

Lesson E

Amortization Models

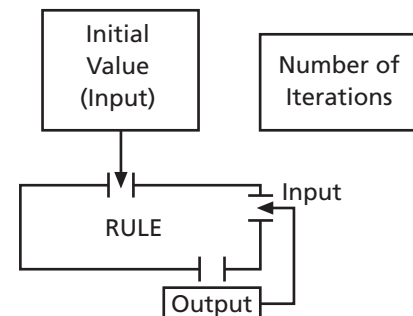
Use after Lesson 11-2

Goals ■ Solve problems involving the amortization of a loan.

Applications Buying Cars, Housing

In mathematics, the term **recursive** is used to describe a process where the value of the previous step is used to determine the new value of the current step. The diagram at the right shows how the recursive process works.

1. If the initial value is 500 and the rule is $n - (100 - 0.1n)$, what are the first four outputs?
2. Write a new initial value and a new rule to be used in the recursive process.
3. Find the first four outputs using the new initial value and the new rule.



BUILD UNDERSTANDING

To make payments on a loan is to **amortize** the loan. Each payment pays for the monthly interest on the last balance and pays part of the money borrowed. Because the interest is paid on the last balance, an **amortization schedule** is recursive.

Example 1

Molly borrows \$20,000 which she plans to pay back over 5 years. Her monthly payment is \$386.66. The monthly interest rate is 0.005. Make an amortization schedule for the first 6 months of the loan.

Solution

Each month, the interest is 0.005 of last month's balance. The rest of the payment goes towards paying off the loan. Use this information to create an amortization schedule. Round to the nearest cent.

| Month | Amount of Loan | Payment | Interest | Loan Reduction | New Balance |
|-------|----------------|----------|---|---------------------------------|---------------------------------------|
| 1 | \$20,000.00 | \$386.66 | $20,000.00 \times 0.005 =$ \$100.00 | $386.66 - 100.00 =$ \$286.66 | $20,000.00 - 286.66 =$ \$19,713.34 |
| 2 | \$19,713.34 | \$386.66 | $19,713.34 \times 0.005 \approx$ \$98.57 | $386.66 - 98.57 =$ \$288.09 | $19,713.34 - 288.09 =$ \$19,425.25 |
| 3 | \$19,425.25 | \$386.66 | $19,425.25 \times 0.005 \approx$ \$97.13 | $386.66 - 97.13 =$ \$289.53 | $19,425.25 - 289.53 =$ \$19,135.72 |
| 4 | \$19,135.72 | \$386.66 | $19,135.72 \times 0.005 \approx$ \$95.68 | $386.66 - 95.68 =$ \$290.98 | $19,135.72 - 290.98 =$ \$18,844.74 |
| 5 | \$18,844.74 | \$386.66 | $18,844.74 \times 0.005 \approx$ \$94.22 | $386.66 - 94.22 =$ \$292.44 | $18,844.74 - 292.44 =$ \$18,552.30 |
| 6 | \$18,552.30 | \$386.66 | $18,552.30 \times 0.005 \approx$ \$92.76 | $386.66 - 92.76 =$ \$293.90 | $18,552.30 - 293.90 =$ \$18,258.40 |

Notice that the amount of interest decreases and the amount of the loan reduction increases each month.

To determine the amount of the monthly payment for a given loan, you can use the following formula.

$$M = \frac{Pr(1+r)^n}{(1+r)^n - 1}$$

In this formula, M = the monthly payment,
 P = the amount of the loan,
 r = the monthly rate of interest and
 n = the number of monthly payments.

Example 2

BUYING CARS Joshua wants to borrow \$18,000 to buy a car. He wants to repay the loan over 4 years. If the monthly interest rate is 0.003%, find his monthly payment.

Solution

Joshua will make 4×12 or 48 payments. Use the formula to find his monthly payment.

$$M = \frac{18,000(0.003)(1+0.003)^{48}}{(1+0.003)^{48} - 1} \quad P = 18,000, r = 0.003 \text{ and } n = 48.$$

$$\approx 403.21$$

Use a calculator and round to the nearest hundredth.

His payment will be \$403.21.

When borrowing money, the total amount of money a person pays to the lender equals the amount of each payment times the number of payments. The total amount of interest a person pays equals the total amount paid minus the original amount of the loan.

Example 3

BUYING CARS What is the total amount Joshua will pay the lender for his \$18,000 loan? How much interest will he pay for the loan?

Solution

Joshua will pay the lender $\$403.21 \times 48$ or \$19,354.08.

The amount of interest will be $\$19,354.08 - \$18,000$ or \$1354.08.

TRY THESE EXERCISES

Make an amortization schedule for the first 3 months of each loan. Round to the nearest cent.

1. Amount of loan: \$1200
Monthly interest rate: 0.01
Monthly payment: \$106.62

2. Amount of loan: \$40,000
Monthly interest rate: 0.008
Monthly payment: \$519.78

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Determine the monthly payment, the total amount paid to the lender and the total amount of the interest for each loan. Round to the nearest cent.

3. Amount of loan: \$1800
Monthly interest rate: 0.009
Length of loan: 1 year
4. Amount of loan: \$35,000
Monthly interest rate: 0.006
Length of loan: 10 years
5. **BUYING CARS** Mia wants to borrow \$22,000 to buy a car. She wants to repay the loan over 5 years. If the monthly interest rate is 0.004%, find her monthly payment.

PRACTICE EXERCISES

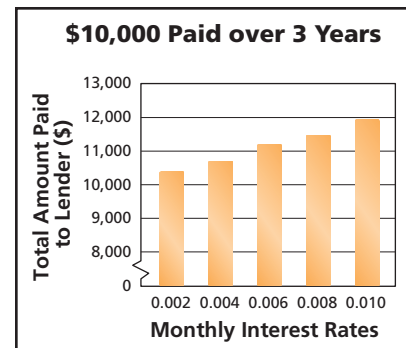
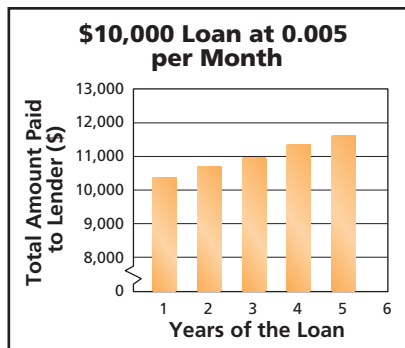
Make an amortization schedule for the first 3 months of each loan. Round to the nearest cent.

6. Amount of loan: \$42,000
Monthly interest rate: 0.006
Monthly payment: \$330.69
7. Amount of loan: \$25,000
Monthly interest rate: 0.005
Monthly payment: \$210.96

Determine the monthly payment, the total amount paid to the lender and the total amount of the interest for each loan. Round to the nearest cent.

8. Amount of loan: \$15,000
Monthly interest rate: 0.004
Length of loan: 3 years
9. Amount of loan: \$80,000
Monthly interest rate: 0.009
Length of loan: 25 years
10. Amount of loan: \$60,000
Monthly interest rate: 0.008
Length of loan: 20 years
11. Amount of loan: \$120,000
Monthly interest rate: 0.007
Length of loan: 30 years

Study the graphs below.



12. About how much more would a \$10,000 loan at 0.005 per month cost if it was paid over 5 years rather than over 1 year?
13. About how much more would a \$10,000 loan paid over 3 years cost if the monthly interest rate is 0.010 rather than 0.002?
14. **BUYING CARS** Bailey wants to borrow \$24,000 to buy a car. She wants to repay the loan over 4 years. If the monthly interest rate is 0.003%, find her monthly payment.
15. **BUYING CARS** Patrick wants to borrow \$18,000 to buy a car. He wants to repay the loan over 5 years. If the monthly interest rate is 0.004%, find the total amount of interest he will pay for the loan.

16. **HOUSING** Mr. Rockwell is seeking a loan for \$125,000 to buy a house. The monthly interest rate is 0.006. If he wants to pay the loan over 20 years, find his monthly payment.

17. **HOUSING** Mr. and Mrs. Ito are seeking a loan for \$140,000 to buy a house. The monthly interest rate is 0.006. If they want to pay the loan over 25 years, what is the total amount they will pay the lender?

HOUSING Mr. and Mrs. Williams want to buy a house. They need a loan for \$150,000. The bank will give them a 30-year loan at a monthly interest rate of 0.006.

18. Find the amount of their monthly payment for this loan. Round to the nearest cent.

19. If they stay in the house for 30 years, how much will they pay for the house?

20. How much interest will they pay for the house?

21. Mr. and Mrs. Williams can sign a year's lease to rent an apartment for \$750 per month. How much would they pay to live in the apartment for a year?

22. How much would Mr. and Mrs. Williams save by living in an apartment for one year rather than buying a house immediately?

23. What are the advantages of buying a house and the advantages of renting an apartment?

BUYING CARS Dylan wants to buy a car that costs \$20,000.

24. Dylan can put \$1000 down and finance the rest of the amount with a 5-year loan. The monthly interest of the loan is 0.003. Find the amount of the monthly payments. Round to the nearest cent.

25. What is the total amount Dylan will pay to buy the car?

26. Dylan can also lease the car by paying a fee of \$1000 and monthly payments of \$279 for 39 months. After 39 months, he can either turn in the car or buy the used car. How much will he pay to use the car for 39 months?

27. Suppose Dylan can lease another new car for the same terms after the 39 months. How much more would Dylan pay for leasing the two cars rather than buy the car and keeping it for $6\frac{1}{2}$ years?

28. What are the advantages of buying a car and the advantages of leasing a car?

EXTENDED PRACTICE EXERCISES

A \$1000 loan is paid over one year at a monthly rate of 0.007.

29. Determine the monthly payment, the total amount paid to the lender and the total amount of the interest for the loan. Round to the nearest cent.

30. **MISSING PAYMENTS** If payments are missed, the month's interest is added to the amount owed. Use an amortization schedule to determine how long it will take to pay off a loan if the second and third payments are missed. How much more interest will be paid?

31. **DOUBLE PAYMENTS** If a double payment is made, the extra amount is applied to the loan reduction. Use an amortization schedule to determine how long it will take to pay off the loan if double payments are made in the second and third month. How much will be saved in interest? About what fraction of the original interest was saved?

32. **WRITING MATH** Describe the effects of missing payments and the effects of making extra payments.

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