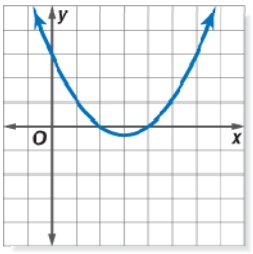
Lesson 12-7

Problem

SCIENCE The results of an experiment are represented by the graph shown at the right. What is the equation of the graph?



Solution

You can determine the equation of a parabola if you know three points on its graph.

Begin with the general form of a quadratic function:
$y = ax^2 + bx + c$, where a, b, and c represent coefficients and constants.

Locate three points on the graph. As you can see, the graph of the parabola passes through (0, 3), (2, 0) and (4, 0)

Substitute the x-values and y-values for each point into the equation to create a system of three equations.

For (0, 3)	For (2, 0)	For (4, 0)
$ax^2 + bx + c = y$	$ax^2 + bx + c = y$	$ax^2 + bx + c = y$
$c = 3$	$4a + 2b + c = 0$	$16a + 4b + c = 0$

Now use any of the methods you have learned for solving system of equations.

$c = 3$
$4a + 2b + c = 0$
$16a + 4b + c = 0$

Use $c = 3$ on the other two equations. Then solve for a.

$4a + 2b + 3 = 0$ ☐ $8a + 4b + 6 = 0$ Multiply by 2.
$16a + 4b + 3 = 0$ ☐ $16a + 4b + 3 = 0$
$-8a + 3 = 0$ Subtract.

Since $-8a + 3 = 0$, $a = \frac{3}{8}$. Substitute $a = \frac{3}{8}$ and $c = 3$ into the second equation.

$$\begin{aligned} 4a + 2b + c &= 0 \\ 4\left(\frac{3}{8}\right)^2 + 2b + 3 &= 0 \\ 2b &= -\frac{9}{2} \\ b &= -\frac{9}{4} \end{aligned}$$

Thus, $a = \frac{3}{8}$, $b = -\frac{9}{4}$, and $c = 3$. Check these values in the third equation. Then use these values in the general quadratic form to write an equation.

The equation of the parabola is $y = \frac{3}{8}x^2 - \frac{9}{4}x + 3$ or $8y = 3x^2 - 18x + 24$.