Assignment 5: Limits, Part I (1.2) **Name____________________**
Please provide a handwritten response.

1a. Many ordinary limits can be found in *Mathematica* using the **`Limit`** command. For example, to evaluate the limit in Example 2.2, execute the command

```
Limit[(3x + 9)/(x^2 - 9), x->-3]
```

and record the result below; is your answer the same as that in the text?

1b. Example 2.4 suggests that $\lim_{x \to 0} \frac{\sin x}{x} = 1$; execute the command

```
Limit[Sin[x]/x, x->0]
```

and record the result below. Does *Mathematica*'s result support the conjecture made in the text?

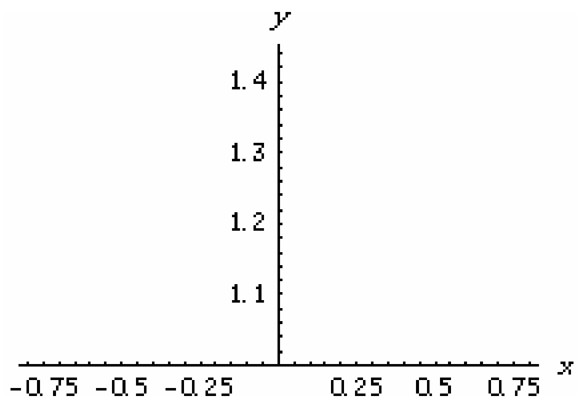
2a. Exercise 13 asks for numerical and graphical evidence regarding $\lim_{x \to 0} \frac{\tan x}{\sin x}$. First execute the command

```
f[x_] = Tan[x]/Sin[x]
```

followed by

```
Plot[f[x], {x, -Pi/4, Pi/4}]
```

and sketch the result on the axes at right. What value for $\lim_{x \to 0} \frac{\tan x}{\sin x}$ does this graph suggest?



2b. Next, execute the commands **`f[0.1]`**, **`f[0.01]`**, etc. to complete the table at right. What value for $\lim_{x \to 0} \frac{\tan x}{\sin x}$ does the table suggest?

x	$f(x)$
0.1	
0.01	
0.001	
–0.1	
–0.01	
–0.001	

2c. Finally, execute the command

```
Limit[f[x], x->0]
```

and record the result below; did all three approaches lead you to the same conclusion?

3a. Exploratory Exercise 2, Section 1.2 uses the example $\lim_{x \to 0} \frac{\cos x - 1}{x^2}$ to show that round–off error can cause very misleading computed results. Execute the command **`Clear[f]`** followed by

```
f[x_] = (Cos[x] - 1)/x^2
```

to define $f(x) = \frac{\cos x - 1}{x^2}$ and then use this *Mathematica* function to complete the table at right. (Be sure to count the zeros!) Then execute the command **`Limit[f[x], x->0]`** and record the result below.

x	$f(x)$
0.1	
0.0001	
0.0000001	
0.00000001	
0.000000001	

3b. Do you think that all of *Mathematica*'s results in parts **a** and **b** are correct? If not, then which one(s) do you think are wrong, and why?

4a. To find one–sided limits we apply the **`Direction`** option to the **`Limit`** command; setting **`Direction->1`** gives the limit from the left, and setting **`Direction->-1`** gives the limit from the right. For example, the function $g(x) = \frac{x}{|x|}$ discussed in Example 2.5 would be written in *Mathematica* using the **`Abs`** function by executing the following command:

```
g[x_] = x/Abs[x]
```

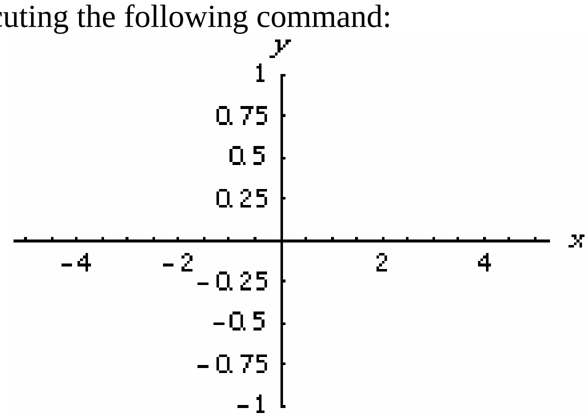
Next execute the command

```
Plot[g[x], {x, -5, 5}]
```

and sketch the result on the axes at right. Now execute the command

```
Limit[g[x], x->0, Direction->1]
```

to find $\lim_{x \to 0^-} g(x)$, and record the result below.



4b. Now execute the command **`Limit[g[x], x->0, Direction->-1]`** to find $\lim_{x \to 0^+} g(x)$, and record the result below.

4c. Finally execute **`Limit[g[x], x->0]`** ; would we have expected this result? Why? (The reason for it is that by default, **`Limit`** assumes the option **`Direction->-1`** .)