times 10.2 \\ \hline \end{array}$

6. $\begin{array}{r} 12.6 \\ \times 3.2 \\ \hline \end{array}$

Estimate the following quotients.

7. $24.7 \div 4.8$

8. $9.2 \div 4.7$

9. $34.2 \div 4.5$

10. $99.1 \div 24.7$

11. $44.9 \div 9.3$

12. $19.4 \div 2.1$

3-2 Study Guide and Intervention***Multiplying Decimals***

When multiplying decimals, **estimation** and **counting decimal places** can help in determining the location of the decimal point in the answer.

Example 1 Find 2.62×3 .**Method 1** Use Estimation.

$$\begin{array}{r} 2.62 \\ \times 3 \\ \hline 7.86 \end{array}$$

Estimate $2.62 \times 3 \approx 3 \times 3$ or 9

Since the estimate is 9, place the decimal point after the 7.

Method 2 Count decimal places.

$$\begin{array}{r} 2.62 \\ \times 3 \\ \hline 7.86 \end{array}$$

2 decimal places

2 decimal places

Example 2 Find 6.24×2.8 .**Method 1** Use Estimation.

$$\begin{array}{r} 6.24 \\ \times 2.8 \\ \hline 4992 \\ \underline{1248} \\ 17.472 \end{array}$$

Estimate $6.24 \times 2.8 \approx 6 \times 3$ or 18

Since the estimate is 18, place the decimal point after the 7.

Method 2 Count decimal places.

$$\begin{array}{r} 6.24 \\ \times 2.8 \\ \hline 4992 \\ \underline{1248} \\ 17.472 \end{array}$$

2 decimal places

+1 decimal place

3 decimal places

Exercises**Multiply.**

1. 4.3×5

2. 7.12×2

3. $\begin{array}{r} 9.8 \\ \times 3 \\ \hline \end{array}$

4. $\begin{array}{r} 3.14 \\ \times 6 \\ \hline \end{array}$

5. 7.1×8

6. 2.04×4

7. 2.45×3.1

8. 5.22×6.12

9. 8.8×3.7

10. $\begin{array}{r} 6.07 \\ \times 2.01 \\ \hline \end{array}$

11. $\begin{array}{r} 0.8 \\ \times 0.6 \\ \hline \end{array}$

12. $\begin{array}{r} 1.9 \\ \times 1.9 \\ \hline \end{array}$

3-3 Study Guide and Intervention***Multiplying by Powers of Ten***

- **When multiplying a decimal by a power of 10**, move the decimal point to the **right** the same number of places as the number of zeros in the power of 10.
- **When multiplying a decimal by a power of 10 less than one**, move the decimal point to the **left** the same number of places as there are after the decimal point.

Example 1 Find $7.24 \times 1,000$.

$$7.24 \times 1,000 = 7240$$

$$= 7,240$$

$$\text{So, } 7.24 \times 1,000 = 7,240$$

1,000 has 3 zeros so move the decimal point 3 places to the right. Annex zeros as needed.

Remove the decimal point and add a comma.

Example 2 Find 36.4×0.001 .

$$36.4 \times 0.001 = .0364$$

$$= 0.0364$$

$$\text{So, } 36.4 \times 0.001 = 0.0364$$

There are 3 places after the decimal point, so move the decimal point 3 places to the left. Annex zeros as needed.

Annex a zero in front of the decimal point.

Exercises**Find each product.**

1. 4.5×100

2. $0.298 \times 1,000$

3. 6.31×10

4. $0.34 \times 1,000$

5. $8.1 \times 10,000$

6. 44.73×10

7. 52.1×0.01

8. 12.21×0.0001

9. 0.56×0.1

10. 12.8×0.001

11. 35.2×0.1

12. 0.7×0.01

3-4 Study Guide and Intervention***Dividing Decimals***

- When dividing a decimal by a whole number, line up the decimal in the quotient with the decimal in the dividend.
- When dividing a decimal by a decimal, change the divisor to a whole number by multiplying both the divisor and dividend by the same power of ten and then divide as with whole numbers.

Example 1 Find $638.4 \div 3$.Estimate: $638.4 \div 3 \approx 639 \div 3$ or 213

$$\begin{array}{r} 212.8 \\ 3 \overline{)638.4} \\ \underline{-6} \\ 03 \\ \underline{-3} \\ 08 \\ \underline{-6} \\ 24 \\ \underline{-24} \\ 0 \end{array}$$

Place the decimal point directly above the decimal point in the dividend.

Divide as with whole numbers.

So, $638.4 \div 3 = 212.8$. Check for Reasonableness: $212.8 \approx 213$ **Example 2 Find $12.36 \div 1.2$.**Estimate: $12.36 \div 1.2 \approx 12 \div 1 = 12$

$$\begin{array}{r} 10.3 \\ 1.2 \overline{)12.36} \\ \underline{-12} \\ 03 \\ \underline{-0} \\ 36 \\ \underline{-36} \\ 0 \end{array}$$

Place the decimal point. Divide as with the whole numbers.

Multiply the same number, 10.

To change to a whole number, multiply by 10.

So, $12.36 \div 1.2$ is 10.3. Check for Reasonableness: $10.3 \approx 12$ **Exercises****Divide.**

1. $15.32 \div 5$ 2. $98.2 \div 4$ 3. $123.2 \div 8$ 4. $13.09 \div 7$

5. $30.45 \div 1.5$ 6. $33.3 \div 0.3$ 7. $370.65 \div 2.5$ 8. $21.8 \div 0.2$

3-5 Study Guide and Intervention***Problem Solving Investigation: Choose the Method of Computation***

When solving problems, if the exact answer is not needed, use **estimation**. If you can see a pattern to find the solution, use **mental math**. When there are only simple calculations, use **paper and pencil**. Lastly, if the calculations are harder to compute, use a **calculator**.

Example **ALLOWANCE** John has \$17 in his savings account. After receiving his allowance for the first week, he has \$22.00. At the end of the second week, he receives his allowance and has \$27.00. IF this pattern continues, and John doesn't spend any of the money he earns, how much money will John have in 5 weeks?

Explore We know how much money John started with and that he receives allowance money each week. We need to find out how much money John will have saved in five weeks.

Plan Since we can see a pattern, use mental math.

Solve

Start	Week 1	Week 2
17	22	27
	+5	+5

Every week John is earning \$5.

In five weeks, John will have earned 5×5 or \$25.

So $17 + 25 = 42$

In five weeks, John will have \$42.

Check Because we know John earns \$5 each week, add 5 to the previous total five times for the five weeks.

Since $\$17 + \$5 = \$22 + \$5 = \$27 + \$5 = \$32 + \$5 = \$37 + \$5 = \$42$, the answer is correct.

Exercises

Choose mental math, estimation, paper or pencil, or a calculator to solve the following problems.

- MONEY** Sylvia earns \$21 each time she babysits for her neighbor. She wants to buy a CD player, which costs \$95.56 after tax. How many times will Sylvia need to baby-sit her neighbor in order to make enough money to buy the CD player?
- CONSTRUCTION** Jerrick is building a tree house for his sister. The plans call for 130 feet of lumber. Jerrick can buy 10 feet of lumber for \$15.00. How much will the lumber Jerrick needs cost?

3-6 Study Guide and Intervention**Measurement: The Metric System**

The table below is a summary of how to convert measures in the metric system.

	Larger Units → Smaller Units	Smaller Units → Larger Units
Units of Length (meter)	km to m – multiply by 1,000 m to cm – multiply by 100 m to mm – multiply by 1,000 cm to mm – multiply by 10	mm to cm – divide by 10 mm to m – divide by 1,000 cm to m – divide by 100 m to km – divide by 1,000
Units of Mass (kilogram)	kg to g – multiply by 1,000 g to mg – multiply by 1,000	mg to g – divide by 1,000 g to kg – divide by 1,000
Units of Capacity (liter)	kL to L – multiply by 1,000 L to mL – multiply by 1,000	mL to L – divide by 1,000 L to kL – divide by 1,000

Examples 1 Complete. $62 \text{ cm} = \underline{\hspace{1cm}} \text{ m}$

To convert from centimeters to meters, divide by 100.

$$62 \div 100 = 0.62$$

$$62 \text{ cm} = 0.62 \text{ m}$$

Example 2 Complete. $2.6 \text{ kL} = \underline{\hspace{1cm}} \text{ L}$

To convert from kiloliters to liters, multiply by 1,000.

$$2.6 \times 1,000 = 2,600$$

$$2.6 \text{ kL} = 2,600 \text{ L}$$

Exercises

Complete.

1. $650 \text{ cm} = \underline{\hspace{1cm}} \text{ m}$

2. $57 \text{ kg} = \underline{\hspace{1cm}} \text{ g}$

3. $751 \text{ mg} = \underline{\hspace{1cm}} \text{ g}$

4. $8.2 \text{ L} = \underline{\hspace{1cm}} \text{ mL}$

5. $52 \text{ L} = \underline{\hspace{1cm}} \text{ kL}$

6. $892 \text{ mm} = \underline{\hspace{1cm}} \text{ m}$

7. $121.4 \text{ kL} = \underline{\hspace{1cm}} \text{ L}$

8. $0.72 \text{ cm} = \underline{\hspace{1cm}} \text{ mm}$

9. $67.3 \text{ g} = \underline{\hspace{1cm}} \text{ kg}$

10. $5.2 \text{ g} = \underline{\hspace{1cm}} \text{ mg}$

11. $0.05 \text{ m} = \underline{\hspace{1cm}} \text{ mm}$

12. $2,500 \text{ mg} = \underline{\hspace{1cm}} \text{ g}$

13. $32 \text{ mm} = \underline{\hspace{1cm}} \text{ cm}$

14. $96 \text{ m} = \underline{\hspace{1cm}} \text{ cm}$

3-7 Study Guide and Intervention**Scientific Notation**

A number is in scientific notation when it is written as the product of a number and a power of ten. The number must be greater than or equal to 1 and less than 10.

- To write a number in standard form, you apply the order of operations. First evaluate the power of ten and then multiply.
- To write a number in scientific notation, move the decimal point to the right of the first nonzero number. Then, find the power of ten by counting the number of places moved.

Example 1 Write 6.1×10^3 in standard form.

$$\begin{aligned} 6.1 \times 10^3 &= 6.1 \times 1,000 & 10^3 &= 1,000 \\ &= \underset{\curvearrowright}{6} . \underset{\curvearrowright}{1} \underset{\curvearrowright}{0} \underset{\curvearrowright}{0} & & \text{Move the decimal point 3 places to the right.} \\ &= 6,100 \end{aligned}$$

Example 2 Write 62,500 in scientific notation.

$$\begin{aligned} 62,500 &= 6.250 \times 10,000 & & \text{Move the decimal point 4 places to get a number between 1 and 10.} \\ &= 6.25 \times 10^4 \end{aligned}$$

Exercises

Write each number in standard form.

1. 7.25×10^2

2. 2.5×10^3

3. 9.95×10^5

4. 8.80×10^4

5. 3.18×10^6

6. 6.12×10^3

Write each number in scientific notation.

7. 325

8. 9,210

9. 200

10. 5,120

11. 561

12. 1,230

13. 21,300

14. 53,000

15. 8,930

4-1 Study Guide and Intervention

Writing Expressions and Equations

The table below shows phrases written as mathematical expressions.

Phrases	Expression	Phrases	Expression
9 more than a number the sum of 9 and a number a number plus 9 a number increased by 9 the total of x and 9	$x + 9$	4 subtracted from a number a number minus 4 4 less than a number a number decreased by 4 the difference of h and 4	$h - 4$
Phrases	Expression	Phrases	Expression
6 multiplied by g 6 times a number the product of g and 6	$6g$	a number divided by 5 the quotient of t and 5 divide a number by 5	$\frac{t}{5}$

The table below shows sentences written as an equation.

Sentences	Equation
Sixty less than three times the amount is \$59. Three times the amount less 60 is equal to 59. 59 is equal to 60 subtracted from three times a number. A number times three minus 60 equals 59.	$3n - 60 = 59$

Exercises

Write each phrase as an algebraic expression.

- 7 less than m
- the quotient of 3 and y
- the total of 5 and c
- the difference of 6 and r
- n divided by 2
- the product of k and 9

Write each sentence as an algebraic equation.

- A number increased by 7 is 11.
- The price decreased by \$4 is \$29.
- Twice as many points as Bob would be 18 points.
- After dividing the money 5 ways, each person got \$67.
- Three more than 8 times as many trees is 75 trees.
- Seven less than a number is 15.

4-2 Study Guide and Intervention***Solving Addition and Subtraction Equations***

Remember, equations must always remain balanced. If you subtract the same number from each side of an equation, the two sides remain equal. Also, if you add the same number to each side of an equation, the two sides remain equal.

Example 1 Solve $x + 5 = 11$. Check your solution.

$$\begin{array}{r} x + 5 = 11 \quad \text{Write the equation.} \\ - 5 = -5 \quad \text{Subtract 5 from each side.} \\ \hline x = 6 \quad \text{Simplify.} \end{array}$$

Check $x + 5 = 11$ Write the equation.
 $6 + 5 \stackrel{?}{=} 11$ Replace x with 6.
 $11 = 11$ ✓ This sentence is true.

The solution is 6.

Example 2 Solve $15 = t - 12$. Check your solution.

$$\begin{array}{r} 15 = t - 12 \quad \text{Write the equation.} \\ + 12 = + 12 \quad \text{Add 12 to each side.} \\ \hline 27 = t \quad \text{Simplify.} \end{array}$$

Check $15 = t - 12$ Write the equation.
 $15 \stackrel{?}{=} 27 - 12$ Replace t with 27.
 $15 = 15$ ✓ This sentence is true.

The solution is 27.

Exercises

Solve each equation. Check your solution.

1. $h + 3 = 14$ 2. $m + 8 = 22$ 3. $p + 5 = 15$ 4. $17 = y + 8$

5. $w + 4 = -1$ 6. $k + 5 = -3$ 7. $25 = 14 + r$ 8. $57 + z = 97$

9. $b - 3 = 6$ 10. $7 = c - 5$ 11. $j - 12 = 18$ 12. $v - 4 = 18$

13. $-9 = w - 12$ 14. $y - 8 = -12$ 15. $14 = f - 2$ 16. $23 = n - 12$

4-3**Study Guide and Intervention**
Solving Multiplication Equations

If each side of an equation is divided by the same non-zero number, the resulting equation is equivalent to the given one. You can use this property to solve equations involving multiplication and division.

Example 1 Solve $45 = 5x$. Check your solution.

$$45 = 5x \quad \text{Write the equation.}$$

$$\frac{45}{5} = \frac{5x}{5} \quad \text{Divide each side of the equation by 5.}$$

$$9 = x \quad 45 \div 5 = 9$$

Check $45 = 5x$ Write the original equation.
 $45 \stackrel{?}{=} 5(9)$ Replace x with 9. Is this sentence true?
 $45 = 45$ ✓

The solution is 9.

Example 2 Solve $-21 = -3y$. Check your solution.

$$-21 = -3y \quad \text{Write the equation.}$$

$$\frac{-21}{-3} = \frac{-3y}{-3} \quad \text{Divide each side by } -3.$$

$$7 = y \quad -21 \div (-3) = 7$$

Check $-21 = -3y$ Write the original equation.
 $-21 \stackrel{?}{=} -3(7)$ Replace y with 7. Is this sentence true?
 $-21 = -21$ ✓

The solution is 7.

Exercises

Solve each equation. Then check your solution.

1. $8q = 56$

2. $4p = 32$

3. $42 = 6m$

4. $104 = 13h$

5. $-6n = 30$

6. $-18x = 36$

7. $48 = -8y$

8. $72 = -3b$

9. $-9a = -45$

10. $-12m = -120$

11. $-66 = -11t$

12. $-144 = -9r$

13. $3a = 4.5$

14. $2h = 3.8$

15. $4.9 = 0.7k$

16. $9.75 = 2.5z$

4-4**Study Guide and Intervention*****Problem-Solving Investigation: Work Backward***

By working backward from where you end to where you began, you can solve problems. Use the four-step problem solving model to stay organized when working backward.

Example 1

Jonah put half of his birthday money into his savings account. Then he paid back the \$10 that he owed his brother for dance tickets. Lastly, he spent \$3 on lunch at school. At then end of the day he was left with \$12. How much money did Jonah receive for his birthday?

Explore	You know that he had \$12 left and the amounts he spent throughout the day. You need to find out how much money he received for his birthday.	
Plan	Start with the amount of money he was left with and work backward.	
Solve	He had \$12 left.	12
	Undo the \$3 he spent on lunch.	<u>+ 3</u>
		15
	Undo the \$10 he gave back to his brother	<u>+ 10</u>
		25
	Undo the half put into his savings account	<u>2</u>
	So, Jonah received \$50 for his birthday.	50
Check	Assume that Jonah receive \$50 for his birthday. After putting half into his savings account he had $\$50 \div 2$ or \$25. Then he gave \$10 to his brother for dance tickets, so he had $\$25 - \10 or \$15. Lastly, he spent #3 on lunch at school, so he had $\$15 - \3 , or \$12. So, our answer of \$50 is correct.	

Exercises

Solve each problem by using the work backward strategy.

- On Monday everyone was present in Mr. Miller's class. At 12:00 5 students left early for doctors' appointments. At 1:15, half of the remaining students went to an assembly. Finally, at 2:00, 6 more students left for a student council meeting. At the end of they day, there were only 5 students in the room. Assuming that no students returned after having left, how many students are in Mr. Miller's class?
- Jordan was trading baseball cards with some friends. He gave 15 cards to Tommy and got 3 back. He gave some of his remaining cards to Elaine and kept the rest for himself. When he got home he counted that he had 25 cards. How many baseball cards did Jordan start with?

4-5**Study Guide and Intervention**
Solving Two-Step Equations

To solve two-step equations, you need to add or subtract first. Then divide to solve the equation.

Example 1 Solve $7v - 3 = 25$. Check your solution.

$$\begin{array}{r} 7v - 3 = 25 \quad \text{Write the equation.} \\ +3 = +3 \quad \text{Add 3 to each side.} \\ \hline 7v = 28 \quad \text{Simplify.} \\ \frac{7v}{7} = \frac{28}{7} \quad \text{Divide each side by 7.} \\ v = 4 \quad \text{Simplify.} \end{array}$$

Check $7v - 3 = 25$ Write the original equation.
 $7(4) - 3 \stackrel{?}{=} 25$ Replace v with 4.
 $28 - 3 \stackrel{?}{=} 25$ Multiply.
 $25 = 25 \checkmark$ The solution checks.

The solution is 4.

Example 2 Solve $-10 = 8 + 3x$. Check your solution.

$$\begin{array}{r} -10 = 8 + 3x \quad \text{Write the equation.} \\ -8 = -8 \quad \text{Subtract 8 from each side.} \\ \hline -18 = 3x \quad \text{Simplify.} \\ \frac{-18}{3} = \frac{3x}{3} \quad \text{Divide each side by 3.} \\ -6 = x \quad \text{Simplify.} \end{array}$$

Check $-10 = 8 + 3x$ Write the original equation.
 $-10 \stackrel{?}{=} 8 + 3(-6)$ Replace x with -6 .
 $-10 \stackrel{?}{=} 8 + (-18)$ Multiply.
 $-10 = -10 \checkmark$ The solution checks.

The solution is -6 .

Exercises

Solve each equation. Check your solution.

1. $4y + 1 = 13$

2. $6x + 2 = 26$

3. $-3 = 5k + 7$

4. $6n + 4 = -26$

5. $7 = -3c - 2$

6. $-8p + 3 = -29$

7. $-5 = -5t - 5$

8. $-9r + 12 = -24$

9. $11 + 7n = 4$

10. $35 = 7 + 4b$

11. $15 + 2p = 9$

12. $49 = 16 + 3y$

13. $2 = 4t - 14$

14. $-9x - 10 = 62$

15. $30 = 12z - 18$

16. $7 + 4g = 7$

17. $24 + 9x = -3$

18. $50 = 16q + 2$

19. $3c - 2.5 = 4.1$

20. $9y + 4.8 = 17.4$

4-6 Study Guide and Intervention

Geometry: Perimeter and Area

The distance around a geometric figure is called the **perimeter**.

To find the perimeter of any geometric figure, you can use addition or a formula.

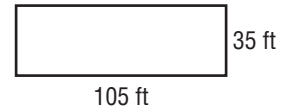
The perimeter of a rectangle is twice the length ℓ plus twice the width w .

$$P = 2\ell + 2w$$

Example 1 Find the perimeter of the figure at right.

$$P = 105 + 105 + 35 + 35 \text{ or } 280$$

The perimeter is 280 inches.



The measure of the surface enclosed by a geometric figure is called the **area**.

The area of a rectangle is the product of the length ℓ and width w .

$$A = \ell \cdot w$$

Example 2 Find the area of the rectangle.

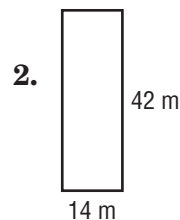
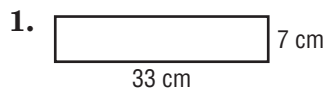
$$\begin{aligned} A &= \ell \cdot w \\ &= 24 \cdot 12 \text{ or } 288 \end{aligned}$$

The area is 288 square centimeters.

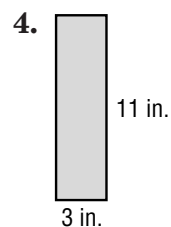
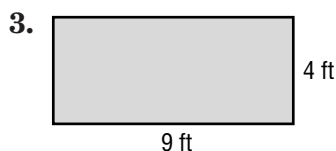


Exercises

Find the perimeter of each figure.



Find the perimeter and area of each rectangle.



5. $\ell = 8 \text{ ft}, w = 5 \text{ ft}$

6. $\ell = 3.5 \text{ m}, w = 2 \text{ m}$

7. $\ell = 8 \text{ yd}, w = 4\frac{1}{3} \text{ yd}$

8. $\ell = 29 \text{ cm}, w = 7.3 \text{ cm}$

4-7 Study Guide and Intervention

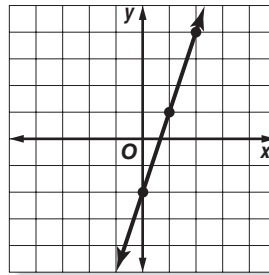
Functions and Graphs

The solution of an equation with two variables consists of two numbers, one for each variable, that make the equation true. The solution is usually written as an ordered pair (x, y) , which can be graphed. If the graph for an equation is a straight line, then the equation is a linear equation.

Example 1 Graph $y = 3x - 2$.

Select any four values for the input x . We chose 3, 2, 0, and -1 . Substitute these values for x to find the output y .

x	$3x - 2$	y	(x, y)
2	$3(2) - 2$	4	$(2, 4)$
1	$3(1) - 2$	1	$(1, 1)$
0	$3(0) - 2$	-2	$(0, -2)$
-1	$3(-1) - 2$	-5	$(-1, -5)$

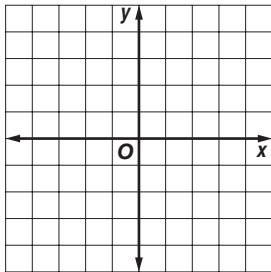


Four solutions are $(2, 4)$, $(1, 1)$, $(0, -2)$, and $(-1, -5)$. The graph is shown at the right.

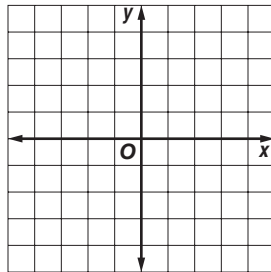
Exercises

Graph each equation.

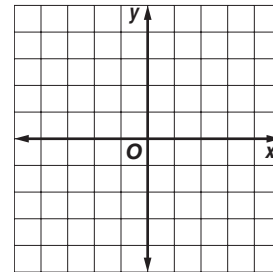
1. $y = x - 1$



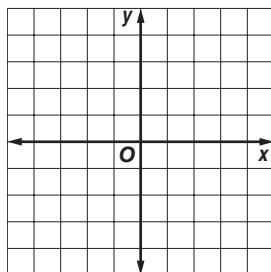
2. $y = x + 2$



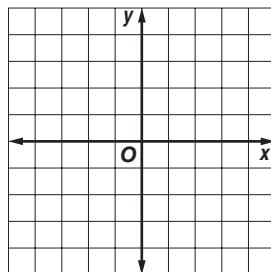
3. $y = -x$



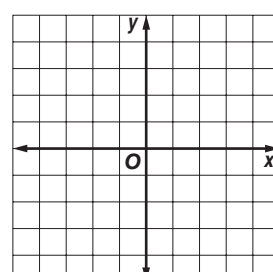
4. $y = 4x$



5. $y = 2x + 4$



6. $y = 3x - 1$



5-1

Study Guide and Intervention

Prime Factorization

A whole number is **prime** if it has exactly two factors, 1 and itself. A whole number is **composite** if it is greater than one and has more than two factors. To determine the **prime factorization** of a number us a **factor tree**.

Example 1 Determine whether each number is *prime* or *composite*.

a. 11

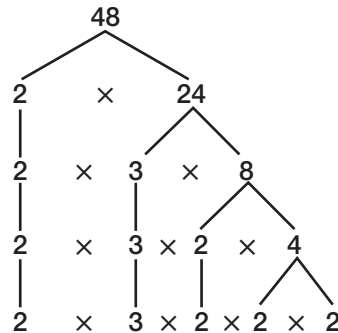
b. 24

a. The number 11 has only two factors, 1 and 11, so it is prime.

b. The number 24 has 8 factors, 1, 2, 3, 4, 6, 8, 12, and 24. So, it is composite.

Example 2 Determine the prime factorization of 48.

Use a factor tree.



The prime factorization of 48 is $2 \times 2 \times 2 \times 2 \times 3$ or $2^3 \times 3$

Exercises

Determine whether each number is prime or composite.

1. 27

2. 31

3. 46

4. 53

5. 11

6. 72

7. 17

8. 51

Determine the prime factorization of the following numbers.

9. 64

10. 100

11. 45

12. 81

5-2**Study Guide and Intervention****Greatest Common Factor**

The **greatest common factor (GCF)** of two or more numbers is the largest number that is a factor of each number. The GCF of prime numbers is 1.

Example 1 Find the GCF of 72 and 108 by listing factors.

factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

factors of 108: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108

common factors: 1, 2, 3, 4, 6, 9, 12, 18, 36

The GCF of 72 and 108 is 36.

Example 2 Find the GCF of 42 and 60 using prime factors.

Method 1 Write the prime factorization.

$$60 = 2 \times \boxed{2} \times \boxed{3} \times 5$$

$$42 = \boxed{2} \times \boxed{3} \times 7$$

Method 2 Divide by prime numbers.

Divide both 42 and 60 by 2.

Then divide the quotients by 3.

$$\begin{array}{r} 7 \quad 10 \\ \hline 3 \overline{)21 \quad 30} \\ 2 \overline{)42 \quad 60} \leftarrow \text{Start here} \end{array}$$

The common prime factors are 2 and 3. The GCF of 42 and 60 is 2×3 , or 6.

Exercises

Find the GCF of each set of numbers.

1. 18, 30

2. 60, 45

3. 24, 72

4. 32, 48

5. 100, 30

6. 54, 36

7. 3, 97, 5

8. 4, 20, 24

9. 36, 9, 45

5-3

Study Guide and Intervention

Problem-Solving Investigation: Make an Organized List

When solving problems often times it is useful to make an organized list. By doing so you can see all the possible solutions to the problem being posed.

Example 1 **LUNCH** Walnut Hills School has a deli line where students are able to select a meat sandwich, a side, and fruit. Meat choices are ham or turkey. The side choices are pretzels or chips. Fruit options are an apple or a pear. How many different combinations are possible?

Explore You know that students can choose a sandwich, a side, and fruit. There are 2 meat choices, 2 side choices, and 2 fruit choices. You need to find all possible combinations.

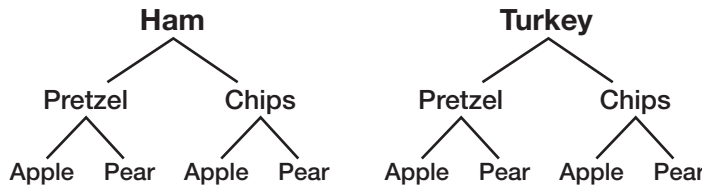
Plan Make an organized list.

Solve

	1	2	3	4	5	6	7	8
Meat	Ham	Ham	Ham	Ham	Turkey	Turkey	Turkey	Turkey
Side	Pretzel	Pretzel	Chips	Chips	Pretzel	Pretzel	Chips	Chips
Fruit	Apple	Pear	Apple	Pear	Apple	Pear	Apple	Pear

There are 8 possibilities.

Check Draw a tree diagram to check the result.



Exercises

- Susan has 3 shirts; red, blue, and green; 2 pants; jeans and khakis; and 3 shoes; white, black, and tan, to choose from for her school outfit. How many different outfits can she create?
- The Motor Speedway is awarding money to the first two finishers in their annual race. If there are four cars in the race numbered 1 through 4, how many different ways can they come in first and second?

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

5-4**Study Guide and Intervention*****Simplifying Fractions***

Fractions that have the same value are called **equivalent fractions**. A fraction is in **simplest form** when the GCF of the numerator and denominator is 1.

Example 1 Write $\frac{36}{54}$ in simplest form.

First, find the GCF of the numerator and denominator.

factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

The GCF of 36 and 54 is 18.

Then, divide the numerator and the denominator by the GCF.

$$\frac{36}{54} = \frac{36 \div 18}{54 \div 18} = \frac{2}{3} \quad \text{So, } \frac{36}{54} \text{ written in simplest form is } \frac{2}{3}.$$

Example 2 Write $\frac{8}{12}$ in simplest form.

Find the GCF of the numerator and the denominator.

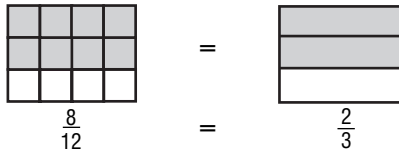
factors of 8 = $4 \cdot 2$

factors of 12 = $4 \cdot 3$

The GCF of 8 and 12 is 4.

$$\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

So, $\frac{8}{12}$ written in simplest form is $\frac{2}{3}$.

**Exercises**

Write each fraction in simplest form.

1. $\frac{42}{72}$

2. $\frac{40}{64}$

3. $\frac{21}{35}$

4. $\frac{25}{100}$

5. $\frac{99}{132}$

6. $\frac{17}{85}$

5-5**Study Guide and Intervention****Fractions and Decimals**

To write a decimal as a fraction, divide the numerator of the fraction by the denominator. Use a power of ten to change a decimal to a fraction.

Example 1 Write $\frac{5}{9}$ as a decimal.

Method 1 Use pencil and paper.

$$\begin{array}{r} 0.555\dots \\ 9 \overline{)5.000} \end{array}$$

$$\underline{45}$$

$$50$$

$$\underline{45}$$

$$50$$

$$\underline{45}$$

$$5$$

The remainder after each step is 5.

Method 2 Use a calculator.

$$5 \div 9 = 0.5555556$$

You can use bar notation $0.\overline{5}$ to indicate that 5 repeats forever. So, $\frac{5}{9} = 0.\overline{5}$.

Example 2 Write 0.32 as a fraction in simplest form.

$$\begin{aligned} 0.32 &= \frac{32}{100} && \text{The 2 is in the hundredths place.} \\ &= \frac{8}{25} && \text{Simplify.} \end{aligned}$$

Exercises

Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.

1. $\frac{8}{10}$

2. $\frac{3}{5}$

3. $\frac{7}{11}$

4. $4\frac{7}{8}$

5. $\frac{13}{15}$

6. $3\frac{47}{99}$

Write each decimal as a fraction in simplest form.

7. 0.14

8. 0.3

9. 0.94

5-6**Study Guide and Intervention****Fractions and Percents**

A **ratio** is a comparison of two numbers by division. When a ratio compares a number to 100, it can be written as a **percent**. To write a ratio or fraction as a percent, find an equivalent fraction with a denominator of 100. You can also use the meaning of percent to change percents to fractions.

Example 1 Write $\frac{19}{20}$ as a percent.

$$\frac{19}{20} \xrightarrow{\times 5} \frac{95}{100} = 95\%$$

Since $100 \div 20 = 5$, multiply the numerator and denominator by 5.

Example 2 Write 92% as a fraction in simplest form.

$$\begin{aligned} 92\% &= \frac{92}{100} && \text{Definition of percent} \\ &= \frac{23}{25} && \text{Simplify.} \end{aligned}$$

Exercises

Write each ratio as a percent.

1. $\frac{14}{100}$

2. $\frac{27}{100}$

3. 34.5 per 100

4. 18 per 100

5. 21:100

6. 96:100

Write each fraction as a percent.

7. $\frac{3}{100}$

8. $\frac{14}{100}$

9. $\frac{2}{5}$

10. $\frac{1}{20}$

11. $\frac{13}{25}$

12. $\frac{4}{10}$

Write each percent as a fraction in simplest form.

13. 35%

14. 18%

15. 75%

16. 80%

17. 16%

18. 15%

5-7**Study Guide and Intervention*****Percents and Decimals***

To write a percent as a decimal, divide the percent by 100 and remove the percent symbol. To write a decimal as a percent, multiply the decimal by 100 and add the percent symbol.

Example 1 Write 42.5% as a decimal.

$$42.5\% = \frac{42.5}{100}$$

Write the percent as a fraction.

$$= \frac{42.5 \times 10}{100 \times 10}$$

Multiply by 10 to remove the decimal in the numerator.

$$= \frac{425}{1,000}$$

Simplify.

$$= 0.425$$

Write the fraction as a decimal.

Example 2 Write 0.625 as a percent.

$$0.625 = 062.5$$

Multiply by 100.

$$= 62.5\%$$

Add the % symbol.

Exercises**Write each percent as a decimal.**

1. 6%

2. 28%

3. 81%

4. 84%

5. 35.5%

6. 12.5%

7. 14.2%

8. 11.1%

Write each decimal as a percent.

9. 0.47

10. 0.03

11. 0.075

12. 0.914

5-8**Study Guide and Intervention*****Least Common Multiple***

A **multiple** of a number is the product of that number and any whole number. The least nonzero multiple of two or more numbers is the **least common multiple (LCM)** of the numbers.

Example 1 Find the LCM of 15 and 20 by listing multiples.

List the multiples.

multiples of 15: 15, 30, 45, **60**, 75, 90, 105, **120**, ...

multiples of 20: 20, 40, **60**, 80, 100, **120**, 140, ...

Notice that 60, 120, ..., are common multiples. So, the LCM of 15 and 20 is 60.

Example 2 Find the LCM of 8 and 12 using prime factors.

Write the prime factorization.

$$8 = 2 \times 2 \times 2 = 2^3$$

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

The prime factors of 8 and 12 are 2 and 3.
Multiply the greatest power of both 2 and 3.

The LCM of 8 and 12 is $2^3 \times 3$, or 24.

Exercises

Find the LCM of each set of numbers.

- | | |
|-------------|-------------|
| 1. 4, 6 | 2. 6, 9 |
| 3. 5, 9 | 4. 8, 10 |
| 5. 12, 15 | 6. 15, 21 |
| 7. 4, 15 | 8. 8, 20 |
| 9. 8, 16 | 10. 6, 14 |
| 11. 12, 20 | 12. 9, 12 |
| 13. 14, 21 | 14. 6, 15 |
| 15. 4, 6, 8 | 16. 3, 5, 6 |

5-9**Study Guide and Intervention****Comparing and Ordering Rational Numbers**

To compare fractions, rewrite them so they have the same denominator. The **least common denominator (LCD)** of two fractions is the LCM of their denominators.

Another way to compare fractions is to express them as decimals. Then compare the decimals.

Example 1 Which fraction is greater, $\frac{3}{4}$ or $\frac{4}{5}$?

Method 1 Rename using the LCD.

$$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

$$\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20}$$

The LCD is 20.

Because the denominators are the same, compare numerators.

Since $\frac{16}{20} > \frac{15}{20}$, then $\frac{4}{5} > \frac{3}{4}$.

Method 2 Write each fraction as a decimal. Then compare decimals.

$$\frac{3}{4} = 0.75$$

$$\frac{4}{5} = 0.8$$

Since $0.8 > 0.75$, then $\frac{4}{5} > \frac{3}{4}$.

Exercises

Find the LCD of each pair of fractions.

1. $\frac{1}{2}, \frac{1}{8}$

2. $\frac{1}{3}, \frac{3}{4}$

3. $\frac{3}{4}, \frac{7}{10}$

Replace each ● with <, >, or = to make a true sentence.

4. $\frac{1}{2} \bullet \frac{4}{9}$

5. $\frac{4}{5} \bullet \frac{8}{10}$

6. $\frac{3}{4} \bullet \frac{7}{8}$

7. $\frac{1}{2} \bullet \frac{5}{9}$

8. $\frac{9}{14} \bullet \frac{10}{17}$

9. $\frac{5}{7} \bullet \frac{6}{11}$

10. $\frac{8}{17} \bullet \frac{1}{2}$

11. $\frac{9}{10} \bullet \frac{17}{19}$

6-1 Study Guide and Intervention**Estimating with Fractions**

Use rounding to estimate with fractions.

Estimating: For mixed numbers, round to the nearest whole number.

$$4\frac{1}{6} + 3\frac{7}{8} \rightarrow 4 + 4 = 8$$

 $4\frac{1}{6} + 3\frac{7}{8}$ is about 8.For fractions, round to 0, $\frac{1}{2}$, or 1.

$$\frac{11}{12} - \frac{4}{9} \rightarrow 1 - \frac{1}{2} = \frac{1}{2}$$

 $\frac{11}{12} - \frac{4}{9}$ is about $\frac{1}{2}$.**Example 1** Estimate $2\frac{2}{3} \times 4\frac{1}{4}$.

$$2\frac{2}{3} \times 4\frac{1}{4} \rightarrow 3 \times 4 = 12$$

The product is about 12.

Example 2 Estimate $\frac{6}{7} - \frac{3}{5}$. $\frac{6}{7}$ is about 1. $\frac{3}{5}$ is about $\frac{1}{2}$.

$$\frac{6}{7} - \frac{3}{5} \rightarrow 1 - \frac{1}{2} = \frac{1}{2}$$

The difference is about $\frac{1}{2}$.**Exercises****Estimate.**

1. $4\frac{1}{3} + 3\frac{4}{5}$

2. $2\frac{1}{6} \times 3\frac{2}{3}$

3. $\frac{7}{12} - \frac{1}{10}$

4. $5\frac{1}{4} - 1\frac{1}{2}$

5. $4\frac{3}{4} + 1\frac{1}{5}$

6. $\frac{5}{9} \times \frac{13}{14}$

7. $\frac{1}{6} \div \frac{8}{9}$

8. $\frac{6}{7} \div \frac{9}{10}$

9. $13\frac{4}{5} \div 1\frac{7}{8}$

10. $12\frac{1}{4} \div 5\frac{7}{8}$

6-2 Study Guide and Intervention**Adding and Subtracting Fractions**

Like fractions are fractions that have the same denominator. To add or subtract like fractions, add or subtract the numerators and write the result over the denominator.

Simplify if necessary.

To add or subtract *unlike fractions*, rename the fractions with a least common denominator. Then add or subtract as with like fractions.

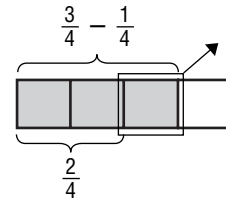
Example 1 Subtract $\frac{3}{4} - \frac{1}{4}$. Write in simplest form.

$$\begin{aligned}\frac{3}{4} - \frac{1}{4} &= \frac{3-1}{4} \\ &= \frac{2}{4} \\ &= \frac{1}{2}\end{aligned}$$

Subtract the numerators.

Write the difference over the denominator.

Simplify.



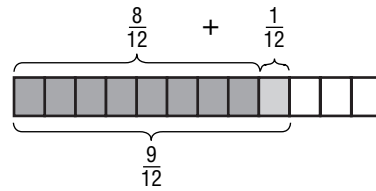
Example 2 Add $\frac{2}{3} + \frac{1}{12}$. Write in simplest form.

The least common denominator of 3 and 12 is 12.

$$\begin{aligned}\frac{2}{3} &= \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \\ \frac{2}{3} &\rightarrow \frac{8}{12} \\ + \frac{1}{12} &\rightarrow + \frac{1}{12} \\ \hline &= \frac{9}{12} \text{ or } \frac{3}{4}\end{aligned}$$

Rename $\frac{2}{3}$ using the LCD.

Add the numerators and simplify.

**Exercises**

Add or subtract. Write in simplest form.

1. $\frac{5}{8} + \frac{1}{8}$

2. $\frac{7}{9} - \frac{2}{9}$

3. $\frac{1}{2} + \frac{3}{4}$

4. $\frac{7}{8} - \frac{5}{6}$

5. $\frac{5}{9} + \frac{5}{6}$

6. $\frac{3}{8} - \frac{1}{12}$

7. $\frac{3}{10} + \frac{7}{12}$

8. $\frac{2}{5} - \frac{1}{3}$

9. $\frac{7}{15} + \frac{5}{6}$

10. $\frac{7}{9} - \frac{1}{2}$

6-3 Study Guide and Intervention***Adding and Subtracting Mixed Numbers***

To add or subtract mixed numbers:

1. Add or subtract the fractions. Rename using the LCD if necessary.
2. Add or subtract the whole numbers.
3. Simplify if necessary.

Example 1 Find $14\frac{1}{2} + 18\frac{2}{3}$.

$$\begin{array}{r} 14\frac{1}{2} \rightarrow 14\frac{3}{6} \\ + 18\frac{2}{3} \rightarrow + 18\frac{4}{6} \\ \hline 32\frac{7}{6} \text{ or } 33\frac{1}{6} \end{array}$$

Rename the fractions.

Add the whole numbers and add the fractions.

Simplify.

Example 2 Find $21 - 12\frac{5}{8}$.

$$\begin{array}{r} 21 \rightarrow 20\frac{8}{8} \\ - 12\frac{5}{8} \rightarrow - 12\frac{5}{8} \\ \hline 8\frac{3}{8} \end{array}$$

Rename 21 as $20\frac{8}{8}$.

First subtract the whole numbers and then the fractions.

Exercises

Add or subtract. Write in simplest form.

1. $7\frac{3}{4} + 2\frac{3}{4}$

2. $14\frac{2}{9} - 6\frac{1}{9}$

3. $9\frac{1}{5} - 4\frac{3}{4}$

4. $7\frac{1}{8} + 5\frac{3}{8}$

5. $7\frac{3}{4} + 2\frac{2}{3}$

6. $5\frac{1}{2} - 5\frac{1}{3}$

7. $5\frac{1}{2} - 3\frac{1}{4}$

8. $6\frac{1}{3} + 2\frac{1}{6}$

9. $9 - 3\frac{2}{5}$

10. $2\frac{2}{3} + 7\frac{1}{2}$

11. $6\frac{1}{2} - 6\frac{1}{3}$

12. $18\frac{1}{2} + 5\frac{5}{8}$

6-4 Study Guide and Intervention**Problem-Solving Investigation: Eliminate Possibilities**

By **eliminating possibilities** when problem solving, you can methodically reduce the number of potential answers.

Example

Joan has \$20 to spend on her sister for her birthday. She has already bought her a DVD for \$9.75. There are three shirts that she likes which cost \$8.75, \$10.00, and \$11.00. Which shirt should she buy so that she spends most of her money without going over \$20?

Explore	You know that the total amount of money she has to spend must be \$20 or less.
Plan	Eliminate answers that are not reasonable.
Solve	She couldn't spend \$11.00 because $\$9.75 + \$11.00 = \$20.75$. So eliminate that choice. Now check \$10.00 $\$9.75 + 10.00 = \19.75 Since this is less than \$20, this is the correct choice. She should buy her sister the \$10.00 shirt.
Check	By buying the \$8.75 shirt, she would only spend a total of $\$9.75 + \$8.75 = \$18.50$. This is less than the \$20 minimum, but not the most she could possibly spend.

Exercises

Solve the following problems by **eliminating possibilities first**.

- TELEPHONE** Susan talked on her cellular telephone for 120 minutes last month. Her plan charges her a \$15.00 fee per month plus \$0.10 a minute after the first 60 minutes, which are included in the \$15 fee. What was her total bill for last month?

A. \$12.00 B. \$27.00 C. \$21.00 D. \$6.00
- HOME SALES** 450 homes sold in your area in the last year. What number shows a good estimate of the number of homes sold per month?

A. 38 homes B. 32.5 homes C. 2 homes D. 45 homes
- CAR SALES** Derrick sells cars for a living. He sells an average of 22 cars a month. What will his total average car sales be in 5 years?

A. 110 cars B. 264 cars C. 1320 cars D. 27 cars
- TELEVISION** Myra is allowed to watch 6 hours of television on a weekend. She watched $2\frac{1}{2}$ hours this morning. How much television will she be allowed to watch at most this afternoon?

A. 4 hours B. $4\frac{1}{2}$ hours C. $2\frac{1}{2}$ hours D. 3 hours

6-5**Study Guide and Intervention*****Multiplying Fractions and Mixed Numbers***

To multiply fractions, multiply the numerators and multiply the denominators.

$$\frac{5}{6} \times \frac{3}{5} = \frac{5 \times 3}{6 \times 5} = \frac{15}{30} = \frac{1}{2}$$

To multiply mixed numbers, rename each mixed number as a fraction. Then multiply the fractions.

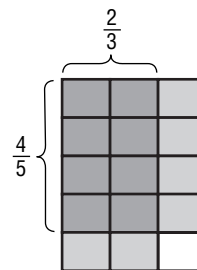
$$2\frac{2}{3} \times 1\frac{1}{4} = \frac{8}{3} \times \frac{5}{4} = \frac{40}{12} = 3\frac{1}{3}$$

Example 1 Find $\frac{2}{3} \times \frac{4}{5}$. Write in simplest form.

$$\begin{aligned} \frac{2}{3} \times \frac{4}{5} &= \frac{2 \times 4}{3 \times 5} && \leftarrow \text{Multiply the numerators.} \\ & && \leftarrow \text{Multiply the denominators.} \\ &= \frac{8}{15} && \text{Simplify.} \end{aligned}$$

Example 2 Find $\frac{1}{3} \times 2\frac{1}{2}$. Write in simplest form.

$$\begin{aligned} \frac{1}{3} \times 2\frac{1}{2} &= \frac{1}{3} \times \frac{5}{2} && \text{Rename } 2\frac{1}{2} \text{ as an improper fraction, } \frac{5}{2}. \\ &= \frac{1 \times 5}{3 \times 2} && \text{Multiply.} \\ &= \frac{5}{6} && \text{Simplify.} \end{aligned}$$

**Exercises**

Multiply. Write in simplest form.

1. $\frac{2}{3} \times \frac{2}{3}$

2. $\frac{1}{2} \times \frac{7}{8}$

3. $\frac{1}{3} \times \frac{3}{5}$

4. $\frac{5}{9} \times 4$

5. $1\frac{2}{3} \times \frac{3}{5}$

6. $3\frac{3}{4} \times 1\frac{1}{6}$

7. $\frac{3}{4} \times 1\frac{2}{3}$

8. $3\frac{1}{3} \times 2\frac{1}{2}$

9. $4\frac{1}{5} \times \frac{1}{7}$

10. $\frac{7}{5} \times 8$

11. $2\frac{1}{3} \times \frac{4}{6}$

12. $\frac{1}{8} \times 2\frac{3}{4}$

6-6 Study Guide and Intervention**Algebra: Solving Equations**

Multiplicative inverses, or reciprocals, are two numbers whose product is 1. To solve an equation in which the coefficient is a fraction, multiply each side of the equation by the reciprocal of the coefficient.

Example 1 Find the multiplicative inverse of $3\frac{1}{4}$.

$$3\frac{1}{4} = \frac{13}{4} \quad \text{Rename the mixed number as an improper fraction.}$$

$$\frac{13}{4} \cdot \frac{4}{13} = 1 \quad \text{Multiply } \frac{13}{4} \text{ by } \frac{4}{13} \text{ to get the product 1.}$$

The multiplicative inverse of $3\frac{1}{4}$ is $\frac{4}{13}$.

Example 2 Solve $\frac{4}{5}x = 8$. Check your solution.

$$\frac{4}{5}x = 8 \quad \text{Write the equation.}$$

$$\left(\frac{5}{4}\right)\frac{4}{5}x = \left(\frac{5}{4}\right)8 \quad \text{Multiply each side by the reciprocal of } \frac{4}{5}, \frac{5}{4}.$$

$$x = 10 \quad \text{Simplify.}$$

The solution is 10.

Exercises

Find the multiplicative inverse of each number.

- | | |
|--------------------|--------------------|
| 1. $\frac{4}{9}$ | 2. $\frac{12}{13}$ |
| 3. $-\frac{15}{4}$ | 4. $6\frac{1}{7}$ |

Solve each equation. Check your solution.

- | | | |
|------------------------|-------------------------|--------------------------|
| 5. $\frac{3}{5}x = 12$ | 6. $16 = \frac{10}{3}a$ | 7. $\frac{c}{2} = 7$ |
| 8. $\frac{15}{7}y = 3$ | 9. $\frac{m}{6} = -4$ | 10. $14 = -\frac{7}{9}b$ |

6-7 Study Guide and Intervention***Dividing Fractions and Mixed Numbers***

To divide by a fraction, multiply by its multiplicative inverse or reciprocal. To divide by a mixed number, rename the mixed number as an improper fraction.

Example 1 Find $3\frac{1}{3} \div \frac{2}{9}$. Write in simplest form.

$$3\frac{1}{3} \div \frac{2}{9} = \frac{10}{3} \div \frac{2}{9}$$

Rename $3\frac{1}{3}$ as an improper fraction.

$$= \frac{10}{3} \cdot \frac{9}{2}$$

Multiply by the reciprocal of $\frac{2}{9}$, which is $\frac{9}{2}$.

$$= \frac{\cancel{10}^5}{\cancel{3}_1} \cdot \frac{\cancel{9}^3}{\cancel{2}_1}$$

Divide out common factors.

$$= 15$$

Multiply.

Exercises

Divide. Write in simplest form.

1. $\frac{2}{3} \div \frac{1}{4}$

2. $\frac{2}{5} \div \frac{5}{6}$

3. $\frac{1}{2} \div \frac{1}{5}$

4. $5 \div \frac{1}{2}$

5. $\frac{5}{8} \div 10$

6. $7\frac{1}{3} \div 2$

7. $\frac{5}{6} \div 3\frac{1}{2}$

8. $36 \div 1\frac{1}{2}$

9. $2\frac{1}{2} \div 10$

10. $5\frac{2}{5} \div 1\frac{4}{5}$

11. $6\frac{2}{3} \div 3\frac{1}{9}$

12. $4\frac{1}{4} \div \frac{3}{8}$

13. $4\frac{6}{7} \div 2\frac{3}{7}$

14. $12 \div 2\frac{1}{2}$

15. $4\frac{1}{6} \div 3\frac{1}{6}$

7-1**Study Guide and Intervention****Ratios**

Any ratio can be written as a fraction. To write a ratio comparing measurements, such as units of length or units of time, both quantities must have the same unit of measure. Two ratios that have the same value are **equivalent ratios**.

Example 1 Write the ratio 15 to 9 as a fraction in simplest form.

$$15 \text{ to } 9 = \frac{15}{9} \quad \text{Write the ratio as a fraction.}$$

$$= \frac{5}{3} \quad \text{Simplify.}$$

Written as a fraction in simplest form, the ratio 15 to 9 is $\frac{5}{3}$.

Example 2 Determine whether the ratios *10 cups of flour in 4 batches of cookies* and *15 cups of flour in 6 batches of cookies* are equivalent ratios.

Compare ratios written in simplest form.

$$10 \text{ cups:}4 \text{ batches} = \frac{10 \div 2}{4 \div 2} \text{ or } \frac{5}{2} \quad \text{Divide the numerator and denominator by the GCF, 2}$$

$$15 \text{ cups:}6 \text{ batches} = \frac{15 \div 3}{6 \div 3} \text{ or } \frac{5}{2} \quad \text{Divide the numerator and denominator by the GCF, 3}$$

Since the ratios simplify to the same fraction, the ratios of cups to batches are equivalent.

Exercises

Write each ratio as a fraction in simplest form.

- | | |
|---------------------------|-----------------------------|
| 1. 30 to 12 | 2. 5:20 |
| 3. 49:42 | 4. 15 to 13 |
| 5. 28 feet:35 feet | 6. 24 minutes to 18 minutes |
| 7. 75 seconds:150 seconds | 8. 12 feet:60 feet |

Determine whether the ratios are equivalent. Explain.

9. $\frac{3}{4}$ and $\frac{12}{16}$	10. 12:17 and 10:15	11. $\frac{25}{35}$ and $\frac{10}{14}$
--------------------------------------	---------------------	---

- | | |
|-------------------------------|--------------------------------|
| 12. 2 lb:36 oz and 3 lb:44 oz | 13. 1 ft:4 in. and 3 ft:12 in. |
|-------------------------------|--------------------------------|

7-2**Study Guide and Intervention****Rates**

A ratio that compares two quantities with different kinds of units is called a **rate**. When a rate is simplified so that it has a denominator of 1 unit, it is called a **unit rate**.

Example 1 **DRIVING** Alita drove her car 78 miles and used 3 gallons of gas. What is the car's gas mileage in miles per gallon?

Write the rate as a fraction. Then find an equivalent rate with a denominator of 1.

$$78 \text{ miles using } 3 \text{ gallons} = \frac{78 \text{ mi}}{3 \text{ gal}}$$

Write the rate as a fraction.

$$= \frac{78 \text{ mi} \div 3}{3 \text{ gal} \div 3}$$

Divide the numerator and the denominator by 3.

$$= \frac{26 \text{ mi}}{1 \text{ gal}}$$

Simplify.

The car's gas mileage, or unit rate, is 26 miles per gallon.

Example 2 **SHOPPING** Joe has two different sizes of boxes of cereal from which to choose. The 12-ounce box costs \$2.54, and the 18-ounce box costs \$3.50. Which box costs less per ounce?

Find the unit price, or the cost per ounce, of each box. Divide the price by the number of ounces.

$$12\text{-ounce box} \quad \$2.54 \div 12 \text{ ounces} \approx \$0.21 \text{ per ounce}$$

$$18\text{-ounce box} \quad \$3.50 \div 18 \text{ ounces} \approx \$0.19 \text{ per ounce}$$

The 18-ounce box costs less per ounce.

Exercises

Find each unit rate. Round to the nearest hundredth if necessary.

1. 18 people in 3 vans

2. \$156 for 3 books

3. 115 miles in 2 hours

4. 8 hits in 22 games

5. 65 miles in 2.7 gallons

6. 2,500 Calories in 24 hours

Choose the best unit price.

7. \$12.95 for 3 pounds of nuts or \$21.45 for 5 pounds of nuts

8. A 32-ounce bottle of apple juice for \$2.50 or a 48-ounce bottle for \$3.84.

7-3**Study Guide and Intervention****Measurement: Changing Customary Units**

Customary Units		
Length	Weight	Capacity
1 foot (ft) = 12 inches (in.)	1 pound (lb) = 16 ounces (oz)	1 cup (c) = 8 fluid ounces (fl oz)
1 yard (yd) = 3 feet	1 ton (T) = 2,000 pounds	1 pint (pt) = 2 cups
1 mile (mi) = 5,280 feet		1 quart (qt) = 2 pints
		1 gallon (gal) = 4 quarts

Example 1 $5\frac{1}{2}$ lb = _____ oz

To change from larger units to smaller units, multiply.

$$5\frac{1}{2} \times 16 = 88 \quad \text{Since 1 pound is 16 ounces, multiply by 16.}$$

$$5\frac{1}{2} \text{ pounds} = 88 \text{ ounces}$$

Example 2 28 fl oz = _____ c

To change from smaller units to larger units, divide.

$$28 \div 8 = 3\frac{1}{2} \quad \text{Since 8 fluid ounces are in 1 cup, divide by 8.}$$

$$28 \text{ fluid ounces} = 3\frac{1}{2} \text{ cups}$$

Exercises

Complete.

- 5 lb = _____ oz
- 48 in. = _____ ft
- 6 yd = _____ ft
- 7 qt = _____ pt
- 8,000 lb = _____ T
- $3\frac{1}{4}$ mi = _____ ft
- 4 c = _____ fl oz
- 6 c = _____ pt
- $\frac{1}{2}$ gal = _____ qt
- 3 ft = _____ in.
- 9 qt = _____ gal
- 30 fl oz = _____ c
- 6,864 ft = _____ mi
- 40 oz = _____ lb
- 9 pt = _____ c
- 18 ft = _____ yd
- 11 pt = _____ qt
- $2\frac{3}{4}$ T = _____ lb

7-4**Study Guide and Intervention****Solving Proportions**

A **proportion** is an equation stating that two ratios are equivalent. Since rates are types of ratios, they can also form proportions. In a proportion, a **cross product** is the product of the numerator of one ratio and the denominator of the other ratio.

Example 1 Determine whether $\frac{2}{3}$ and $\frac{10}{15}$ form a proportion.

$$\frac{2}{3} \stackrel{?}{=} \frac{10}{15}$$

Write a proportion.

$$2 \times 15 \stackrel{?}{=} 3 \times 10$$

Find the cross products.

$$30 = 30 \quad \checkmark$$

Multiply.

The cross products are equal, so the ratios form a proportion.

Example 2 Solve $\frac{8}{a} = \frac{10}{15}$.

$$\frac{8}{a} = \frac{10}{15}$$

Write the proportion.

$$8 \times 15 = a \times 10$$

Find the cross products.

$$120 = 10a$$

Multiply.

$$\frac{120}{10} = \frac{10a}{10}$$

Divide each side by 10.

$$12 = a$$

Simplify.

The solution is 12.

Exercises

Determine if the quantities in each pair of ratios are proportional.

Explain.

1. $\frac{8}{10} = \frac{4}{5}$

2. $\frac{9}{4} = \frac{11}{6}$

3. $\frac{6}{14} = \frac{9}{21}$

4. $\frac{15}{12} = \frac{9}{6}$

5. $\frac{\$2.48}{4 \text{ oz}} = \frac{\$3.72}{6 \text{ oz}}$

6. $\frac{125 \text{ mi}}{5.7 \text{ gal}} = \frac{120 \text{ mi}}{5.6 \text{ gal}}$

Solve each proportion.

7. $\frac{y}{7} = \frac{16}{28}$

8. $\frac{5}{15} = \frac{15}{w}$

9. $\frac{20}{b} = \frac{70}{28}$

10. $\frac{52}{8} = \frac{m}{9}$

7-5**Study Guide and Intervention****Problem-Solving Investigation: Draw a Diagram**

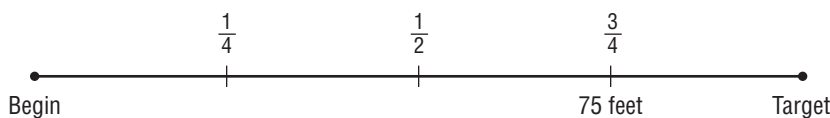
When solving problems, draw a diagram to show what you have and what you need to find.

Example **CARNIVAL** Jim has to reach a target at a carnival game to win a prize. After 3 throws he has gone 75 feet, which is $\frac{3}{4}$ of the way to the target. How far away is the target?

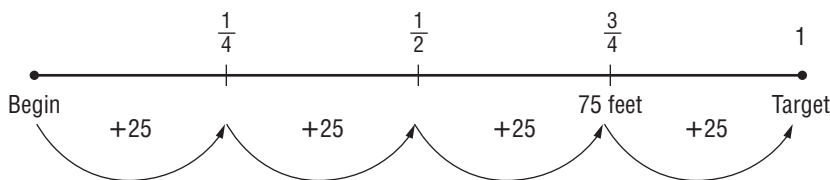
Explore We know that 75 feet is $\frac{3}{4}$ of the way to the target.

Plan Draw a diagram to show the distance already thrown and the fraction it represents.

Solve



If $\frac{3}{4}$ of the distance is 75 feet, then $\frac{1}{4}$ of the distance is 25 feet. So, the missing $\frac{1}{4}$ must be another 25 feet.



The total distance that Jim must throw to hit the target is 100 feet.

Check Since $\frac{3}{4}$ of the total distance is 75 feet, the equation $\frac{3}{4}x = 75$ represents this problem. Solving, we get $x = 100$ feet. So, the solution checks.

Exercises

- SALES** Sharon wants to buy a new car. She has saved up \$ 1,500, which is approximately $\frac{1}{5}$ of the price of the car. How much does she need to save in order to buy the new car?
- TRAVEL** The Jones family has traveled 360 miles. They are $\frac{4}{5}$ of the way to their destination. How far away is their destination?

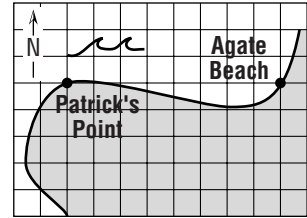
7-6

Study Guide and Intervention

Scale Drawings

A **scale drawing** represents something that is too large or too small to be drawn or built at actual size. Similarly, a **scale model** can be used to represent something that is too large or built too small for an actual-size model. The **scale** gives the relationship between the drawing/model measure and the actual measure.

Example On this map, each grid unit represents 50 yards. Find the distance from Patrick's Point to Agate Beach.



	Scale		Patrick's Point to Agate Beach		
map →	$\frac{1 \text{ unit}}{50 \text{ yards}}$	=	$\frac{8 \text{ units}}{x \text{ yards}}$	←	map
actual →				←	actual
	$1 \times x =$		50×8		Cross products
	$x =$		400		Simplify.

It is 400 yards from Patrick's Point to Agate Beach.

Exercises

Find the actual distance between each pair of cities. Round to the nearest tenth if necessary.

	Cities	Map Distance	Scale	Actual Distance
1.	Los Angeles and San Diego, California	6.35 cm	1 cm = 20 mi	
2.	Lexington and Louisville, Kentucky	15.6 cm	1 cm = 5 mi	
3.	Des Moines and Cedar Rapids, Iowa	16.27 cm	2 cm = 15 mi	
4.	Miami and Jacksonville, Florida	11.73 cm	$\frac{1}{2}$ cm = 20 mi	

Suppose you are making a scale drawing. Find the length of each object on the scale drawing with the given scale. Then find the scale factor.

5. an automobile 16 feet long; 1 inch:6 inches
6. a lake 85 feet across; 1 inch = 4 feet
7. a parking lot 200 meters wide; 1 centimeter:25 meters
8. a flag 5 feet wide; 2 inches = 1 foot

7-7**Study Guide and Intervention****Fractions, Decimals, and Percents****Example**Write $4\frac{3}{8}\%$ as a fraction in simplest form.

$$4\frac{3}{8}\% = \frac{4\frac{3}{8}}{100}$$

Write a fraction.

$$= 4\frac{3}{8} \div 100$$

Divide.

$$= \frac{35}{8} \div 100$$

Write $4\frac{3}{8}$ as an improper fraction.

$$= \frac{35}{8} \times \frac{1}{100}$$

Multiply by the reciprocal of 100, which is $\frac{1}{100}$.

$$= \frac{35}{800} \text{ or } \frac{7}{160}$$

Simplify.

Example 2Write $\frac{5}{16}$ as a percent.

$$\frac{5}{16} = \frac{n}{100}$$

Write a proportion using $\frac{n}{100}$.

$$500 = 16n$$

Find the cross products.

$$\frac{500}{16} = \frac{16n}{16}$$

Divide each side by 16.

$$31\frac{1}{4} = n$$

Simplify.

$$\text{So, } \frac{5}{16} = 31\frac{1}{4}\% \text{ or } 31.25\%.$$

Exercises

Write each percent as a fraction in simplest form.

1. 60%

2. $68\frac{3}{4}\%$

3. $27\frac{1}{2}\%$

4. 37.5%

Write each fraction as a percent. Round to the nearest hundredth if necessary.

5. $\frac{2}{5}$

6. $\frac{5}{8}$

7. $\frac{9}{16}$

8. $\frac{2}{3}$

7-8**Study Guide and Intervention****Percents Greater Than 100% and Percents Less Than 1%**

A percent greater than 100% equals a number greater than 1. A percent less than 1% equals a number less than 0.01 or $\frac{1}{100}$.

Examples

Write each percent as a decimal and as a mixed number or fraction in simplest form.

1 280%

$$\begin{aligned} 280\% &= \frac{280}{100} && \text{Definition of percent} \\ &= 2.8 \text{ or } 2\frac{4}{5} \end{aligned}$$

2 0.12%

$$\begin{aligned} 0.12\% &= \frac{0.12}{100} && \text{Definition of percent} \\ &= 0.0012 \text{ or } \frac{3}{2,500} \end{aligned}$$

Examples

Write each decimal as a percent.

3 2.17

$$\begin{aligned} 2.17 &= \underbrace{217.}_{\text{Multiply by 100.}} \\ &= 217\% \end{aligned}$$

4 0.0034

$$\begin{aligned} 0.0034 &= \underbrace{000.34}_{\text{Multiply by 100.}} \\ &= 0.34\% \end{aligned}$$

Exercises

Write each percent as a decimal and as a mixed number or fraction in simplest form.

1. 200%

2. 750%

3. 325%

4. 0.3%

5. 0.8%

6. 0.48%

Write each decimal as a percent.

7. 2.6

8. 19

9. 5.14

10. 0.008

11. 0.0014

12. 0.0067

8-1 Study Guide and Intervention***Percent of a Number***

You can use a proportion or multiplication to find the percent of a number.

Example 1 Find 25% of 80.

$$25\% = \frac{25}{100} \text{ or } \frac{1}{4}$$

Write 25% as a fraction and reduce to lowest terms.

$$\frac{1}{4} \text{ of } 80 = \frac{1}{4} \times 80 \text{ or } 20$$

Multiply.

So, 25% of 80 is 20.

Example 2 What number is 15% of 200?

$$\begin{aligned} 15\% \text{ of } 200 &= 15\% \times 200 \\ &= 0.15 \times 200 \\ &= 30 \end{aligned}$$

Write a multiplication expression.

Write 15% as a decimal.

Multiply.

So, 15% of 200 is 30.

Exercises**Find each number.**

1. Find 20% of 50.
2. What is 55% of \$400?
3. 5% of 1,500 is what number?
4. Find 190% of 20.
5. What is 24% of \$500?
6. 8% of \$300 is how much?
7. What is 12.5% of 60?
8. Find 0.2% of 40.
9. Find 3% of \$800.
10. What is 0.5% of 180?
11. 0.25% of 42 is what number?
12. What is 0.02% of 280?

8-2 Study Guide and Intervention***The Percent Proportion***

A percent proportion compares part of a quantity to a whole quantity for one ratio and lists the percent as a number over 100 for the other ratio.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

Example 1 What percent of 24 is 18?

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100} \quad \text{Percent proportion}$$

Let $n\%$ represent the percent.

$$\frac{18}{24} = \frac{n}{100} \quad \text{Write the proportion.}$$

$$18 \times 100 = 24 \times n \quad \text{Find the cross products.}$$

$$1,800 = 24n \quad \text{Simplify.}$$

$$\frac{1,800}{24} = \frac{24n}{24} \quad \text{Divide each side by 24.}$$

$$75 = n$$

So, 18 is 75% of 24

Example 2 What number is 60% of 150?

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100} \quad \text{Percent proportion}$$

Let a represent the part.

$$\frac{a}{150} = \frac{60}{100} \quad \text{Write the proportion.}$$

$$a \times 100 = 150 \times 60 \quad \text{Find the cross products.}$$

$$100a = 9,000 \quad \text{Simplify.}$$

$$\frac{100a}{100} = \frac{9,000}{100} \quad \text{Divide each side by 100.}$$

$$a = 90$$

So, 90 is 60% of 150.

Exercises

Find each number. Round to the nearest tenth if necessary.

1. What number is 25% of 20?
2. What percent of 50 is 20?
3. 30 is 75% of what number?
4. 40% of what number is 36?
5. What number is 20% of 625?
6. 12 is what percent of 30?

8-3 Study Guide and Intervention

Percent and Estimation

To estimate the percent of a number, you can use a fraction or a multiple of 10% or 1%.

Example 1 Estimate 77% of 800.

77% is about 75% or $\frac{3}{4}$.

$$\begin{aligned} 77\% \text{ of } 800 &\approx \frac{3}{4} \cdot 800 && \text{Use } \frac{3}{4} \text{ to estimate.} \\ &\approx 600 && \text{Multiply.} \end{aligned}$$

So, 77% of 800 is about 600.

Example 2 Estimate 137% of 50.

137% is more than 100%, so 137% of 50 is greater than 50.
137% is about 140%.

$$\begin{aligned} 140\% \text{ of } 50 &= (100\% \text{ of } 50) + (40\% \text{ of } 50) && 140\% = 100\% + 40\% \\ &= (1 \cdot 50) + \left(\frac{2}{5} \cdot 50\right) && 100\% = 1 \text{ and } 40\% = \frac{2}{5} \\ &= 50 + 20 \text{ or } 70 && \text{Simplify.} \end{aligned}$$

So, 137% of 50 is about 70.

Example 3 Estimate 0.5% of 692.

0.5% is half of 1%. 692 is about 700.

$$\begin{aligned} 1\% \text{ of } 700 &= 0.01 \cdot 700 && \text{To multiply by 1\%, move the decimal point two places to the left.} \\ &= 7 \end{aligned}$$

One half of 7 is $\frac{1}{2} \cdot 7$ or 3.5.

So, 0.5% of 697 is about 3.5.

Exercises

Estimate.

- | | | |
|----------------|---------------|-----------------|
| 1. 24% of 36 | 2. 81% of 25 | 3. 11% of 67 |
| 4. 150% of 179 | 5. 67% of 450 | 6. 79% of 590 |
| 7. 0.4% of 200 | 8. 42% of 61 | 9. 19% of 41 |
| 10. 129% of 54 | 11. 32% of 66 | 12. 0.2% of 150 |

8-4 Study Guide and Intervention

Problem-Solving Investigation: Determine Reasonable Answers

When solving problems, often times it is helpful to determine reasonable answers by using rounding and estimation. Checking answers with a calculator is always helpful in determining if the answer found is in fact reasonable.

Example **SALES TAX** There is 4.8% sales tax on all clothing items purchased. Danielle wants to buy a shirt, which costs \$18.95. Danielle figures that if she has \$20 she will have enough to buy the shirt. After adding in sales tax, is \$20 a reasonable amount for Danielle to bring?

Explore The cost of the shirt is \$18.95. Sales tax is 4.8%. Danielle has \$20.

Plan Round \$18.95 to \$19.00 and 4.8% to 5%. Then use mental math to find 5% of \$19.00.

Solve Round \$18.95 to \$19.00

Round 4.8% to 5%

10% of \$19.00 = 0.1×19 or \$1.90 Use mental math. 10% = 0.1

Round \$1.90 to \$2.00

5% is $\frac{1}{2}$ of 10%

So $\frac{1}{2}$ of \$2.00 is \$1.00

\$1.00 is the amount of sales tax.

$\$19.00 + \$1.00 = \$20.00$

Add \$1.00 to \$19.00.

So \$20 is a reasonable amount of money for Danielle to bring to pay for the shirt.

Check Use a calculator to check.

$0.048 \times 18.95 = 0.9096$

Since 0.9096 is close to 1, the answer is reasonable.

Exercises

- TIP** The total bill at a restaurant for a family of 5 is \$64.72. They want to leave a 20% tip. They decide to leave \$10.00. Is this estimate reasonable? Explain your reasoning.
- TELEVISION** A recent survey shows that 67% of students watch 3 or more hours of television a night. Suppose there are 892 students in your school. What would be a reasonable estimate of the number of students in your school who watch 3 or more hours of television a night? Explain your reasoning.

8-4 Study Guide and Intervention***Problem-Solving Investigation:
Determine Reasonable Answers***

Determine reasonable answers for each.

- 1. MONEY** Gillian and Roger have lunch at a restaurant and Gillian needs to determine how much tip to leave based on their bill. If their bill was \$21.87 and Gillian wants to leave a 15% tip, what is a reasonable estimate for how much she should leave?
- 2. SPORTS** Of the 82,000 fans that attended a bowl game between Ohio State and Notre Dame, 60% were Ohio State fans. About how many fans at the game were for Notre Dame?
- 3. ICE CREAM** A survey of 1,950 people found that 39% preferred chocolate ice cream to vanilla. About how many people preferred chocolate ice cream according to the survey?
- 4. EARTH** The surface area of Earth is approximately 70% water. If the surface area is about 510,000,000 square kilometers, about how many square kilometers are water?
- 5. COLLEGE** Of 7,450 first-year college students interviewed, 72% had changed their major area of study since the beginning of the academic year. About how many students had kept the same major?
- 6. MONEY** While shopping, Hilary spent \$149. If the amount she spent was 20% of her savings, how much savings did she have before she shopped?

8-5 Study Guide and Intervention***The Percent Equation***

To solve any type of percent problem, you can use the **percent equation**, $\text{part} = \text{percent} \cdot \text{base}$, where the percent is written as a decimal.

Example 1 600 is what percent of 750?

600 is the part and 750 is the whole. Let n represent the percent.

$$\underbrace{\text{part}} = \underbrace{\text{percent}} \cdot \underbrace{\text{whole}}$$

$$600 = n \cdot 750 \quad \text{Write an equation.}$$

$$\frac{600}{750} = \frac{750n}{750} \quad \text{Divide each side by 750.}$$

$$0.8 = n \quad \text{Simplify.}$$

$$80\% = n \quad \text{Write 0.8 as a percent.}$$

So, 600 is 80% of 750.

Example 2 45 is 90% of what number?

45 is the part and 90% or 0.9 is the percent. Let n represent the whole.

$$\underbrace{\text{part}} = \underbrace{\text{percent}} \cdot \underbrace{\text{whole}}$$

$$45 = 0.9 \cdot n \quad \text{Write an equation.}$$

$$\frac{45}{0.9} = \frac{0.9n}{0.9} \quad \text{Divide each side by 0.9.}$$

$$50 = n \quad \text{The whole is 50.}$$

So, 45 is 90% of 50.

Exercises

Write an equation for each problem. Then solve. Round to the nearest tenth if necessary.

1. What percent of 56 is 14?

2. 36 is what percent of 40?

3. 80 is 40% of what number?

4. 65% of what number is 78?

5. What percent of 2,000 is 8?

6. What is 110% of 80?

7. 85 is what percent of 170?

8. Find 30% of 70.

8-7 Study Guide and Intervention**Percent of Change**

A **percent of change** is a ratio that compares the change in quantity to the original amount. If the original quantity is increased, it is a **percent of increase**. If the original quantity is decreased, it is a **percent of decrease**.

Example 1 Last year, 2,376 people attended the rodeo. This year, attendance was 2,950. What was the percent of change in attendance to the nearest whole percent?

Since this year's attendance is greater than last year's attendance, this is a percent of increase.

The amount of increase is $2,950 - 2,376$ or 574.

$$\begin{aligned} \text{percent of increase} &= \frac{\text{amount of increase}}{\text{original amount}} && \leftarrow \begin{array}{|l|} \hline \text{new amount} - \\ \text{original amount} \\ \hline \end{array} \\ &= \frac{574}{2,376} && \text{Substitution} \\ &\approx 0.24 \text{ or } 24\% && \text{Simplify.} \end{aligned}$$

Rodeo attendance increased by about 24%.

Example 2 John's grade on the first math exam was 94. His grade on the second math exam was 86. What was the percent of change in John's grade to the nearest whole percent?

Since the second grade is less than the first grade, this is a percent of decrease. The amount of decrease is $94 - 86$ or 8.

$$\begin{aligned} \text{percent of decrease} &= \frac{\text{amount of decrease}}{\text{original amount}} && \leftarrow \begin{array}{|l|} \hline \text{original amount} - \\ \text{new amount} \\ \hline \end{array} \\ &= \frac{8}{94} && \text{Substitution} \\ &\approx 0.09 \text{ or } 9\% && \text{Simplify.} \end{aligned}$$

John's math grade decreased by about 9%.

Exercises

Find each percent of change. Round to the nearest whole percent if necessary. State whether the percent of change is an *increase* or *decrease*.

1. original: 4
new: 5

2. original: 1.0
new: 1.3

3. original: 15
new: 12

4. original: \$30
new: \$18

5. original: 60
new: 63

6. original: 160
new: 136

7. original: 7.7
new: 10.5

8. original: 9.6
new: 5.9

8-8 Study Guide and Intervention**Sales Tax and Discount**

Sales tax is a percent of the purchase price and is an amount paid in addition to the purchase price.
Discount is the amount by which the regular price of an item is reduced.

Example 1 **SOCCER** Find the total price of a \$17.75 soccer ball if the sales tax is 6%.

Method 1

First, find the sales tax.

$$6\% \text{ of } \$17.75 = 0.06 \cdot 17.75 \\ \approx 1.07$$

The sales tax is \$1.07.

Next, add the sales tax to the regular price.

$$1.07 + 17.75 = 18.82$$

The total cost of the soccer ball is \$18.82.

Method 2

$$100\% + 6\% = 106\% \quad \text{Add the percent of tax to 100\%.}$$

The total cost is 106% of the regular price.

$$106\% \text{ of } \$17.75 = 1.06 \cdot 17.75 \\ \approx 18.82$$

Example 2 **TENNIS** Find the price of a \$69.50 tennis racket that is on sale for 20% off.

First, find the amount of the discount d .

$$\underbrace{\text{part}} = \underbrace{\text{percent}} \cdot \underbrace{\text{whole}}$$

$$d = 0.2 \cdot 69.50 \quad \text{Use the percent equation.}$$

$$d = 13.90 \quad \text{The discount is } \$13.90.$$

So, the sale price of the tennis racket is \$69.50 – \$13.90 or \$55.60.

Exercises

Find the total cost or sale price to the nearest cent.

- | | |
|--------------------------------|-------------------------------------|
| 1. \$22.95 shirt; 7% sales tax | 2. \$39.00 jeans; 25% discount |
| 3. \$35 belt; 40% discount | 4. \$115.48 watch; 6% sales tax |
| 5. \$16.99 book; 5% off | 6. \$349 television; 6.5% sales tax |

8-9 Study Guide and Intervention**Simple Interest**

Simple interest is the amount of money paid or earned for the use of money. To find simple interest I , use the formula $I = prt$. Principal p is the amount of money deposited or invested. Rate r is the annual interest rate written as a decimal. Time t is the amount of time the money is invested in years.

Example 1 Find the simple interest earned in a savings account where \$136 is deposited for 2 years if the interest rate is 7.5% per year.

$$I = prt \quad \text{Formula for simple interest}$$

$$I = 136 \cdot 0.075 \cdot 2 \quad \text{Replace } p \text{ with } \$136, r \text{ with } 0.075, \text{ and } t \text{ with } 2.$$

$$I = 20.40 \quad \text{Simplify.}$$

The simple interest earned is \$20.40.

Example 2 Find the simple interest for \$600 invested at 8.5% for 6 months.

$$6 \text{ months} = \frac{6}{12} \text{ or } 0.5 \text{ year} \quad \text{Write the time as years.}$$

$$I = prt \quad \text{Formula for simple interest}$$

$$I = 600 \cdot 0.085 \cdot 0.5 \quad p = \$600, r = 0.085, t = 0.5$$

$$I = 25.50 \quad \text{Simplify.}$$

The simple interest is \$25.50.

Exercises

Find the interest earned to the nearest cent for each principal, interest rate, and time.

1. \$300, 5%, 2 years
2. \$650, 8%, 3 years
3. \$575, 4.5%, 4 years
4. \$735, 7%, $2\frac{1}{2}$ years
5. \$1,665, 6.75%, 3 years
6. \$2,105, 11%, $1\frac{3}{4}$ years
7. \$903, 8.75%, 18 months
8. \$4,275, 19%, 3 months

9-1

Study Guide and Intervention

Line Plots

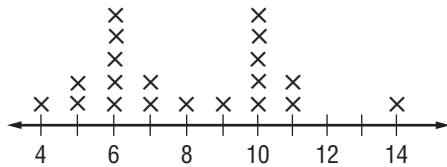
A **line plot** is a diagram that shows the frequency of data on a number line.

Example 1 **SHOE SIZE** The table shows the shoe size of students in Mr. Kowa’s classroom. Make a line plot of the data.

Shoe Sizes			
10	6	4	6
5	11	10	10
6	9	6	8
7	11	7	14
5	10	6	10

Step 1 Draw a number line. Because the smallest size is 4 and the largest size is 14, you can use a scale of 4 to 14 and an interval of 2.

Step 2 Put an “x” above the number that represents the shoe size of each student.



Example 2 Use the line plot in Example 1. Identify any clusters, gaps, or outliers and analyze the data by using these values. What is the range of data?

Many of the data cluster around 6 and 10. You could say that most of the shoe sizes are 6 or 10. There is a gap between 11 and 14, so there are no shoe sizes in this range. The number 14 appears removed from the rest of the data, so it would be considered an outlier. This means that the shoe size of 14 is very large and is not representative of the whole data set.

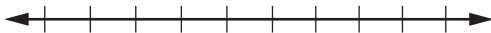
The greatest shoe size is 14, and the smallest is 4. The range is $14 - 4$ or 10.

Exercises

PETS For Exercises 1–3 use the table at the right that shows the number of pets owned by different families.

Number of Pets			
2	1	2	0
3	1	1	2
8	3	1	4

1. Make a line plot of the data.



2. Identify any clusters, gaps, or outliers.

3. What is the range of the data?

9-2**Study Guide and Intervention*****Mean, Median, and Mode***

The **mean** is the sum of the data divided by the number of data items. The **median** is the middle number of the ordered data, or the mean of the middle two numbers. The **mode** is the number (or numbers) that occur most often. The mean, median, and mode are each **measures of central tendency**.

Example The table shows the number of hours students spent practicing for a music recital. Find the mean, median, and mode of the data.

$$\text{mean} = \frac{3 + 12 + 10 + \dots + 12}{20} = \frac{160}{20} \text{ or } 8.$$

To find the median, the data must be ordered.

0, 1, 2, 3, 3, 5, 6, 7, 8, 8, 8, 9, 10, 10, 11, 12, 12, 12, 15, 18

$$\frac{8 + 8}{2} = 8$$

To find the mode, look for the number that occurs most often. Since 8 and 12 each occur 3 times, the modes are 8 and 12.

Numbers of Hours Spent Practicing				
3	12	10	8	7
18	11	12	10	3
8	6	0	1	5
8	2	15	9	12

Exercises

Find the mean, median, and mode for each set of data. Round to the nearest tenth if necessary.

1. 27, 56, 34, 19, 41, 56, 27, 25, 34, 56

2. 7, 3, 12, 4, 6, 3, 4, 8, 7, 3, 20

3. 1, 23, 4, 6, 7, 20, 7, 5, 3, 4, 6, 7, 11, 6

4. 3, 3, 3, 3, 3, 3, 3

5. 2, 4, 1, 3, 5, 6, 1, 1, 3, 4, 3, 1

6. 4, 0, 12, 10, 0, 5, 7, 16, 12, 10, 12, 12

9-3**Study Guide and Intervention*****Stem-and-Leaf Plots***

In a **stem-and-leaf plot**, the data are organized from least to greatest. The digits of the least place value usually form the **leaves**, and the next place value digits form the **stems**.

Example Make a stem-and-leaf plot of the data below. Then find the range, median, and mode of the data.
 42, 45, 37, 46, 35, 49, 47, 35, 45, 63, 45

Order the data from least to greatest.

35, 35, 37, 42, 45, 45, 45, 46, 47, 49, 63

The least value is 35, and the greatest value is 63.

So, the tens digits form the stems, and the ones digits form the leaves.

range: greatest value – least value = $63 - 35$ or 28

median: middle value, or 45

mode: most frequent value, or 45

Stem	Leaf
3	5 5 7
4	2 5 5 5 6 7 9
5	
6	3

$$6 | 3 = 63$$

Exercises

Make a stem-and-leaf plot for each set of data. Then find the range, median, and mode of the data.

1. 15, 25, 16, 28, 1, 27, 16, 19, 28

2. 1, 2, 3, 2, 3, 1, 4, 2, 5, 7, 12, 11, 11, 3, 10

3. 3, 5, 1, 17, 11, 45, 17

4. 4, 7, 10, 5, 8, 12, 7, 6

9-4

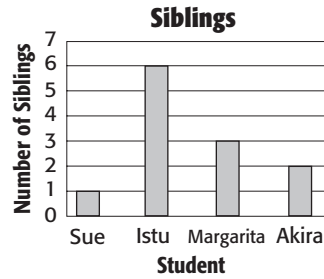
Study Guide and Intervention

Bar Graphs and Histograms

A **bar graph** is one method of comparing data by using solid bars to represent quantities. A **histogram** is a special kind of bar graph. It uses bars to represent the frequency of numerical data that have been organized into intervals.

Example 1 **SIBLINGS** Make a bar graph to display the data in the table below.

Student	Number of Siblings
Sue	1
Isfu	6
Margarita	3
Akira	2

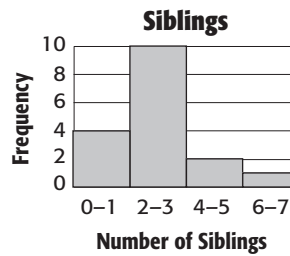


Step 1 Draw a horizontal and a vertical axis. Label the axes as shown. Add a title.

Step 2 Draw a bar to represent each student. In this case, a bar is used to represent the number of siblings for each student.

Example 2 **SIBLINGS** The number of siblings of 17 students have been organized into a table. Make a histogram of the data.

Number of Siblings	Frequency
0–1	4
2–3	10
4–5	2
6–7	1



Step 1 Draw and label horizontal and vertical axes. Add a title.

Step 2 Draw a bar to represent the frequency of each interval.

Exercises

1. Make a bar graph for the data in the table.

Student	Number of Free Throws
Luis	6
Laura	10
Opal	4
Gad	14

2. Make a histogram for the data in the table.

Number of Free Throws	Frequency
0–1	1
2–3	5
4–5	10
6–7	4

9-5

Study Guide and Intervention

Venn Diagrams

A **Venn diagram** is an arrangement of overlapping circles used to show how sets of data are related.

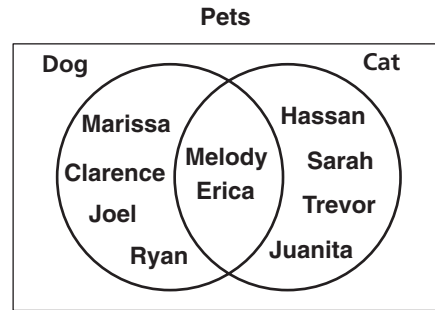
Example **PETS** Mr. Morris took a survey of his class to determine what type of pet each of his 10 students had. Display the results shown below in a Venn diagram.

Dog	Marissa, Clarence, Melody, Joel, Erica, Ryan
Cat	Hassan, Melody, Sarah, Trevor, Juanita, Erica

First draw two overlapping circles to represent dogs and cats. Label each circle.

Since Melody and Erica have both pets, write their names in the overlapping region.

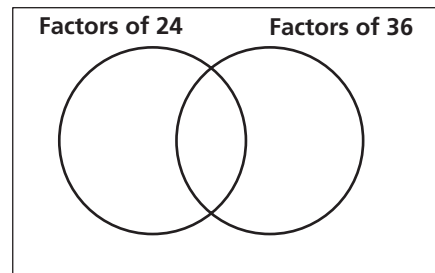
Write each name in the appropriate region. Marissa, Clarence, Joel, and Ryan have only dogs. Hassan, Sarah, Trevor, and Juanita have only cats.



Exercises

- NUMBERS** The table below shows the factors of 24 and 36. Use a Venn diagram to show how the factors are related.

24	36
1, 2, 3, 4, 6, 8, 12, 24	1, 2, 3, 4, 6, 9, 12, 18, 36

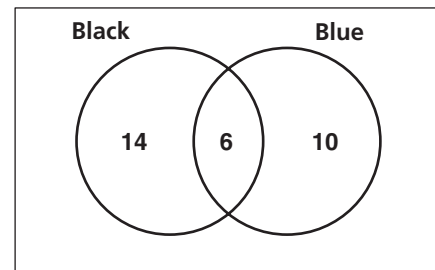


- SPORTS** Kevin took a survey of his six friends to see who plays soccer and who plays rugby. Display the results shown below in a Venn Diagram.

Soccer	Rick, Toby, Jeremy, Ronald, Bobby
Rugby	Jamal, Bert, Toby, Alex

Use the Venn diagram at the right to answer Questions 3–5.

- How many students like either color or blue only?
- How many total students were surveyed?
- How many more students like black or either color than blue or either color?



9-6

Study Guide and Intervention

Problem-Solving Investigation: Use a Graph

When solving problems, a **graph** can show a visual representation of the situation and help you make conclusions about the particular set of data.

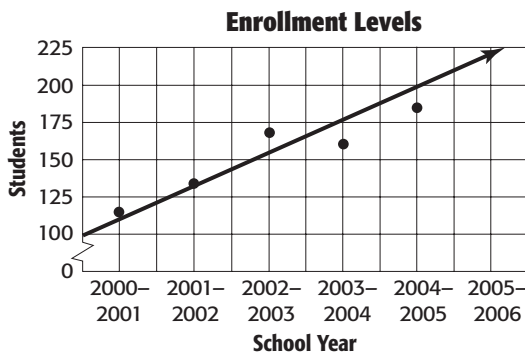
Example **POPULATION** The table below shows the enrollment of Mill High School students over five years. Estimate the enrollment was for the 2005–2006 school year.

Mill High School Enrollment				
00–01	01–02	02–03	03–04	04–05
115	134	168	160	185

Explore You know the enrollment of students for five years. You need to estimate the enrollment for the 2005–2006 school year.

Plan Organize the data in a graph so that you can see a trend in the enrollment levels.

Solve



The graph shows that the enrollment increases over the years. By using the graph you can conclude that Mill High School had about 225 students enrolled for the 2005–2006 school year.

Check Draw a line through as close to as many points as possible. The estimate is close to the line so the answer is reasonable.

Exercises

1. TEMPERATURE The chart to the right shows the average December temperatures in Fahrenheit over four years. Predict the average temperature for the next year.

December Temperatures (F°)			
2002	2003	2004	2005
22°	17°	18°	16°

2. POPULATION Every five years the population of your neighborhood is recorded. What do you predict the population will be in 2010?

Neighborhood Population		
1995	2000	2005
2,072	2,250	2,376

9-7

Study Guide and Intervention

Line Graphs and Scatter Plots

A **line graph** shows trends over time and can be useful for predicting future events. A **scatter plot** displays two sets of data on a graph and can be useful for predictions by showing trends in the data.

Example Use the line graph of the Moralez family car trip shown below to answer the following questions.

- After 250 miles, how much gas did the Moralez family have left?

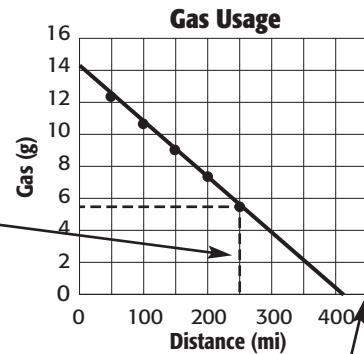
Draw a dotted line up from 250 m until it reaches the graph and then find the corresponding gas measure.

They will have about 5.5 g left.

- How far can the Moralez family travel before they run out of gas?

When they run out of gas, the tank will be at 0 so find where the line reaches 0.

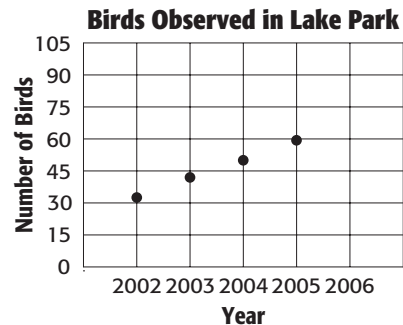
They can travel about 430 miles.



Exercises

Use the scatter plot to answer the questions.

- How many birds were there in 2004?
- What relationship do you see between the number of birds and year?
- Predict the number of birds there were in the year 2001?
- Predict the number of birds there will be in in the year 2006?
- In what year do you think the bird population will reach 100?



9-8**Study Guide and Intervention****Select an Appropriate Display**

Display	Statistical Displays
Bar Graph	Shows the number of items in specific categories
Circle Graph	Compares parts of the data to the whole
Histogram	Shows frequency of data divided into equal intervals
Line Graph	Shows change over a period of time
Line Plot	Shows how many times each number occurs in the data
Stem-and-Leaf Plot	Lists all individual numerical data in a condensed form
Venn Diagram	Shows how two or more sets of data are related

Example 1 Select the appropriate display for the given data example. Explain.

- A survey asking students if they play in the band or play sports
Venn Diagram because it can show overlapping of band and sports.
- A list of the age of people walking into a mall for a one hour time period.
A stem-and-leaf plot because it puts the data into categories.

Example 2 MONEY The table shows what percent of Joanne's paycheck goes to each category of her budget. Select an appropriate display to visually show the information provided in the chart. Explain your choice.

A circle graph is the best choice because it shows how the parts compare to her total paycheck and the sum of the percents is 100%

Category	Percent
Savings	10%
Bills	55%
Car	10%
Miscellaneous	25%

Exercises

Select the appropriate display for the following examples. Explain.

- The distance a runner has traveled each mile of a 5-mile race
- The number of boys in each classroom for 10 rooms
- The ages of the next 50 people who walk come through your checkout line
- A collection of data involving animals that are black and nocturnal

Create an appropriate display for the following sets of data.

5.

City	Number Of Skyscrapers
Ontario	23
New York City	55
Chicago	17
Los Angeles	41

6.

Distance (m)	Time (h)
50	1
92	2
121	3
165	4

9-9

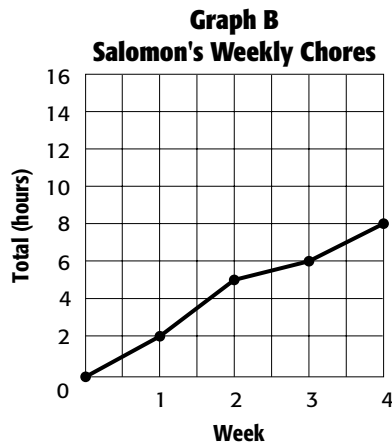
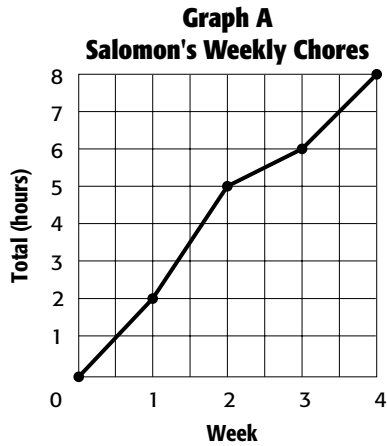
Study Guide and Intervention

Misleading Statistics

Graphs can be misleading for many reasons: there is no title, the scale does not include 0; there are no labels on either axis; the intervals on a scale are not equal; or the size of the graphics misrepresents the data.

Example

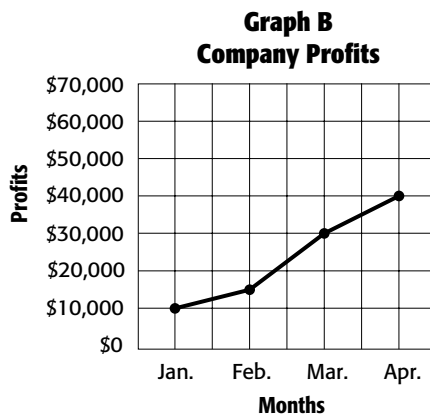
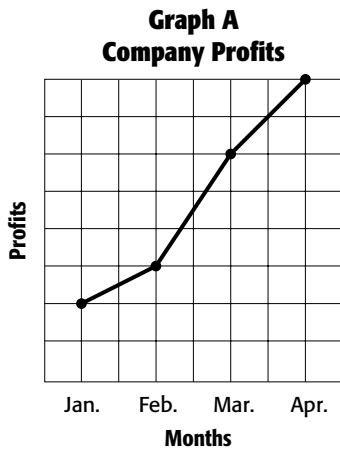
WEEKLY CHORES The line graphs below show the total hours Salomon spent doing his chores one month. Which graph would be best to use to convince his parents he deserves a raise in his allowance? Explain.



He should use graph A because it makes the total hours seem much larger.

Exercises

PROFITS For Exercises 1 and 2, use the graphs below. It shows a company's profits over a four-month period.



- Which graph would be best to use to convince potential investors to invest in this company?
- Why might the graph be misleading?

10-1**Study Guide and Intervention****Simple Events**

The **probability** of a simple event is a ratio that compares the number of favorable outcomes to the number of possible outcomes. Outcomes occur at **random** if each outcome occurs by chance.

Two events that are the only ones that can possibly happen are **complementary events**. The sum of the probabilities of complementary events is 1.

Example 1 What is the probability of rolling a multiple of 3 on a number cube marked with 1, 2, 3, 4, 5, and 6 on its faces.

$$\begin{aligned} P(\text{multiple of } 3) &= \frac{\text{multiples of } 3 \text{ possible}}{\text{total numbers possible}} \\ &= \frac{2}{6} && \text{Two numbers are multiples of } 3: 3 \text{ and } 6. \\ &= \frac{1}{3} && \text{Simplify.} \end{aligned}$$

The probability of rolling a multiple of 3 is $\frac{1}{3}$ or about 33.3%.

Example 2 What is the probability of *not* rolling a multiple of 3 on a number cube marked with 1, 2, 3, 4, 5, and 6 on its faces?

$$\begin{aligned} P(A) + P(\text{not } A) &= 1 \\ \frac{1}{3} + P(\text{not } A) &= 1 && \text{Substitute } \frac{1}{3} \text{ for } P(A). \\ -\frac{1}{3} & \quad -\frac{1}{3} && \text{Subtract } \frac{1}{3} \text{ from each side} \\ \hline P(\text{not } A) &= \frac{2}{3} && \text{Simplify.} \end{aligned}$$

The probability of *not* rolling a multiple of 3 is $\frac{2}{3}$ or about 66.7%.

Exercises

A set of 30 cards is numbered 1, 2, 3, ..., 30. Suppose you pick a card at random without looking. Find the probability of each event. Write as a fraction in simplest form.

- $P(12)$
- $P(2 \text{ or } 3)$
- $P(\text{odd number})$
- $P(\text{a multiple of } 5)$
- $P(\text{not a multiple of } 5)$
- $P(\text{less than or equal to } 10)$

10-2 Study Guide and Intervention

Sample Spaces

A game in which players of equal skill have an equal chance of winning is a **fair game**. A **tree diagram** or table is used to show all of the possible outcomes, or **sample space**, in a probability experiment.

Example 1 **WATCHES** A certain type of watch comes in brown or black and in a small or large size. Find the number of color-size combinations that are possible.

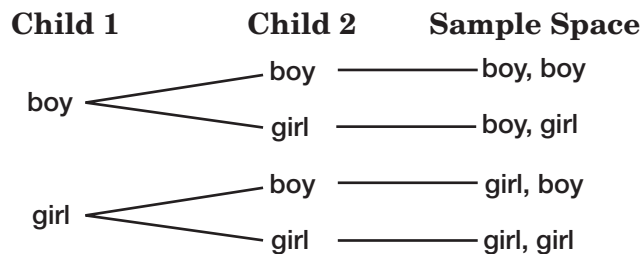
Make a table to show the sample space. Then give the total number of outcomes.

Color	Size
Brown	Small
Brown	Large
Black	Small
Black	Large

There are four different color and size combinations.

Example 2 **CHILDREN** The chance of having either a boy or a girl is 50%. What is the probability of the Smiths having two girls?

Make a tree diagram to show the sample space. Then find the probability of having two girls.



The sample space contains 4 possible outcomes. Only 1 outcome has both children being girls. So, the probability of having two girls is $\frac{1}{4}$.

Exercises

For each situation, make a tree diagram or table to show the sample space. Then give the total number of outcomes.

- choosing an outfit from a green shirt, blue shirt, or a red shirt, and black pants or blue pants
- choosing a vowel from the word COUNTING and a consonant from the word PRIME

10-3**Study Guide and Intervention*****The Fundamental Counting Principle***

If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by N can occur in $m \times n$ ways. This is called the **Fundamental Counting Principle**.

Example 1 **CLOTHING** Andy has 5 shirts, 3 pairs of pants, and 6 pairs of socks. How many different outfits can Andy choose with a shirt, pair of pants, and pair of socks?

$$\begin{array}{ccccccc}
 \text{number of shirts} & & \text{number of pants} & & \text{number of socks} & & \text{total number of outfits} \\
 \underbrace{\hspace{2cm}} & \cdot & \underbrace{\hspace{2cm}} & \cdot & \underbrace{\hspace{2cm}} & = & \underbrace{\hspace{2cm}} \\
 5 & & 3 & & 6 & & 90
 \end{array}$$

Andy can choose 90 different outfits.

Exercises

Use the **Fundamental Counting Principle** to find the total number of outcomes in each situation.

1. rolling two number cubes
2. tossing 3 coins
3. picking one consonant and one vowel
4. choosing one of 3 processor speeds, 2 sizes of memory, and 4 sizes of hard drive
5. choosing a 4-, 6-, or 8-cylinder engine and 2- or 4-wheel drive
6. rolling 2 number cubes and tossing 2 coins
7. choosing a color from 4 colors and a number from 4 to 10

10-4 Study Guide and Intervention**Permutations**

The expression n **factorial** ($n!$) is the product of all counting numbers beginning with n and counting backward to 1. A **permutation** is an arrangement, or listing, of objects in which order is important. You can use the Fundamental Counting Principle to find the number of possible arrangements.

Example 1 Find the value of $5!$.

$$\begin{aligned} 5! &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 && \text{Definition of factorial} \\ &= 120 && \text{Simplify.} \end{aligned}$$

Example 2 Find the value of $4! \cdot 2!$.

$$\begin{aligned} 4! \cdot 2! &= 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1 && \text{Definition of factorial} \\ &= 48 && \text{Simplify.} \end{aligned}$$

Example 3 BOOKS How many ways can 4 different books be arranged on a bookshelf?

This is a permutation that can be written as $4!$. Suppose the books are placed on the shelf from left to right.

There are 4 choices for the first book.

There are 3 choices that remain for the second book.

There are 2 choices that remain for the third book.

There is 1 choice that remains for the fourth book.

$$\begin{aligned} 4! &= 4 \cdot 3 \cdot 2 \cdot 1 && \text{Definition of factorial} \\ &= 24 && \text{Simplify.} \end{aligned}$$

So, there are 24 ways to arrange 4 different books on a bookshelf.

Exercises

Find the value of each expression.

1. $3!$
2. seven factorial
3. $6! \cdot 3!$
4. $9 \cdot 8 \cdot 7$
5. How many ways can you arrange the letters in the word GROUP?
6. How many different 4-digit numbers can be created if no digit can be repeated? Remember, a number cannot begin with 0.

10-5 Study Guide and Intervention**Combinations**

An arrangement, or listing, of objects in which order is *not* important is called a **combination**. You can find the number of combinations of objects by dividing the number of permutations of the entire set by the number of ways each smaller set can be arranged.

Example 1 Jill was asked by her teacher to choose 3 topics from the 8 topics given to her. How many different three-topic groups could she choose?

There are $8 \cdot 7 \cdot 6$ permutations of three-topic groups chosen from eight. There are $3!$ ways to arrange the groups.

$$\frac{8 \cdot 7 \cdot 6}{3!} = \frac{336}{6} = 56$$

So, there are 56 different three-topic groups.

Tell whether each situation represents a *permutation* or *combination*. Then solve the problem.

Example 2 On a quiz, you are allowed to answer any 4 out of the 6 questions. How many ways can you choose the questions?

This is a combination because the order of the 4 questions is not important. So, there are $6 \cdot 5 \cdot 4 \cdot 3$ permutations of four questions chosen from six. There are $4!$ or $4 \cdot 3 \cdot 2 \cdot 1$ orders in which these questions can be chosen.

$$\frac{6 \cdot 5 \cdot 4 \cdot 3}{4!} = \frac{360}{24} = 15$$

So, there are 15 ways to choose the questions.

Example 3 Five different cars enter a parking lot with only 3 empty spaces. How many ways can these spaces be filled?

This is a permutation because each arrangement of the same 3 cars counts as a distinct arrangement. So, there are $5 \cdot 4 \cdot 3$ or 60 ways the spaces can be filled.

Exercises

Tell whether each situation represents a *permutation* or *combination*. Then solve the problem.

1. How many ways can 4 people be chosen from a group of 11?
2. How many ways can 3 people sit in 4 chairs?
3. How many ways can 2 goldfish be chosen from a tank containing 15 goldfish?

10-6 Study Guide and Intervention**Problem-Solving Investigation: Act It Out**

By acting out a problem, you are able to see all possible solutions to the problem being posed.

Example **CLOTHING** Ricardo has two shirts and three pairs of pants to choose from for his outfit to wear on the first day of school. How many different outfits can he make by wearing one shirt and one pair of pants?

Explore We know that he has two shirts and three pairs of pants to choose from. We can use a coin for the shirts and an equally divided spinner labeled for the pants.

Plan Let's make a list showing all possible outcomes of tossing a coin and then spinning a spinner.

Solve H = Heads
T = Tails
Spinner = 1, 2, 3

Flip a Coin	Spin a Spinner
H	1
H	2
H	3
T	1
T	2
T	3

There are six possible outcomes of flipping a coin and spinning a spinner. So, there are 6 different possible outfits that Ricardo can wear for the first day of school.

Check Flipping a coin has two outcomes and there are two shirts. Spinning a three-section spinner has three outcomes and there are three pairs of pants. Therefore, the solution of 6 different outcomes with a coin and spinner represent the 6 possible outfit outcomes for Ricardo.

Exercises

- 1. SCIENCE FAIR** There are 4 students with projects to present at the school science fair. How many different ways can these 4 projects be displayed on four tables in a row?
- 2. GENDER** Determine whether flipping a coin is a good way to predict the gender of the next 5 babies born at General Hospital. Justify your answer.
- 3. OLYMPICS** Four runners are entered in the first hurdles heat of twelve heats at the Olympics. The first two move on to the next round. Assuming no ties, how many different ways can the four runners come in first and second place?

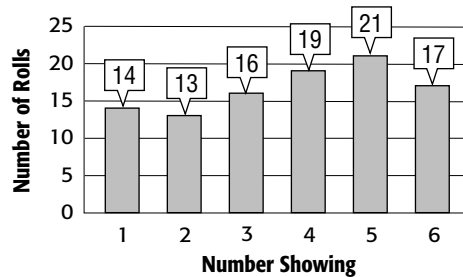
10-7

Study Guide and Intervention

Theoretical and Experimental Probability

Experimental probability is found using frequencies obtained in an experiment or game. **Theoretical probability** is the expected probability of an event occurring.

Example 1 The graph shows the results of an experiment in which a number cube was rolled 100 times. Find the experimental probability of rolling a 3 for this experiment.



$$P(3) = \frac{\text{number of times 3 occurs}}{\text{number of possible outcomes}}$$

$$= \frac{16}{100} \text{ or } \frac{4}{25}$$

The experimental probability of rolling a 3 is $\frac{4}{25}$, which is close to its theoretical probability of $\frac{1}{6}$.

Example 2 In a telephone poll, 225 people were asked for whom they planned to vote in the race for mayor. What is the experimental probability of Juarez being elected?

Candidate	Number of People
Juarez	75
Davis	67
Abramson	83

Of the 225 people polled, 75 planned to vote for Juarez.

So, the experimental probability is $\frac{75}{225}$ or $\frac{1}{3}$.

Example 3 Suppose 5,700 people vote in the election. How many can be expected to vote for Juarez?

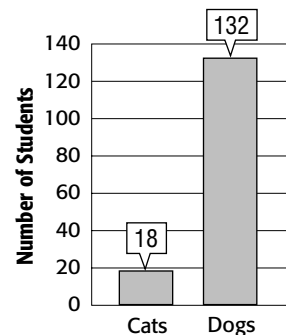
$$\frac{1}{3} \cdot 5,700 = 1,900$$

About 1,900 will vote for Juarez.

Exercises

For Exercises 1–3, use the graph of a survey of 150 students asked whether they prefer cats or dogs.

1. What is the probability of a student preferring dogs?
2. Suppose 100 students were surveyed. How many can be expected to prefer dogs?
3. Suppose 300 students were surveyed. How many can be expected to prefer cats?



10-8 Study Guide and Intervention***Independent Events***

A **compound event** consists of two or more simple events. If the outcome of one event does not affect the outcome of a second event, the events are called **independent events**. The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.

Example 1 A coin is tossed and a number cube is rolled. Find the probability of tossing tails and rolling a 5.

$$P(\text{tails}) = \frac{1}{2} \qquad P(5) = \frac{1}{6}$$

$$P(\text{tails and } 5) = \frac{1}{2} \cdot \frac{1}{6} \text{ or } \frac{1}{12}$$

So, the probability of tossing tails and rolling a 5 is $\frac{1}{12}$.

Example 2 **MARBLES** A bag contains 7 blue, 3 green, and 3 red marbles. If Agnes randomly draws two marbles from the bag, replacing the first before drawing the second, what is the probability of drawing a green and then a blue marble?

$$P(\text{green}) = \frac{3}{13} \qquad 13 \text{ marbles, } 3 \text{ are green}$$

$$P(\text{blue}) = \frac{7}{13} \qquad 13 \text{ marbles, } 7 \text{ are blue}$$

$$P(\text{green, then blue}) = \frac{3}{13} \cdot \frac{7}{13} = \frac{21}{169}$$

So, the probability that Agnes will draw a green, then a blue marble is $\frac{21}{169}$.

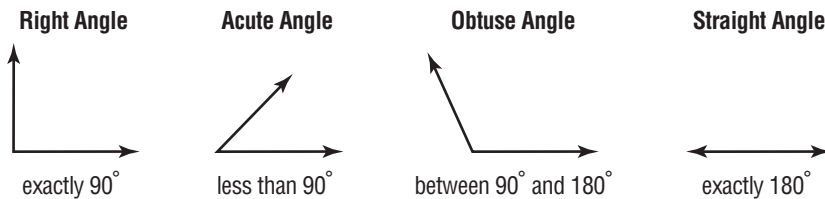
Exercises

- Find the probability of rolling a 2 and then an even number on two consecutive rolls of a number cube.
- A penny and a dime are tossed. What is the probability that the penny lands on heads and the dime lands on tails?
- Lazlo's sock drawer contains 8 blue and 5 black socks. If he randomly pulls out one sock, what is the probability that he picks a blue sock?

11-1 Study Guide and Intervention

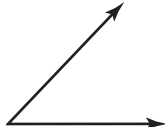
Angle Relationships

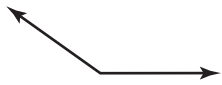
- An **angle** has two sides that share a common endpoint. The point where the sides meet is called the **vertex**. Angles are measured in **degrees**, where 1 degree is one of 360 equal parts of a circle.
- Angles are classified according to their measure.



- Two angles are **complementary** if the sum of their measures is 90° .
- Two angles are **supplementary** if the sum of their measures is 180° .
- To **find a missing angle measure**, first determine if the angles are complementary or supplementary. Then write an equation and subtract to find the missing measure.

Example 1 Classify each angle as *acute*, *obtuse*, *right*, or *straight*.

A.  The angle is less than 90° , so it is an acute angle.

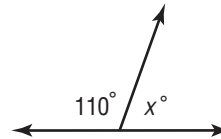
B.  The angle is greater than 180° , so it is an obtuse angle.

Example 2 Find the value of x .

The two angles form a straight line or 180° , so they are supplementary,

$$\begin{array}{r} 110 + x = 180 \\ -110 \quad -110 \\ \hline x = 70 \end{array}$$

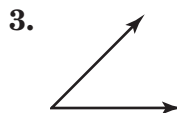
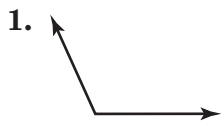
Write the equation.
Subtract 110 from each side.



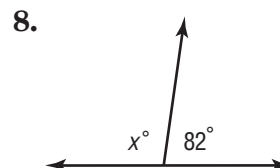
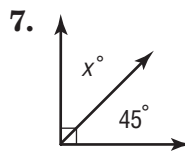
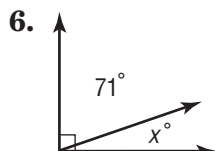
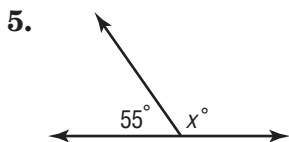
So, the value of x is 70° .

Exercises

Classify each angle as *acute*, *obtuse*, *right*, or *straight*.



Find the missing angle measure.



11-2 Study Guide and Intervention

Statistics: Display Data in a Circle Graph

A graph that shows data as parts of a whole circle is called a **circle graph**. In a circle graph, the percents add up to 100. When percents are not given, you must first determine what part of the whole each item represents.

Example 1 **ENERGY** Make a circle graph of the data in the table.

Nuclear Reactors in Operation, 2001	
Country	Number of Reactors
United States	104
France	59
Japan	54
Other Countries	222

Step 1 Find the total number of reactors:
 $104 + 59 + 54 + 222 = 439$.

Step 2 Find the ratio that compares each number with the total. Write the ratio as a decimal rounded to the nearest hundredth.

United States: $\frac{104}{439} \approx 0.24$

Japan: $\frac{54}{439} \approx 0.12$

France: $\frac{59}{439} \approx 0.13$

Other: $\frac{222}{439} \approx 0.51$

Step 3 Find the number of degrees for each section of the graph.

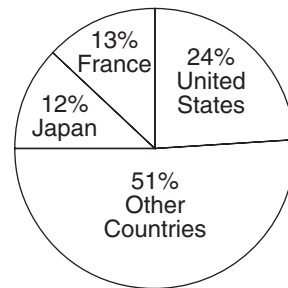
United States: $0.24 \cdot 360^\circ \approx 86^\circ$ Japan: $0.12 \cdot 360^\circ \approx 43^\circ$

France: $0.13 \cdot 360^\circ \approx 47^\circ$ Other: $0.51 \cdot 360^\circ \approx 183^\circ$

Step 4 Use a compass to construct a circle and draw a radius. Then use a protractor to draw an 86° angle. This represents the percent of nuclear reactors in the United States.

Step 5 From the new radius, draw a 47° angle for France. Repeat this step for the other two sections. Label each section and give the graph a title.

Nuclear Reactors in Operation, 2001

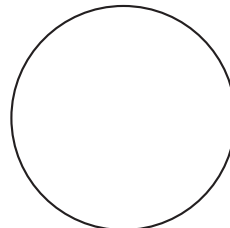


Exercises

1. **SWIMMING** The table shows the number of members of the swim team who competed at the swim meet. Each competed in only one event. Make a circle graph of the data.

Swim Team Member Participation	
Event	Number
Freestyle	18
Breaststroke	7
Backstroke	5
Butterfly	2

Swim Team Member Participation



11-3 Study Guide and Intervention

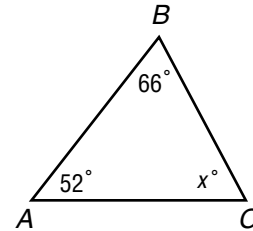
Triangles

A **triangle** is a figure with three sides and three angles. The symbol for triangle is \triangle . The sum of the measures of the angles of a triangle is 180° . You can use this to find a missing angle measure in a triangle.

Example 1 Find the value of x in $\triangle ABC$.

$$\begin{array}{r} x + 66 + 52 = 180 \\ x + 118 = 180 \\ - 118 \quad - 118 \\ \hline x = 62 \end{array}$$

The sum of the measures is 180.
Simplify.
Subtract 118 from each side.



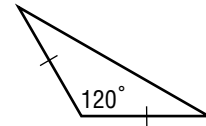
The missing angle is 62° .

Triangles can be classified by the measures of their angles. An **acute triangle** has three acute angles. An **obtuse triangle** has one obtuse angle. A **right triangle** has one right angle.

Triangles can also be classified by the lengths of their sides. Sides that are the same length are **congruent segments** and are often marked by tick marks. In a **scalene triangle**, all sides have different lengths. An **isosceles triangle** has at least two congruent sides. An **equilateral triangle** has all three sides congruent.

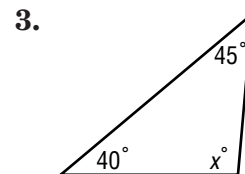
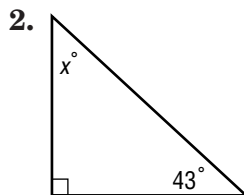
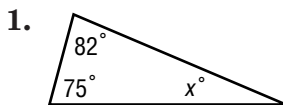
Example 2 Classify the triangle by its angles and by its sides.

The triangle has one obtuse angle and two sides the same length. So, it is an obtuse, isosceles triangle.

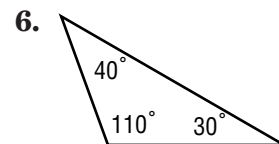
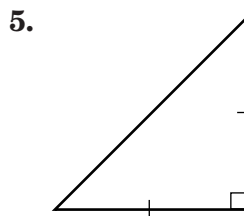
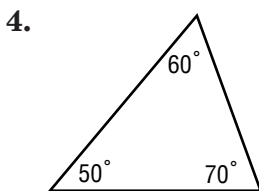


Exercises

Find the missing measure in each triangle. Then classify the triangle as *acute*, *right*, or *obtuse*.



Classify each triangle by its angles and by its sides.



11-4 Study Guide and Intervention***Problem-Solving Investigation: Use Logical Reasoning***

Logical reasoning is a method of problem solving that uses **inductive reasoning**, making a rule after seeing several examples or **deducting reasoning**, use a rule to make a decision.

Example Use the formula $d = gt$ where d is distance, g is gravity, and t is time to determine how far a ball has dropped after 8 seconds if the effect of gravity is 10 m/s.

Explore You know the ball has dropped for 8 seconds and that gravity is 10 m/s.

Plan Try a few examples to find a pattern. Make a table.

Solve

Seconds Passed	Distance Traveled
1	10
2	20
3	30
4	40
5	50
t	$10t$

After each second, the ball has traveled 10 more meters. So after 8 seconds the ball will have traveled 80m.

Examine The formula is $d = gt$ so $d = 10 \times 8$ or 80 m.

Exercises

Solve the following problems using logical reasoning.


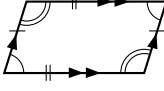



- TRAVEL** Use the formula $d = rt$ where d is the distance, r is the rate, and t is the time to determine how far the Morales family has traveled if they are driving at a rate of 72 miles per hour for 9 hours.
- CELL PHONES** Determine the cost per phone call if Maria made 30 calls last month and her total bill for the month was \$45.00.
- MUSIC** Sarah, Juan, and Derrick play the piano, trumpet, and violin, but not in that order. Sarah and Derrick sit on either side of the trumpet player. Sarah does not play the violin. Who plays the violin?

11-5

Study Guide and Intervention

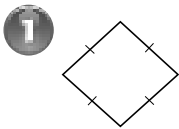
Quadrilaterals

Quadrilaterals can be classified using their angles and sides. The best description of a quadrilateral is the one that is the most specific.

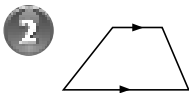
 <p>Trapezoid one pair of parallel sides</p>	 <p>Parallelogram opposite sides parallel and opposite sides congruent</p>	 <p>Rectangle parallelogram with 4 right angles</p>	 <p>Rhombus parallelogram with 4 congruent sides</p>	 <p>Square parallelogram with 4 right angles and 4 congruent sides</p>
--	--	---	--	--

Examples

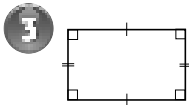
Classify the quadrilateral using the name that *best* describes it.



The quadrilateral is a parallelogram with 4 congruent sides. It is a rhombus.



The quadrilateral has one pair of parallel sides. It is a trapezoid.



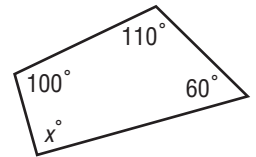
The quadrilateral is a parallelogram with 4 right angles. It is a rectangle.

Example

Find the missing measure in the quadrilateral.

$$\begin{array}{r}
 100 + 110 + 6 + x = 360 \\
 270 + x = 360 \\
 \underline{- 270 \quad - 270} \\
 x = 90
 \end{array}$$

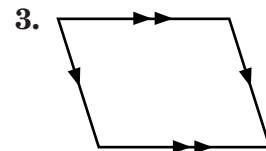
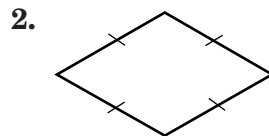
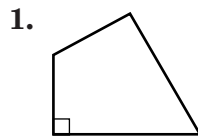
Write the equation.
Simplify.
Subtract 270 from each side.



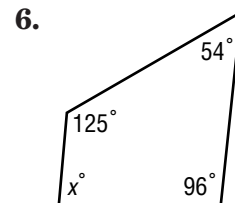
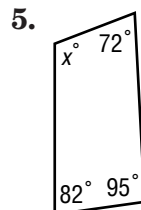
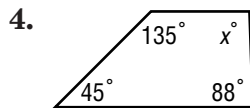
So, the missing measure is 90°.

Exercises

Classify the quadrilateral using the name that *best* describes it.



Find the missing angle measure in each quadrilateral.



11-6 Study Guide and Intervention

Similar Figures

Figures that have the same shape but not necessarily the same size are *similar figures*. The symbol \sim means *is similar to*. You can use proportions to find the missing length of a side in a pair of similar figures.

For example $\triangle ABC \sim \triangle DEF$.

Corresponding angles

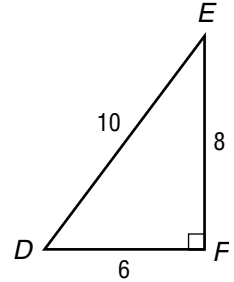
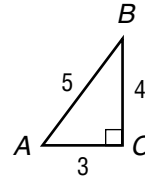
$$\angle A \cong \angle D$$

$$\angle B \cong \angle E$$

$$\angle C \cong \angle F$$

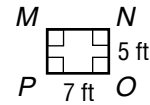
Corresponding sides

$$\frac{5}{10} = \frac{4}{8} = \frac{3}{6}$$



Example 1 If $MNOP \sim RSTU$, find the length of \overline{ST} .

Since the two figures are similar, the ratios of their corresponding sides are equal. You can write and solve a proportion to find \overline{ST} .



$$\frac{PO}{UT} = \frac{NO}{ST}$$

Write a proportion.

$$\frac{7}{28} = \frac{5}{n}$$

Let n represent the length of ST . Then substitute.

$$7n = 28(5)$$

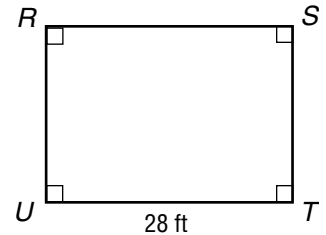
Find the cross products.

$$7n = 140$$

Simplify.

$$n = 20$$

Divide each side by 7.

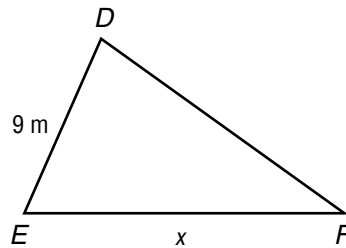
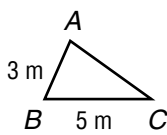


The length of \overline{ST} is 20 feet.

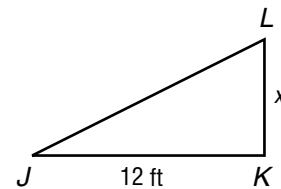
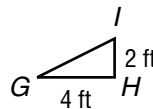
Exercises

Find the value of x in each pair of similar figures.

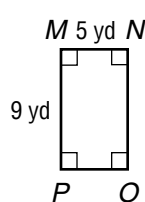
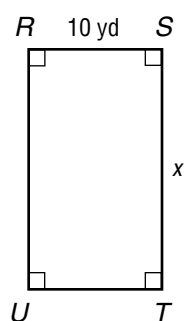
1.



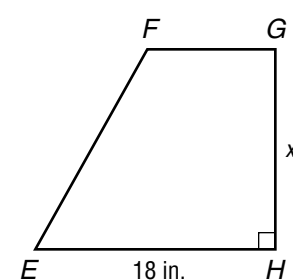
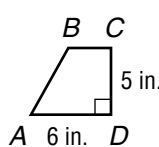
2.



3.



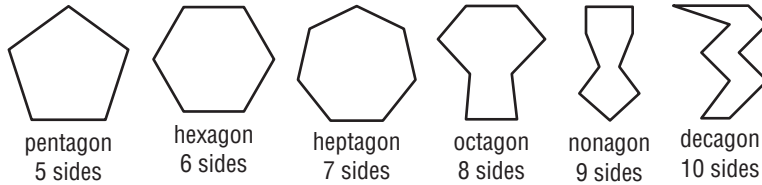
4.



11-7 Study Guide and Intervention

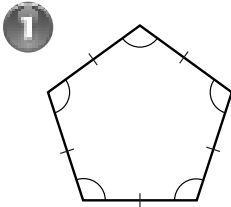
Polygons and Tessellations

A **polygon** is a simple, closed figure formed by three or more straight lines. A simple figure does not have lines that cross each other. You have drawn a closed figure when your pencil ends up where it started. Polygons can be classified by the number of sides they have.

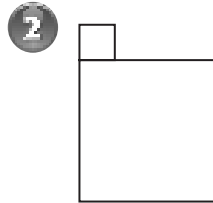


A polygon that has all sides congruent and all angles congruent is called a **regular polygon**.

Examples Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



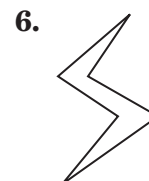
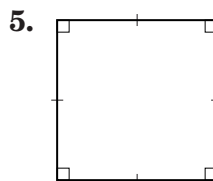
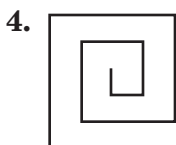
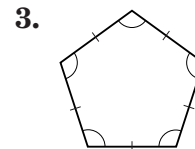
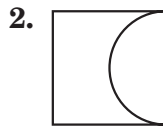
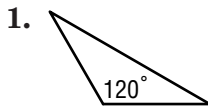
The figure has 5 congruent sides and 5 congruent angles. It is a regular pentagon.



The figure is not a polygon because it has sides that overlap.

Exercises

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



11-8 Study Guide and Intervention

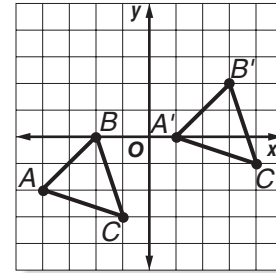
Translations

A **translation** is the movement of a geometric figure in some direction without turning the figure. When translating a figure, every point of the original figure is moved the same distance and in the same direction. To graph a translation of a figure, move each vertex of the figure in the given direction. Then connect the new vertices.

Example Triangle ABC has vertices $A(-4, -2)$, $B(-2, 0)$, and $C(-1, -3)$. Find the vertices of triangle $A'B'C'$ after a translation of 5 units right and 2 units up.

Add 5 to each x -coordinate. Add 2 to each y -coordinate.

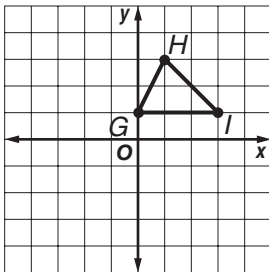
Vertices of $\triangle ABC$	$(x + 5, y + 2)$	Vertices of $\triangle A'B'C'$
$A(-4, -2)$	$(-4 + 5, -2 + 2)$	$A'(1, 0)$
$B(-2, 0)$	$(-2 + 5, 0 + 2)$	$B'(3, 2)$
$C(-1, -3)$	$(-1 + 5, -3 + 2)$	$C'(4, -1)$



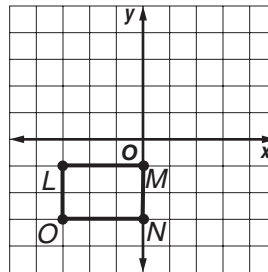
The coordinates of the vertices of $\triangle A'B'C'$ are $A'(1, 0)$, $B'(3, 2)$, and $C'(4, -1)$.

Exercises

1. Translate $\triangle GHI$ 1 unit left and 5 units down.

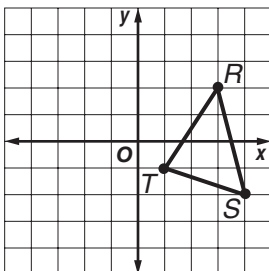


2. Translate rectangle $LMNO$ 4 units right and 3 units up.

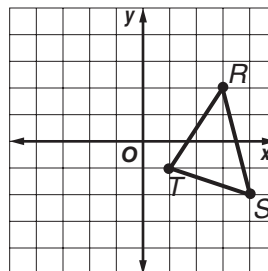


Triangle RST has vertices $R(3, 2)$, $S(4, -2)$, and $T(1, -1)$. Find the vertices of $R'S'T'$ after each translation. Then graph the figure and its translated image.

3. 5 units left, 1 unit up



4. 3 units left, 2 units down



11-9 Study Guide and Intervention

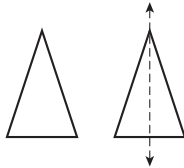
Reflections

Figures that match exactly when folded in half have **line symmetry**. Each fold line is called a **line of symmetry**. Some figures have more than one line of symmetry.

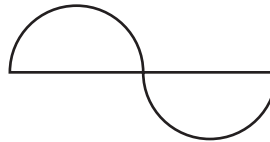
Examples

Determine whether each figure has line symmetry. If so, draw all lines of symmetry.

1



2

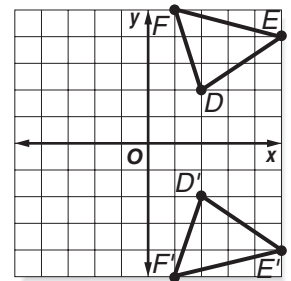


no symmetry

A type of transformation where a figure is flipped over a line of symmetry is a **reflection**. To draw the reflection of a polygon, find the distance from each vertex of the polygon to the line of symmetry. Plot the new vertices the same distance from the line of symmetry but on the other side of the line. Then connect the new vertices to complete the reflected image.

Example 3

Triangle DEF has vertices $D(2, 2)$, $E(5, 4)$, and $F(1, 5)$. Find the coordinates of the reflected image. Graph the figure and its reflected image over the x -axis.

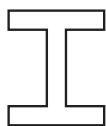


Plot the vertices and connect to form $\triangle DEF$. The x -axis is the line of symmetry. The distance from a point on $\triangle DEF$ to the line of symmetry is the same as the distance from the line of symmetry to the reflected image. The image coordinates are $D'(2, -2)$, $E'(5, -4)$, and $F'(1, -5)$.

Exercises

For Exercises 1 and 2, determine which figures have line symmetry. Write *yes* or *no*. If *yes*, draw all lines of symmetry.

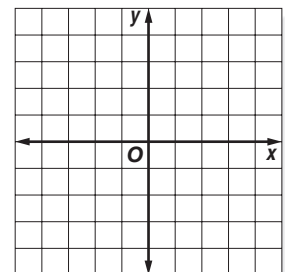
1.



2.



3. Triangle ABC has vertices $A(0, 4)$, $B(2, 1)$, and $C(4, 3)$. Find the coordinates of the vertices of ABC after a reflection over the x -axis. Then graph the figure and its reflected image.



12-1

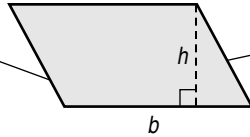
Study Guide and Intervention

Area of Parallelograms

The area A of a parallelogram equals the product of its base b and its height h .

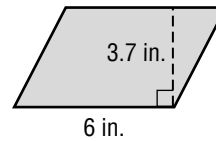
$$A = bh$$

The **base** is any side of a parallelogram.



The **height** is the length of the segment perpendicular to the base with endpoints on opposite sides.

Example 1 Find the area of a parallelogram if the base is 6 inches and the height is 3.7 inches.



Estimate $A = 6 \cdot 4$ or 24 in^2

$A = bh$ Area of a parallelogram

$A = 6 \cdot 3.7$ Replace b with 6 and h with 3.7.

$A = 22.2$ Multiply.

The area of the parallelogram is 22.2 square inches. This is close to the estimate.

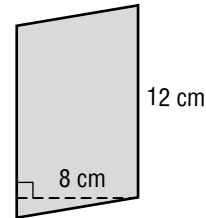
Example 2 Find the area of the parallelogram at the right.

Estimate $A = 10 \cdot 10$ or 100 cm^2

$A = bh$ Area of a parallelogram

$A = 12 \cdot 8$ Replace b with 12 and h with 8.

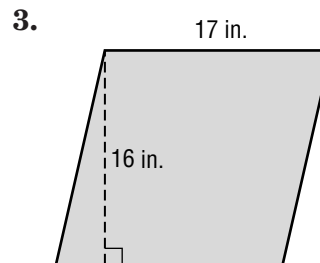
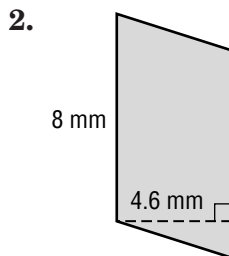
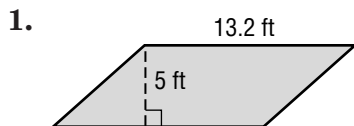
$A = 96$ Multiply.



The area of the parallelogram is 96 square centimeters. This is close to the estimate.

Exercises

Find the area of each parallelogram. Round to the nearest tenth if necessary.



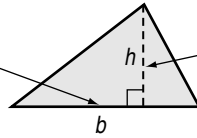
12-2 Study Guide and Intervention

Area of Triangles and Trapezoids

The area A of a triangle equals half the product of its base b and its height h .

$$A = \frac{1}{2}bh$$

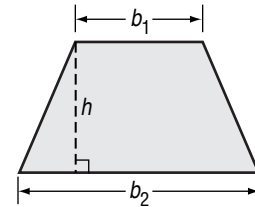
The **base** of a triangle can be any of its sides.



The **height** is the distance from a base to the opposite vertex.

A trapezoid has two bases, b_1 and b_2 . The height of a trapezoid is the distance between the two bases. The area A of a trapezoid equals half the product of the height h and the sum of the bases b_1 and b_2 .

$$A = \frac{1}{2}h(b_1 + b_2)$$



Example 1 Find the area of the triangle.

Estimate $\frac{1}{2}(6)(5) = 15$

$$A = \frac{1}{2}bh$$

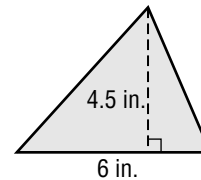
Area of a triangle

$$A = \frac{1}{2}6 \cdot 4.5$$

Replace b with 6 and h with 4.5.

$$A = 13.5$$

Multiply.



The area of the triangle is 13.5 square inches. This is close to the estimate.

Example 2 Find the area of the trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2)$$

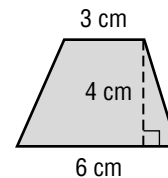
Area of a trapezoid

$$A = \frac{1}{2}(4)(3 + 6)$$

Replace h with 4, b_1 with 3, and b_2 with 6.

$$A = 18$$

Simplify.

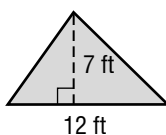


The area of the trapezoid is 18 square centimeters.

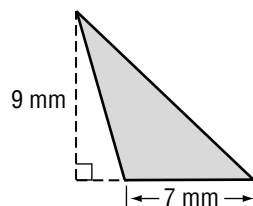
Exercises

Find the area of each figure. Round to the nearest tenth if necessary.

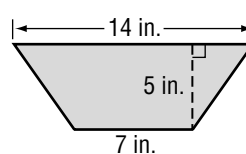
1.



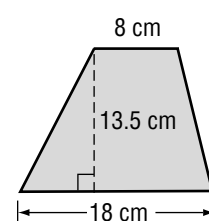
2.



3.



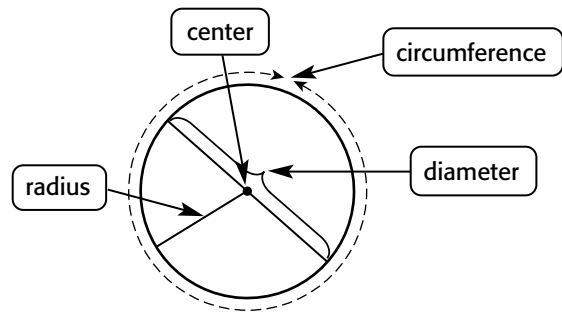
4.



12-3 Study Guide and Intervention

Circles and Circumference

A **circle** is the set of all points in a plane that are the same distance from a given point, called the **center**. The **diameter** d is the distance across the circle through its center. The **radius** r is the distance from the center to any point on the circle. The **circumference** C is the distance around the circle. The circumference C of a circle is equal to its diameter d times π , or 2 times its radius r times π .



Example 1 Find the circumference of a circle with a diameter of 7.5 centimeters.

$$C = \pi d$$

$$C \approx 3.14 \times 7.5 \quad \text{Use 3.14 for } \pi.$$

$$C \approx 23.55 \quad \text{The circumference of the circle is about 23.55 centimeters.}$$

Example 2 If the radius of a circle is 14 inches, what is its circumference?

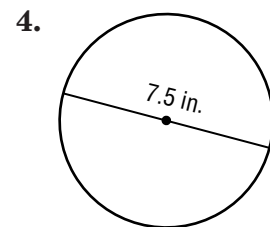
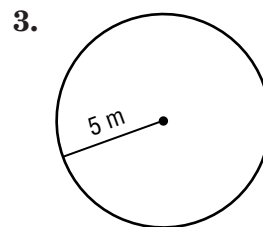
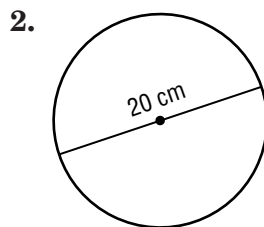
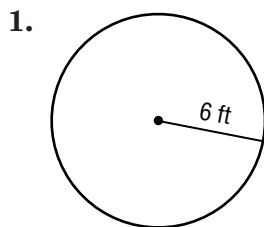
$$C = 2\pi r$$

$$C \approx 2 \times \frac{22}{7} \times 14 \quad \text{Use } \frac{22}{7} \text{ for } \pi.$$

$$C \approx 88 \quad \text{The circumference of the circle is about 88 inches.}$$

Exercises

Find the circumference of each circle. Use 3.14 or $\frac{22}{7}$ for π . Round to the nearest tenth if necessary.



5. diameter = 15 km

6. radius = 21 mi

7. radius = 50 m

8. diameter = 600 ft

9. radius = 62 mm

10. diameter = 7 km

11. radius = 95 in.

12. diameter = 6.3 m

13. diameter = $5\frac{1}{4}$ cm

12-4 Study Guide and Intervention**Area of Circles**

The area A of a circle equals the product of pi (π) and the square of its radius r .

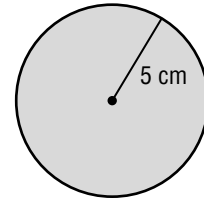
$$A = \pi r^2$$

Example 1 Find the area of the circle.

$$A = \pi r^2 \quad \text{Area of circle}$$

$$A = \pi \cdot 5^2 \quad \text{Replace } r \text{ with } 5.$$

$$\pi \times 5^2 = 78.53981634$$



The area of the circle is approximately 78.5 square centimeters.

Example 2 Find the area of a circle that has a diameter of 9.4 millimeters.

$$A = \pi r^2 \quad \text{Area of a circle}$$

$$A = \pi \cdot 4.7^2 \quad \text{Replace } r \text{ with } 9.4 \div 2 \text{ or } 4.7.$$

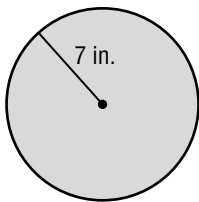
$$A \approx 69.4 \quad \text{Use a calculator.}$$

The area of the circle is approximately 69.4 square millimeters.

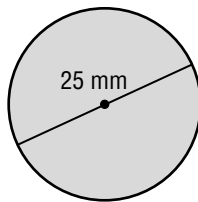
Exercises

Find the area of each circle. Round to the nearest tenth.

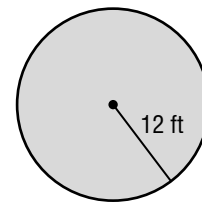
1.



2.



3.



4. radius = 2.6 cm

5. radius = 14.3 in.

6. diameter = $5\frac{1}{2}$ yd

7. diameter = $4\frac{3}{4}$ mi

8. diameter = 7.9 mm

9. radius = $2\frac{1}{5}$ ft

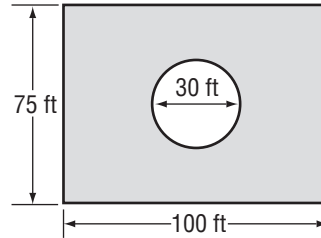
12-5 Study Guide and Intervention

Problem-Solving Investigation: Solve a Simpler Problem

When problem solving, sometimes it is easier to **solve a simpler problem** first to find the correct strategy for solving a more difficult problem.

Example

SPORTS West High School wants to paint field blue, but not the center. The diagram below shows the dimensions of the field and center circle. How much area will they need to paint blue?



Explore

You know that the field is one large rectangle and the center symbol is a large circle.

Plan

You can find the area of the rectangle and the area of the circle and subtract.

Solve

Area of rectangle: $A = \ell w$
 $A = 100 \times 75$ or 7500

Area of circle: $A = \pi r^2$
 $A = 3.14 \times 15^2$ or 706.5

Subtract: $7500 - 706.5$ or 6793.5ft²

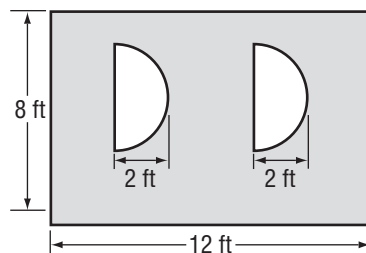
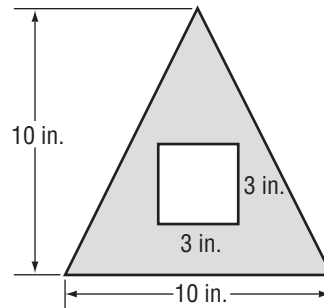
So, they would need to paint 6,793.5 square feet of field.

Check

Use estimation to check. The area of the entire field is 7500ft and the circle is approximately 700 feet so the area should be less than 6,800 feet. Since 6,793.5 is less than 6,800ft, the answer is reasonable.

Exercises

- FRAMES** Joan wants to paint her favorite picture frame. How much paint would she need to use in order to cover just the frame?
- WALLPAPER** Richard wants to wallpaper one wall of his bathroom. He has two semi-circular windows along the wall. How much wallpaper must he purchase?



12-6 Study Guide and Intervention

Area of Composite Figures

Complex figures are made of triangles, quadrilaterals, semicircles, and other two-dimensional figures. To find the area of a complex figure, separate it into figures whose areas you know how to find, and then add the areas.

Example 1 Find the area of the figure at the right in square feet.

The figure can be separated into a rectangle and a trapezoid. Find the area of each.

Area of Rectangle

$$A = \ell w$$

Area of a rectangle

$$A = 12 \cdot 8$$

Replace ℓ with 12 and w with 8.

$$A = 96$$

Multiply.

Area of Trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$

Area of a trapezoid

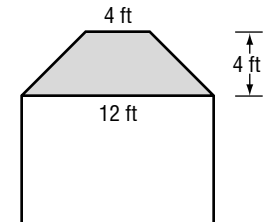
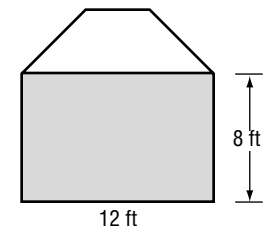
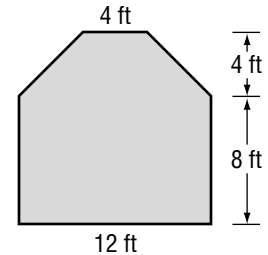
$$A = \frac{1}{2}(4)(4 + 12)$$

Replace h with 4, b_1 with 4, and b_2 with 12.

$$A = 32$$

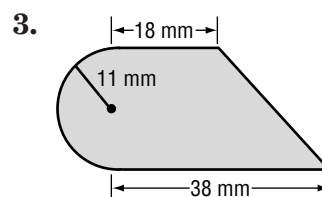
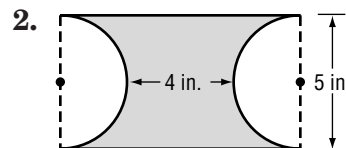
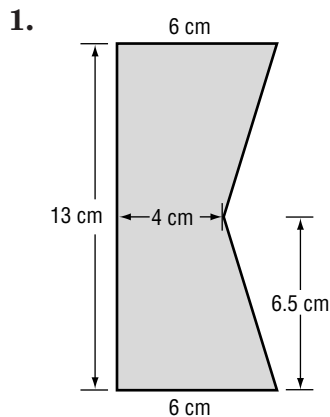
Multiply.

The area of the figure is $96 + 32$ or 128 square feet.



Exercises

Find the area of each figure. Round to the nearest tenth if necessary.



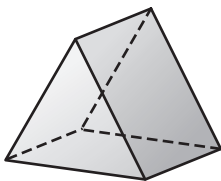
12-7 Study Guide and Intervention

Three Dimensional Figures

Prisms	At least 3 rectangular lateral faces	Top and bottom bases are parallel	Shape of the base tells the name of the prism
Pyramids	At least three triangular lateral faces	One base shaped like any 3-sided closed figure	Shape of the base tells the name of the pyramid
Cones	Only one base	Base is a circle	One vertex and no edges
Cylinders	Only two bases	Bases are circles	No vertices and no edges
Spheres	All points are the same distance from the center	No faces or bases	No edges or vertices

Example For each figure, name the shape of the base(s). Then classify each figure.

A.



B.



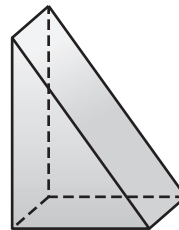
Exercises

For each figure name the shape of the base(s). Then classify each figure.

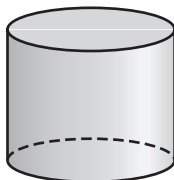
1.



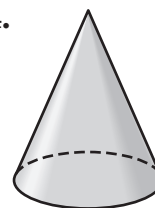
2.



3.



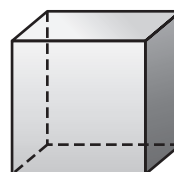
4.



5.



6.



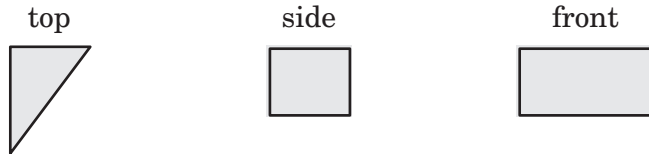
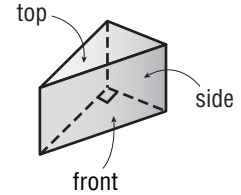
12-8 Study Guide and Intervention

Drawing Three-Dimensional Figures

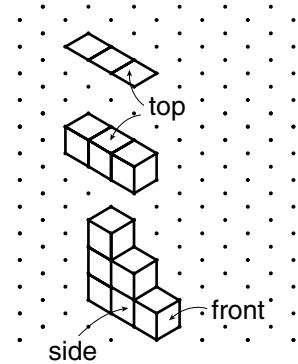
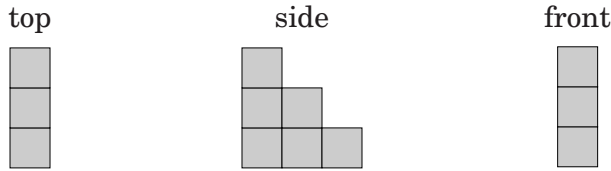
A solid is a three-dimensional figure.

Example 1 Draw a top, a side, and a front view of the solid at the right.

The top view is a triangle. The side and front views are rectangles.



Example 2 Draw the solid using the top, side, and front views shown below.



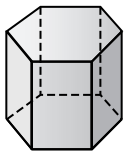
Step 1 Use the top view to draw the base of the figure, a 1-by-3 rectangle.

Step 2 Add edges to make the base a solid figure.

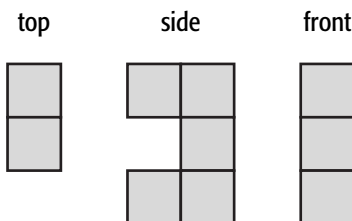
Step 3 Use the side and front views to complete the figure.

Exercises

1. Draw a top, a side, and front view of the solid.



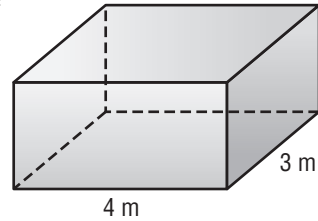
2. Draw the solid whose top, side, and front views are shown. Use isometric dot paper.



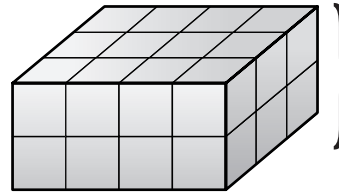
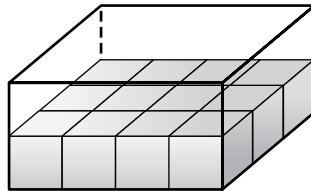
12-9 Study Guide and Intervention

Volume of Prisms

The **volume** of a three-dimensional figure is the measure of space occupied by it. It is measured in cubic units such as cubic centimeters (cm^3) or cubic inches (in^3). The volume of the figure at the right can be shown using cubes.



The bottom layer, or base, has $4 \cdot 3$ or 12 cubes.



There are two layers.

It takes $12 \cdot 2$ or 24 cubes to fill the box. So, the volume of the box is 24 cubic meters.

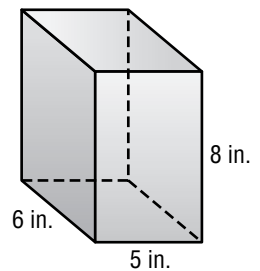
A **rectangular prism** is a three-dimensional figure that has two parallel and congruent sides, or bases, that are rectangles. To find the volume of a rectangular prism, multiply the area of the base and the height, or find the product of the length ℓ , the width w , and the height h .

$$V = Bh \text{ or } V = \ell wh$$

Example Find the volume of the rectangular prism.

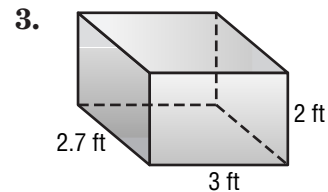
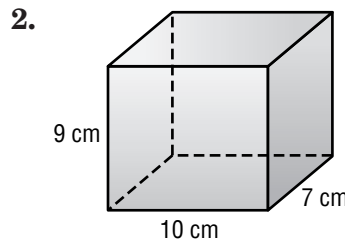
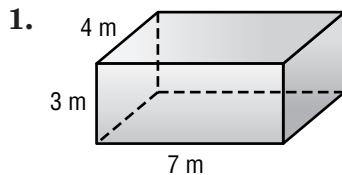
- $V = \ell wh$ Volume of a rectangular prism
 $V = 5 \cdot 6 \cdot 8$ Replace ℓ with 5, w with 6, and h with 8.
 $V = 240$ Multiply.

The volume is 240 cubic inches.



Exercises

Find the volume of each rectangular prism. Round to the nearest tenth if necessary.

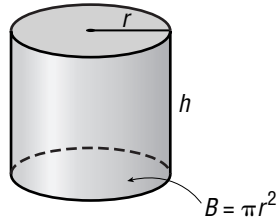


12-10 Study Guide and Intervention

Volume of Cylinders

As with prisms, the area of the base of a **cylinder** tells the number of cubic units in one layer. The height tells how many layers there are in the cylinder. The volume V of a cylinder with radius r is the area of the base B times the height h .

$$V = Bh \text{ or } V = \pi r^2 h, \text{ where } B = \pi r^2$$



Example Find the volume of the cylinder. Round to the nearest tenth.

$$V = \pi r^2 h$$

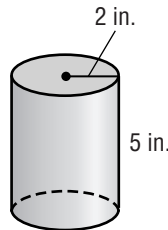
Volume of a cylinder

$$V = \pi(2)^2(5)$$

Replace r with 2 and h with 5.

$$V \approx 62.8$$

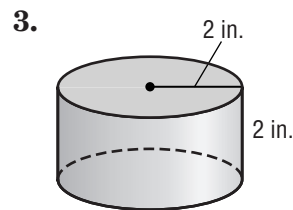
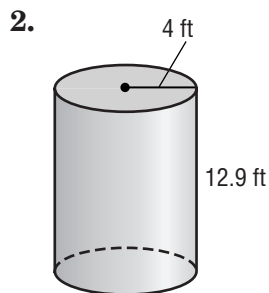
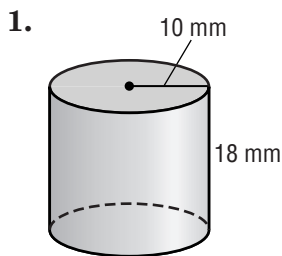
Simplify.



The volume is approximately 62.8 cubic inches. Check by using estimation.

Exercises

Find the volume of each cylinder. Round to the nearest tenth.



4. radius = 9.5 yd
height = 2.2 yd

5. diameter = 6 cm
height = 11 cm

6. diameter = $3\frac{2}{5}$ m
height = $1\frac{1}{4}$ m

13-1**Study Guide and Intervention****Estimating Square Roots**

Recall that a perfect square is a square of a rational number. In Lesson 5-8, you learned that any number that can be written as a fraction is a rational number. A number that cannot be written as a fraction is an **irrational number**.

Example 1 Estimate $\sqrt{40}$ to the nearest whole number.

List some perfect squares.

1, 4, 9, 16, 25, 36, 49, ...

$$36 < 40 < 49$$

40 is between the perfect squares 36 and 49.

$$\sqrt{36} < \sqrt{40} < \sqrt{49}$$

Find the square root of each number.

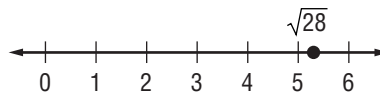
$$6 < \sqrt{40} < 7$$

$$\sqrt{36} = 6 \text{ and } \sqrt{49} = 7$$

So, $\sqrt{40}$ is between 6 and 7. Since 40 is closer to 36 than to 49, the best whole number estimate is 6.

Example 2 Use a calculator to find the value of $\sqrt{28}$ to the nearest tenth.

2nd $\sqrt{\quad}$ 28 ENTER = 5.291502622



$$\sqrt{28} \approx 5.3$$

Check Since $5^2 = 25$ and 25 is close to 28, the answer is reasonable.

Exercises

Estimate each square root to the nearest whole number.

1. $\sqrt{3}$

2. $\sqrt{8}$

3. $\sqrt{26}$

4. $\sqrt{41}$

5. $\sqrt{61}$

6. $\sqrt{94}$

7. $\sqrt{152}$

8. $\sqrt{850}$

Use a calculator to find each square root to the nearest tenth.

9. $\sqrt{2}$

10. $\sqrt{27}$

11. $\sqrt{73}$

12. $\sqrt{82}$

13. $\sqrt{105}$

14. $\sqrt{395}$

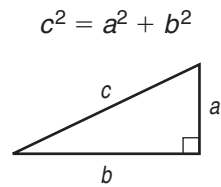
15. $\sqrt{846}$

16. $\sqrt{2,298}$

13-2 Study Guide and Intervention

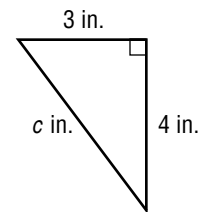
The Pythagorean Theorem

The sides of a right triangle have special names. The sides adjacent to the right angle are the **legs**. The side opposite the right angle is the **hypotenuse**. The **Pythagorean Theorem** describes the relationship between the length of the hypotenuse and the lengths of the legs. In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.



Example 1 Find the missing measure of a right triangle if $a = 4$ inches and $b = 3$ inches.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 4^2 + 3^2$	Replace a with 4 and b with 3.
$c^2 = 16 + 9$	Evaluate 4^2 and 3^2 .
$c^2 = 25$	Add.
$\sqrt{c^2} = \sqrt{25}$	Take the positive square root of each side.
$c = 5$	Simplify.



The length of the hypotenuse is 5 inches.

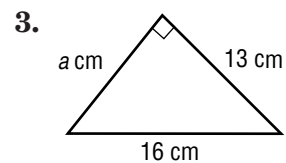
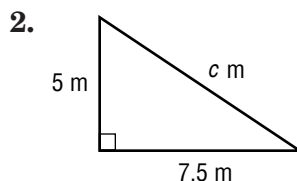
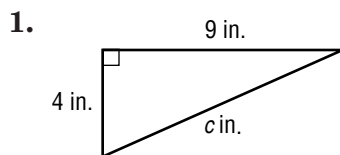
Example 2 Determine whether a triangle with side lengths of 6 meters, 9 meters, and 12 meters is a right triangle.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$12^2 \stackrel{?}{=} 6^2 + 9^2$	Replace a with 6, b with 9, and c with 12.
$144 \stackrel{?}{=} 36 + 81$	Simplify.
$144 \neq 117$	Add.

The triangle is *not* a right triangle.

Exercises

Find the missing measure of each right triangle. Round to the nearest tenth if necessary.



Determine whether each triangle with the given side lengths is a right triangle. Write *yes* or *no*.

4. 15 ft, 8 ft, 17 ft 5. 5 in., 13 in., 17 in. 6. 9 yd, 40 yd, 41 yd

13-3**Study Guide and Intervention****Problem-Solving Investigation: Make a Model**

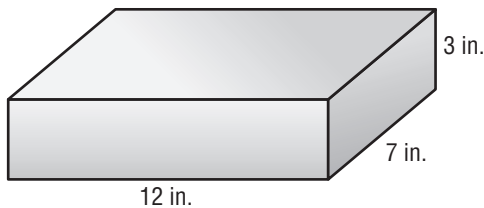
When solving problems, make a model to represent the given situation in order to determine the best plan for a solution.

Example **GIFT WRAP** Rita wants to wrap a rectangular box. The box is 12 inches by 7 inches by 3 inches high. What must be the area of the paper so that she has a 1 inch overlap to neatly wrap the paper?

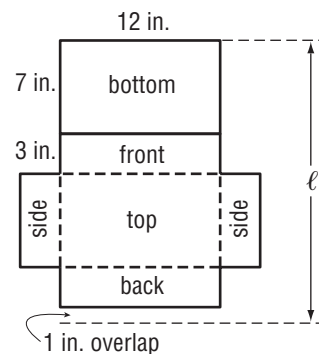
Explore You know that the box is $12 \times 7 \times 3$ and that you need to add 1 inch to some measures for the overlap. You also know that the wrapping paper will be a rectangle.

Plan Draw a sketch of the box and then make a model of the box if it were cut apart and laid flat. You need the overlap going around the box.

Solve Sketch the box.



Make a model of the box unfolded.



The length of the paper needed is the distance around the box plus 1 inch. So, $\ell = 7 + 3 + 7 + 3 + 1$ or 21 inches.

The width of the paper would be $3 + 12 + 3$ or 18 inches.

The area would be 21×18 or 378 in^2 .

Check Make a box using centimeters instead of inches. Then cut a piece of paper 18 centimeters by 21 centimeters to see if you can wrap the box neatly.

Exercises

- GARDENING** Peg wants to put a stone path 3 feet wide around her rectangular garden measuring 10 feet by 15 feet. What will be the perimeter of her garden including the stone path?
- DRAWING** Dante is making a full-size drawing of his favorite cartoon character. If the figure is 1 inch by 0.5 inches and his scale is 1 inch = 10 inches, how large will the full size character be?

13-4 Study Guide and Intervention

Surface Area of Rectangular Prisms

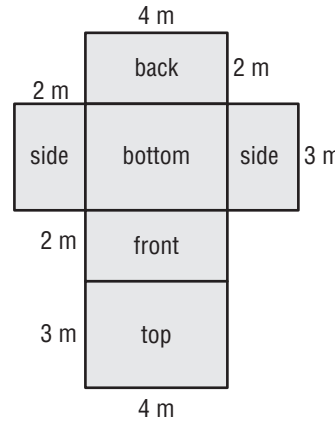
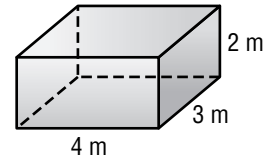
The sum of the areas of all the surfaces, or faces, of a three-dimensional figure is the **surface area**. The surface area S of a rectangular prism with length ℓ , width w , and height h is found using the following formula.

$$S = 2\ell w + 2\ell h + 2wh$$

Example Find the surface area of the rectangular prism.

You can use the net of the rectangular prism to find its surface area. There are three pairs of congruent faces in a rectangular prism:

- top and bottom
- front and back
- two sides



Faces	Area
top and bottom	$(4 \cdot 3) + (4 \cdot 3) = 24$
front and back	$(4 \cdot 2) + (4 \cdot 2) = 16$
two sides	$(2 \cdot 3) + (2 \cdot 3) = 12$
Sum of the areas	$24 + 16 + 12 = 52$

Alternatively, replace ℓ with 4, w with 3, and h with 2 in the formula for surface area.

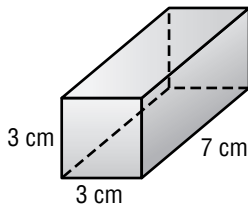
$$\begin{aligned}
 S &= 2\ell w + 2\ell h + 2wh \\
 &= 2 \cdot 4 \cdot 3 + 2 \cdot 4 \cdot 2 + 2 \cdot 3 \cdot 2 && \text{Follow order of operations.} \\
 &= 24 + 16 + 12 \\
 &= 52
 \end{aligned}$$

So, the surface area of the rectangular prism is 52 square meters.

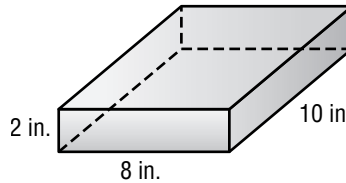
Exercises

Find the surface area of each rectangular prism.

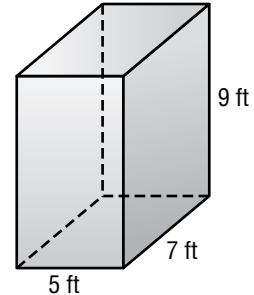
1.



2.



3.



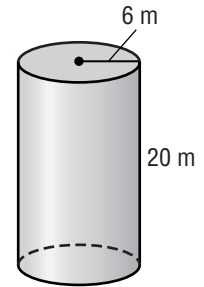
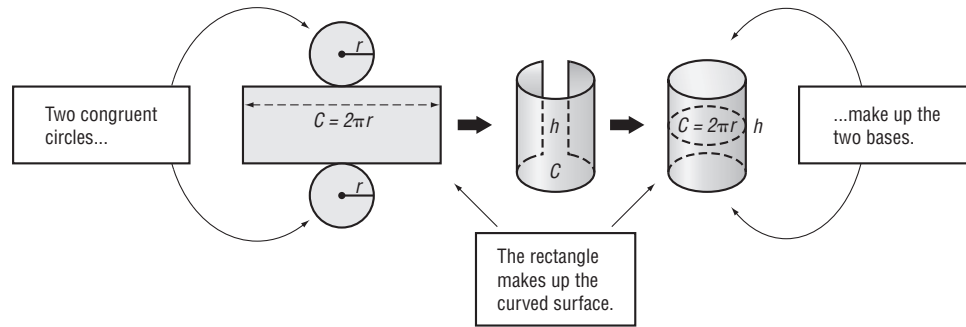
13-5 Study Guide and Intervention

Surface Area of Cylinders

The diagram below shows how you can put two circles and a rectangle together to make a cylinder.

The surface area of a cylinder equals the area of two bases plus the area of the curved surface.

$$S = 2(\pi r^2) + (2\pi r)h$$



In the diagram above, the length of the rectangle is the same as the circumference of the circle. Also, the width of the rectangle is the same as the height of the cylinder.

Example Find the surface area of the cylinder. Round to the nearest tenth.

$$S = 2\pi r^2 + 2\pi rh$$

Surface area of a cylinder.

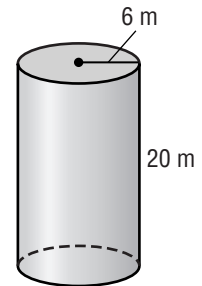
$$S = 2\pi(6)^2 + 2\pi(6)(20)$$

Replace r with 6 and h with 20.

$$\approx 980.2$$

Simplify.

The surface area is about 980.2 meters.



Exercises

Find the surface area of each cylinder. Round to the nearest tenth.

