

## Lesson 13-4

## Example 1

Graph the equation  $12x^2 + 3y^2 = 48$ .

## Solution

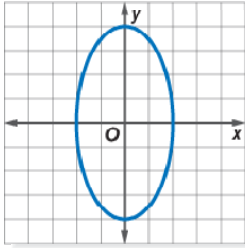
Divide both sides of the equation by 48 to change it to standard form.

$$12x^2 + 3y^2 = 48$$
$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

$$a^2 = 4, a = \pm 2$$

$$b^2 = 16, b = \pm 4$$

The  $x$ -intercepts are  $(2, 0)$  and  $(-2, 0)$ . The  $y$ -intercepts are  $(0, 4)$  and  $(0, -4)$ .  
Locate the points and draw a smooth curve.



**Example 2**

**ASTRONOMY** Sara is using a computer to model the orbit of a moon. After placing a grid over a drawing of the orbit, she finds that the foci of the ellipse are (3, 0) and (-3, 0), and the  $x$ -intercepts are (4, 0) and (-4, 0). She needs to enter the equation of the ellipse into the computer to finish her work. Find the equation of the ellipse.

**Solution**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a = \pm 4, c = \pm 3$$

$$b^2 = a^2 - c^2$$

$$b^2 = 16 - 9$$

$$b^2 = 7$$

$$\frac{x^2}{16} + \frac{y^2}{7} = 1$$

$$7x^2 + 16y^2 = 112$$

Use the Pythagorean Theorem to find  $b^2$ .

Substitute in standard form.

Multiply by 112 ( $16 \cdot 7$ ).

The equation of the ellipse with foci (3, 0) and (-3, 0), and  $x$ -intercepts are (4, 0) and (-4, 0) is  $7x^2 + 16y^2 = 112$ .

**Example 3**

**Graph the hyperbola with equation  $9x^2 - 4y^2 = 36$ .**

**Solution**

$$9x^2 - 4y^2 = 36$$

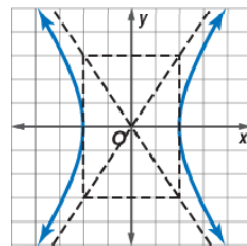
$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

$$a^2 = 4, a = \pm 2$$

$$b^2 = 9, b = \pm 3$$

Divide by 36 to write the equation in standard form.

To locate the rectangle, use the points (2, 0), (-2, 0), (0, 3), and (0, -3). Draw lines perpendicular to the axes through these points to form a rectangle. Draw extended diagonals of the rectangle; these are asymptotes. Sketch the hyperbola.



**Example 4**

**OCEANOGRAPHY** An object propelled through water travels along one branch of a hyperbola with an experimental submarine at its center  $(0, 0)$  and in which  $a = 5$  and  $b = 4$  and the foci are on the  $x$ -axis. Find the equation for the hyperbola.

**Solution**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{5^2} - \frac{y^2}{4^2} = 1$$

$$\frac{x^2}{25} - \frac{y^2}{16} = 1 \quad \text{Substitute in the standard equation.}$$

$$16x^2 - 25y^2 = 400 \quad \text{Multiply by 400 (16} \cdot \frac{25}{4} \text{).}$$

The equation is  $16x^2 - 25y^2 = 400$ .