

Lesson 12-4

Example 1

Complete the square for $x^2 - 6x$.

Solution

MODELING Use Algeblocks to illustrate $x^2 - 6x$. Add blocks to make a perfect square. Write the expression.

$$x^2 - 6x + ?$$

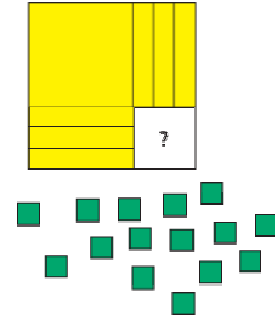
You have learned that for perfect squares in the form $ax^2 + bx + c$, the constant $c = \left(\frac{b}{2}\right)^2$. Thus, by substitution, you know that $ax^2 + bx + c = ax^2 + bx + \left(\frac{b}{2}\right)^2$.

You can use this relationship to complete the square for $x^2 - 6x$.

$$x^2 - 6x + \left(\frac{b}{2}\right)^2 \quad \text{Add } \left(\frac{b}{2}\right)^2 \text{ to complete the square.}$$

$$x^2 - 6x + (-3)^2 \quad \text{Find } \frac{b}{2}, \frac{-6}{2}, = -3.$$

$$x^2 - 6x + 9 \quad \text{Square } \frac{b}{2}, (-3)^2 = 9.$$



Example 2

Solve by completing the square.

$$x^2 + 6x + 5 = 0$$

Solution

$$x^2 + 6x + 5 = 0$$

$$x^2 + 6x = -5$$

Add -5 to each side.

$$x^2 + 6x + 9 = -5 + 9$$

Add $\left(\frac{b}{2}\right)^2$ to each side.

$$(x + 3)^2 = 4$$

Factor.

$$x + 3 = \sqrt{4}$$

Simplify.

$$x + 3 = \pm 2$$

$$x = -3 + 2 \quad x = -3 + (-2)$$

$$x = -1 \quad x = -5$$

The solutions to the equation $x^2 + 6x + 5 = 0$ are $x = -1$, $x = -5$.

Example 3

SCIENCE The number of seconds (x) that it takes a chemical reaction to begin and end can be found by solving the equation $3x^2 - 4x + 1 = 0$. Find the solutions.

Solution

$$3x^2 - 4x + 1 = 0$$

$$x^2 - \frac{4}{3}x + \frac{1}{3} = 0$$

$$x^2 - \frac{4}{3}x = -\frac{1}{3}$$

$$x^2 - \frac{4}{3}x + \frac{4}{9} = -\frac{1}{3} + \frac{4}{9}$$

$$\left(x - \frac{2}{3}\right)^2 = \frac{1}{9}$$

$$x - \frac{2}{3} = \pm\sqrt{\frac{1}{9}}$$

$$x = \frac{2}{3} \pm \frac{1}{3}$$

$$x = \frac{1}{3}, x = 1$$

To make the coefficient of the x^2 term 1, multiply by $\frac{1}{3}$.

Add $\left(\frac{b}{2}\right)^2$ to each side.

Factor.

Simplify.

The solutions to the equation $3x^2 - 4x + 1 = 0$ are $x = \frac{1}{3}$ and $x = 1$.