Chapter 3: The Falling Moon

Student Worksheet

Objective:
To retrace Newton's idea that the gravity of Earth is pulling the Moon in its orbit.

Engage: Think for a few moments about gravity. It is quite a thought provoking and complex subject. In the space below, note your personal definition of gravity. What do you think is the source of gravity? What questions do you have about gravity?

Introduction:
Isaac Newton wondered if the force pulling the Moon around the Earth was the same force that pulled an apple to the Earth. Newton did some calculations to figure this out. He knew through observation that the time of the Moon's orbit is about 27 days. Could he use his 2^{nd} Law (F=ma) to solve for the period of the Moon's orbit if indeed Earth's gravity were the force keeping the Moon in orbit? If his formulas worked out to show a time that matched the observed period of the Moon, he would show that the Moon is simply falling around the Earth, like the apple falling to the Earth.

Your Task:
Retrace the progression of how Newton came to realize that the Moon's orbit around the Earth is due to gravity.

Procedure:
Follow the calculations and fill in the blanks as you go. Have your calculator ready.

1. Newton had a good estimate of the radius of the Earth from Eratosthenes's experiment. That radius is 6,371,000 m. Newton knew via parallax that the distance from the Earth to the Moon was about 60 times the distance of the radius of the Earth.

   What is the distance from the Earth to the Moon in meters?
   ________________________m.

   Use this distance as the value R, radius of the Moon's orbit.

2. Newton suspected gravity worked in accordance with the inverse square law. So at a distance of 60 Earth radii, gravity would be $1/60^2$ the strength.

   How many times weaker would Earth's gravity be at the distance to the Moon?

   $1/60^2$ = _______________ the strength, or _____________ times weaker.

   This means that the force of gravity acting on the Moon is the Moon's mass, m, multiplied by the acceleration due to gravity on Earth, g,(which has a value of 9.8 m/s^2) divided by 60^2. Simply put, the force of gravity acting on the moon is: $F = \frac{mg}{60^2} = \frac{mg}{3600}$

3. According to Newton's 2nd law, $F = ma$, and in the case of circular motion the acceleration is centripetal, $a = \frac{v^2}{r}$ so $F = \frac{mv^2}{r}$. In step two we saw that the force acting on the Moon was $F = \frac{mg}{3600}$. This force we can set equal to the centripetal force, $F = F$. Bringing us to $\frac{mg}{3600} = \frac{mv^2}{r}$. Setting centripetal force to gravitational force is the key set- up to solving this problem. How can this

equation be simplified? Write the simplified version in the space provided.

_____________________________________. Check with your instructor to verify your result.

4. Now let's determine v (velocity) and r (radius). The radius is easy since you already established that Newton knew the Moon was as far as 60 Earth radii. You solved for this distance in step 1. (You will use this distance as r in step 6.)

   We also have to look at velocity. The simplest definition for velocity is distance traveled divided by the time taken for the travel. So, $v = \frac{d}{t}$. In this case the distance is the circumference of the Moon's orbit around the Earth. The circumference of a circle is $C = 2\pi r$ so we can say $d = 2\pi r$. Thus, $v = \frac{2\pi r}{t}$.

   Plug this v in for the v in the formula from step 3: $\frac{g}{3600} = \frac{v^2}{r}$. You should get:

   $$\frac{g}{3600} = \frac{4\pi^2 r^2}{rt^2}$$

   Simplify this formula in the space below.

5. Remember, we are solving for time to see how it matches up to the easily observed time from Earth. Rearrange the simplified formula in problem 4 to solve for time. Do not plug in numerical values just yet.

   $t=$

6. Now go ahead and plug your values for r and g into the formula. Recall that you solved for r in Step 1. Make sure you use 9.81 m/s^2 for g. Since this value is expressed in terms of meters, we need our radius to be in meters too. Convert the radius of the moon from *km* to *m*. The time you find as your answer will be in

seconds. Convert seconds into days.

7. The accepted value for the period of the moon is 27.3217 days. How close is your result?

8. Find the percent error in Newton's calculation. % error = (your value – accepted value)/accepted value.

Conclusion:

1. Though you used the speed of the Moon in this calculation to find the time of orbit, you did not actually solve for the speed. Go ahead and do so now. Make sure your answer is in meters per second.

2. If a car with a top speed of 100 miles per hour (or 45 m/s) were driving the orbit of the Moon, how long would it take the car to complete the orbit?

3. What other natural occurrences observe the inverse square law?

Extend:

- How did the Moon form? Research the leading hypothesis.
- Newton's laws of motion gave rise to formulae for the force of gravity on a given body, the launch speed an object would need to orbit a body, as well as the escape speed an object would need to leave a body. Make calculations for each of these quantities.

- How does Einstein's gravity differ from Newton's gravity?