Quantitative Aside 12.2--Using Binomial Probability

If a man and a woman who are both carriers for cystic fibrosis have four children, what is the probability of their having only one unaffected child?

There are several ways you can do this problem. The most obvious is to simply determine the number of ways this can occur and the probability of each way, then add these up (addition rule for probability of any of these possible mutually exclusive events). There are four possible birth orders that can lead to this outcome: (using unaffected = U, and affected = A) UUUA, UUAU, UAUU, AUUU. The respective probabilities are (3/4)(3/4)(3/4)(1/4), (3/4)(3/4)(1/4)(3/4), (3/4)(1/4)(3/4)(3/4), (1/4)(3/4)(3/4)(3/4). These can all be simplified to: $(3/4)^3(1/4)^1$, using the rule of addition, the probability of any of these orders occurring is the sum of the individual probabilities, or $4(3/4)^3(1/4)^1=0.42$.

A general method for solving this kind of problem is to use binomial probability. This allows you to calculate the probability of any two mutually exclusive events happening a defined number of times.

If you have two mutually exclusive outcomes, then the probability of *s* outcomes with probability *p,* and *t* outcomes with probability *q,* of *N* total events is:

$$(N!/s!t!)p^s q^t$$

where

$N = s + t$
$p + q = 1$

For the cystic fibrosis example, if the probability of being unaffected (*p*) is 3/4 and the probability of being affected (*q*) is 1/4, *s* = 3 (3 unaffected) and *t* = 1 (1 unaffected). Overall this gives us:

$$\begin{aligned}\text{Probability(3 unaffected, 1 affected)} &= (4!/3!1!)(3/4)^3(1/4)^1\\ &= (4)(3/4)^3(1/4)^1\\ &= 0.42\end{aligned}$$

Notice that the factor (*N*!/*s*!*t*!) in the equation above produced the same number as our manual calculation of numbers of birth orders. It is a general solution to this problem that does not require manually counting possible orders. It is also the coefficient for the second term of the fourth-order binomial expansion. For this reason it is also called the binomial coefficient.

We can also express this in terms of percentages: given a large sampling of families with 4 offspring, 42% will have 3 unaffected, and 1 affected offspring.