

MATH APPENDIX

NAMES FOR NUMBERS

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	•	tenths	hundredths	thousandths
10^6	10^5	10^4	10^3	10^2	10^1	10^0	•	10^{-1}	10^{-2}	10^{-3}
3	2	5	6	4	5	8		5	5	7

Each digit in a number has a value that depends on its place. The 2 in 3,256,458.557 stands for 2×10^5 .

$$2 \times 10^5 = 2 \times 100,000 = 200,000$$

200,000 two hundred thousand
8.557 eight and five hundred fifty-seven thousandths
\$39.95 thirty-nine dollars and ninety-five cents

ESTIMATION STRATEGIES

Rounding Numbers

Rounding creates numbers that are easier to work with in your head. Use rounding to get an answer that is close but that does not have to be exact.

To round numbers, look at the digit to the right of the place you are rounding to. If it is 4 or less, round down by changing it and all digits to the right to zero. If it is greater than 4, round up by adding 1 to the place you are rounding to. Then change all the digits to the right to zeros.

Round 4765 to the nearest hundred.

4765
└───┬───> The digit in the hundreds place is 7. The digit to the right is 6. Since 6 is greater than 4, add 1 and change the digits to the right to zeros.

4800

Round 0.843 to the nearest tenth.

0.843
└───┬───> The digit in the tenths place is 8. The digit to the right is 4. Round down by changing digits to the right to zeros.

0.800 = 0.8

Front-End Estimation

Front end estimation usually produces a closer estimate of sums or differences than the answer produced when you use rounded numbers.

To estimate a sum or difference, add or subtract the digits of the two highest place values. Then change the digits to the right to zeros.

$$296 + 4525 = ?$$

$$290 + 4500 = 4790$$

$$2396 - 745 = ?$$

$$2300 - 740 = 1560$$

OPERATIONS

Whole Numbers and Decimals

Add or subtract decimals as you do whole numbers making sure to line up numbers on their decimal points. Add zeros in empty places. Use dollar signs with money amounts.

$$34.5 - 4.04 = ?$$

$$\begin{array}{r} 34.5 \\ - 4.04 \\ \hline \end{array}$$

$$\begin{array}{r} 34.50 \\ - 4.04 \\ \hline 30.46 \end{array}$$

Multiply decimals as you do whole numbers. Then, count the decimal places in the factors. This will be the number of decimal places in the product. Count decimal places starting at the right of the product and add zeros if necessary.

$$\begin{array}{r} 0.05 \\ \times 0.25 \\ \hline 0.0125 \end{array}$$

When multiplying a number by 10 or a multiple of 10, count the zeros. Move the decimal point to the right the same number of spaces. When dividing by 10 or a multiple of 10, count the zeros. Move the decimal point to the left the same number of spaces.

$$375.5 \times 1000 = 375,500$$

$$375.5 \div 1000 = 0.3755$$

MATH APPENDIX

Fractions

To add or subtract fractions, find a common denominator if they are not the same. One easy way to find a common denominator is to multiply both denominators.

$$\frac{4}{5} - \frac{1}{6} = \frac{24}{30} - \frac{5}{30} = \frac{19}{30}$$

To multiply fractions, multiply the numerators, multiply the denominators, and express the product in simplest form. To simplify a fraction, divide numerator and denominator by their greatest common factor (GCF). To divide fractions, invert the divisor and multiply.

$$\frac{3}{4} \times \frac{7}{8} = \frac{21}{32}$$

$$\frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \times \frac{8}{7} = \frac{24}{28} = \frac{6}{7}$$

Signed Numbers

- The sum of two positive integers is always positive.

$$\text{positive} + \text{positive} = \text{positive}$$

- The sum of two negative integers is always negative.

$$\text{negative} + \text{negative} = \text{negative}$$

- When adding a negative and a positive integer, subtract the numbers and keep the sign from the larger integer.

$$-5 + +3 = -2$$

- When subtracting integers, change the sign of the number being subtracted and then add.

$$6 - -2 = 8$$

- The product of two integers with the same sign is always positive.

$$\text{positive} \times \text{positive} = \text{positive} \quad \text{negative} \times \text{negative} = \text{positive}$$

- The product of two integers with different signs is always negative.

$$\text{positive} \times \text{negative} = \text{negative} \quad \text{negative} \times \text{positive} = \text{negative}$$

- When dividing two integers with the same sign, the quotient is always positive.

$$\text{positive} \div \text{positive} = \text{positive} \quad \text{negative} \div \text{negative} = \text{positive}$$

- When dividing two integers with different signs, the quotient is always negative.

$$\text{positive} \div \text{negative} = \text{negative} \quad \text{negative} \div \text{positive} = \text{negative}$$

SOLVING EQUATIONS

Evaluating Expressions With One Variable

To evaluate an expression with a variable, simply substitute the value of the variable into the expression and simplify.

$$\begin{aligned} \text{Evaluate } 3x + 4 \text{ for } x = 2 \\ 3 \times 2 + 4 = 6 + 4 = 10 \end{aligned}$$

Evaluating an Expression With Two Variables

A mathematical expression can have variables as part of the expression. If $x = 3$, and $y = 5$, the expression $7x + y - 4$ becomes $7 \times 3 + 5 - 4$ which is equal to $21 + 5 - 4$ or 22. To evaluate an expression with two or more variables, substitute the value of the variables into the expression and simplify.

$$\begin{aligned} \text{Evaluate } 6x - y + 5 \text{ for } x = 5 \text{ and } y = 3 \\ 6 \times 5 - 3 + 5 = 32 \end{aligned}$$

Balance in Equations

An equation has two expressions separated by an equal sign. One or both of the expressions may contain variables. Solving an equation means finding the value of the variables. We find these values by balancing: whatever we do to one side of an equation, we must do the same to the other side.

$$\begin{aligned} \text{Solve for } x: x - 3 = 5 \\ \text{Adding 3 to both sides, } x - 3 + 3 = 5 + 3 \\ x = 8 \end{aligned}$$

Order of Operations

When solving an equation, PEMDAS can help you decide what to do first.

Solve for x :	$(\frac{1}{2} - \frac{1}{4})4^2 + 2(x - 2) = 1$
P (parentheses)	$(\frac{1}{4})4^2 + 2x - 4 = 1$
E (exponents)	$(\frac{1}{4})16 + 2x - 4 = 1$
M D(multiply or divide from left to right)	$4 + 2x - 4 = 1$
A S (add or subtract from left to right)	$2x = 1$
	$x = \frac{1}{2}$

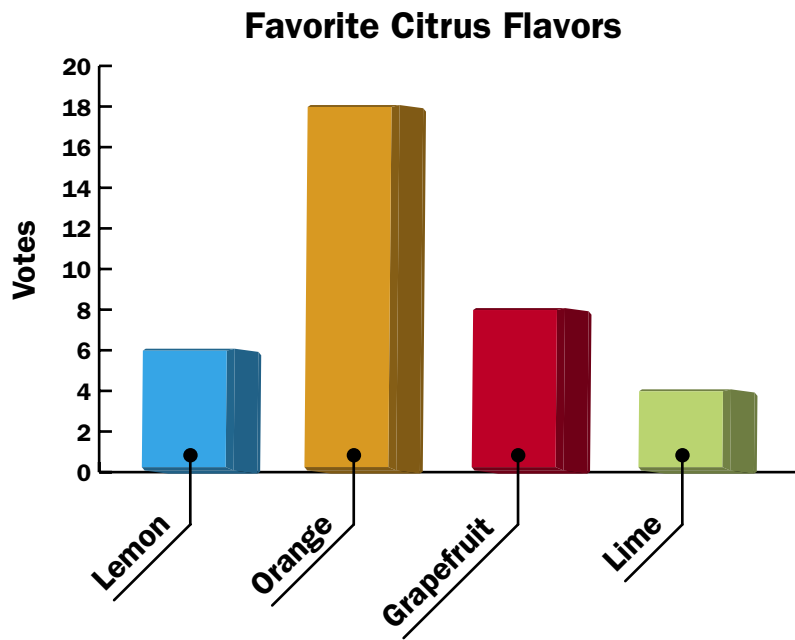
Solving an Inequality

In an inequality, two expressions are separated by a symbol that indicates how one is related to the other. In $5x > 35$, the $>$ sign indicates that the left side is larger than the right side. To solve inequalities, follow the same rules as for equations.

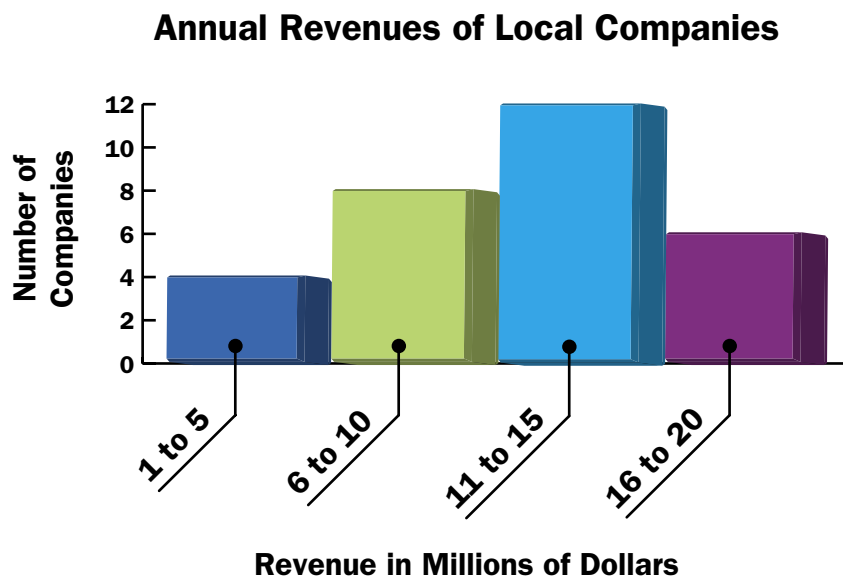
$$\begin{aligned} \text{Solve for } x: 5x > 35, \\ 5x/5 > 35/5 \\ x > 7 \end{aligned}$$

GRAPHS AND CHARTS

Bar graphs can be used to compare data about different groups. The x -axis has labels for each category and the y -axis has numbers showing amounts.



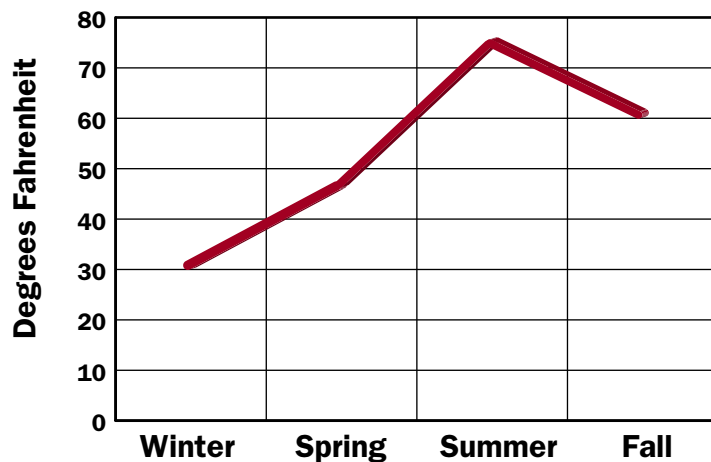
A **histogram** is like a bar graph except that each bar is like a sorting bin that represents a range.



MATH APPENDIX

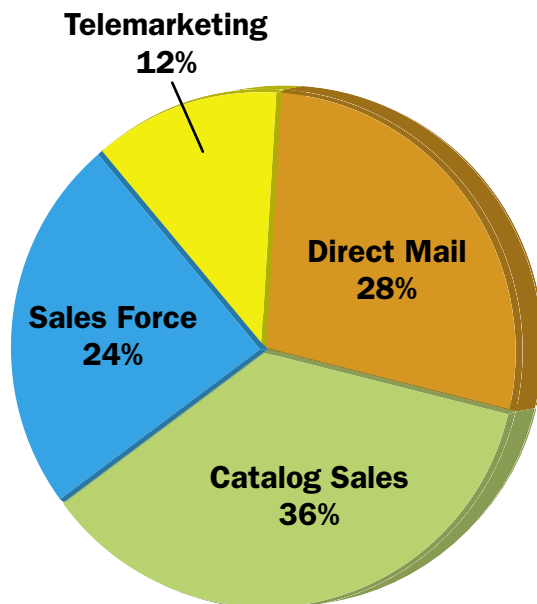
Line graphs can be used to show how things change over time. The x -axis has numbers showing time intervals and the y -axis has numbers showing amounts.

Annual Daily High Temperature



Pie charts, or circle graphs, can be used to show parts of a whole. Each section represents a percentage of the whole.

Revenue Sources



STATISTICAL TERMS

Measures of Central Tendency are used to interpret the meaning of data. There are three commonly used measures of central tendency: mean, median, and mode. Each gives a different look at a set of data.

When asked to rate, on a scale of 1 to 5, the effectiveness of an ad for a new car, the nine members of a focus group produced these results (1 = poor; 2 = fair; 3 = good; 4 = excellent; 5 = outstanding):

2 3 3 3 4 5 5 5 5

The **mean** is the most common measure of central tendency. The mean is the sum of the numbers divided by the number of numbers. It is also called the average.

$$2 + 3 + 3 + 3 + 4 + 5 + 5 + 5 + 5 = 35$$

$35 \div 9 = 3.89$. Interpretation: On the average, the group thought the ad was good to excellent.

The **median** is the midpoint of a set of values: the same number of scores are above the median as below it. It organizes data into two equal groups around a midpoint.

The median is 4. Interpretation: Half the group thought the ad was poor or fair. The other half thought it was outstanding.

The **mode** is the most frequently occurring value in a set of values.

The mode is 5. Interpretation: Quite a few respondents thought the ad was outstanding.

The **range** of a set of values is the difference between the greatest and least values in the set. $5 - 2 = 3$. The range of the data is 3.

SOLVING PROBLEMS

Four-Step Approach

Most real-world problems can be solved using this four-step approach.

- **Read** the problem carefully.
- **Plan** your approach. Assign letters to the unknown quantities, and write an equation or a set of equations to solve the problem.
- **Solve** the equations.
- **Check** your solution. Write a sentence explaining your answer.

Problem Solving Strategies

- Find a pattern
- Make an organized list or table
- Draw a picture or diagram
- Use ratios
- Solve a simpler problem
- Estimate
- Work backward

Using Formulas

To use a formula to solve a problem, substitute known values in the formula, then solve for the unknown values.

How long will it take a shipment of ball bearings to arrive at the factory if the manufacturer is 150 miles away and the truck travels at 50 mph?

Formula: $Distance = Rate \times Time$

$$150 \text{ miles} = 50 \text{ mph} \times T$$

$$T = 150/50$$

$$T = 3 \text{ hours}$$

Using Proportions

To find what percent one number is of another, write and solve a proportion.

What percent of 75 is 60?

$$n\% \text{ of } 75 = 60$$

$$n/100 = 60/75$$

To solve a proportion, cross-multiply. That means multiply the numerator of the first fraction by the denominator of the second fraction. Set that equal to the numerator of the second fraction multiplied by the denominator of the first fraction.

$$75 \times n = 6,000$$

Now, solve for the unknown quantity.

$$n = 6000/75$$

$$n = 80$$