Lesson 11-2

Example 1

CALCULATOR Use inductive reasoning to make a conjecture about the ones digit of 7^{11}.

Solution

Use the calculator to find several successive powers of 7. Look for a pattern in the ones digits.

$7^1 =$	7	$7^2 =$	49	$7^3 =$	343	$7^4 =$	2401
$7^5 =$	16,807	$7^6 =$	117,649	$7^7 =$	823,543	$7^8 =$	5,764,801

In these examples, the ones digits repeat in the pattern 7, 9, 3, 1.

Next, make a conjecture based on the pattern.

When the exponent of 7 is 1 less than a multiple of 4, the ones digit is 3.

Finally, make a specific prediction based on the conjecture and test it.

Since 11 is 1 less than a multiple of 4, the ones digit for 7^{11} is 3.

In fact, $7^{11} = 1{,}977{,}326{,}743$, a number whose ones digit is 3.

Example 2

Lori observed that 3, 5, and 7 are all prime numbers.

Based on these three examples, she made the following conjecture:

Every odd integer greater than 1 is prime.

Is Lori's conjecture true?

Solution

If possible, find a case in which Lori's conjecture is false.

$9 = 3 \cdot 3$, so 9 is not prime.

This is a counterexample. Lori's conjecture is false.

Example 3

CALCULATOR Alec took a job in which he agreed to be paid only 1 cent his first day, but with the amount he is paid doubling every day he works, so that he is paid 2 cents the second day, 4 cents the third day, and so on. How many work days will it take until he is paid more than $1 million a day?

Solution

1 cent = $0.01

Enter 0.01 into a calculator and press Enter. Then press ×2 to get "Ans * 2." Repeatedly press Enter until the value 1,342,177.28 is attained. Counting the number of Enters gives 28 work days.

Alec will first be paid over $1 million a day on his 28th work day.