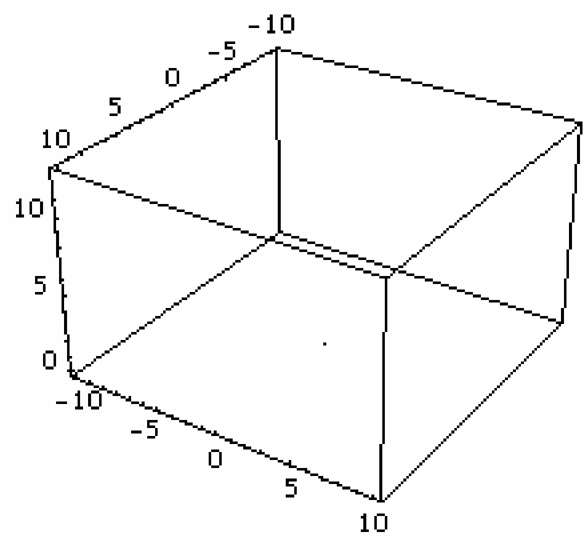
Assignment 32: Vector Fields in Space (14.6–8) **Name___________________**
Please provide a handwritten response.

1a. To graph the surface S defined parametrically by $x = u\cos v$, $y = u\sin v$, $z = v$ over $0 \le u \le 10$, $0 \le v \le 4\pi$ execute `r[u_, v_] = {u Cos[v], u Sin[v], v}` and then

```
ParametricPlot3D[Evaluate[r[u, v]], {u, 0, 10}, {v, 0, 4Pi},
        ViewPoint->{3, 2, 2}, PlotPoints->{15, 50}]
```

(The specification for `PlotPoints` causes fewer points to be sampled for u than for v.) Sketch the result in the box at right and describe the surface.



1b. We will study the flux integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z)$ is the vector field $\langle y,-x,1\rangle$ of Exercise 39, Section 14.6. Execute

```
Needs["Graphics`PlotField3D`"]
```

and then draw **F** using the commands

```
F[x_,y_,z_] = {y, -x, 1}
PlotVectorField3D[F[x, y, z], {x,
  -10, 10}, {y, -10, 10}, {z, 0,
     10}, Axes->True, ScaleFunction->(1.5&), ScaleFactor->None,
          Viewpoint->{3,2,2}, VectorHeads->True]
```

Suppose $\mathbf{F}(x,y,z)$ were the velocity vector of the wind at the point (x,y,z) and you released a leaf into this wind; in your own words, where would it go?

1c. Taking the unit normal **n** to have positive z–component, would we expect $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ to be positive, negative or zero? Why?

1d. To find a normal vector $\mathbf{r}_u \times \mathbf{r}_v$ execute

```
norm[u_, v_] = Cross[D[r[u, v], u], D[r[u, v], v]]
```

and explain how we know that this is the "correct" normal vector.

1e. To find the integrand $\mathbf{F}\cdot\mathbf{n}dS$ execute

```
F[r[u, v][[1]], r[u, v][[2]], r[u, v][[3]]].norm[u, v]
```

followed by `Integrate[%, {v, 0, 4Pi}, {u, 0, 10}]` and record the result below. Were your expectations borne out?

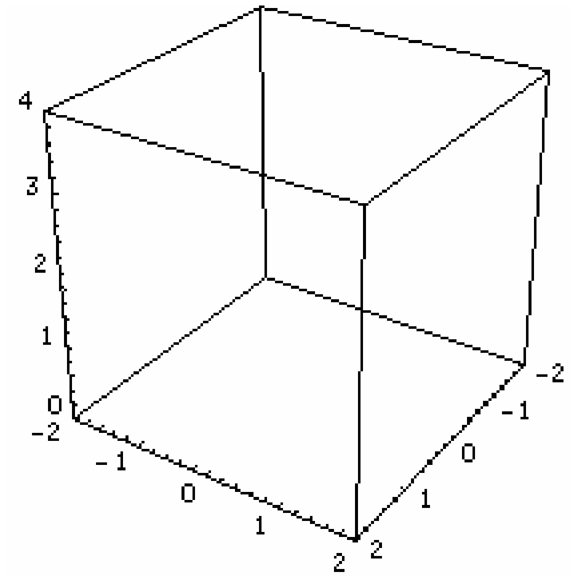
2a. To graph the region Q of Exercise 9, Section 14.7 using cylindrical cöordinates execute

```
Needs["Graphics`ParametricPlot3D`"]
```

followed by the commands

```
b = CylindricalPlot3D[r^2,{r,0,2},{t,0,2Pi},ViewPoint->{3,2,2}]
t = CylindricalPlot3D[4,{r,0,2},{t,0,2Pi},ViewPoint->{3,2,2}]
```

and `Show[b, t]` ; sketch the result in the box at right. Next clear and redefine `F` to be the vector field $\mathbf{F}(x,y,z) = \langle x^3, y^3 - z, xy^2 \rangle$ and execute the commands




```
Needs["Calculus`VectorAnalysis`"]
SetCoordinates[Cartesian[x,y,z]]
```

Execute `Div[F[x, y, z]]` and `Curl[F[x, y, z]]` to find $\nabla \cdot \mathbf{F}$ and $\nabla \times \mathbf{F}$, and record the results below.

2b. Now set up an iterated integral giving $\iiint_Q \nabla \cdot \mathbf{F}(x,y,z)\,dV$ and use *Mathematica* to evaluate it; record your answers below.

2c. By Stokes' Theorem, $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\,dS$ is the same whether S is the "bowl" or the "lid" of ∂Q . Clear variables, parameterize the "bowl" using $\mathbf{r}(u,v) = \langle u\cos v, u\sin v, u^2 \rangle$, $0 \le u \le 2$, $0 \le v \le 2\pi$ and use Question **1d** to find $\mathbf{r}_u \times \mathbf{r}_v$. Execute

```
delF[u_, v_] = Curl[F[x,y,z]]/.{x->r[u,v][[1]],
    y->r[u,v][[2]],z->r[u,v][[3]]}
```

and then find $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\,dS$ for the "bowl" by executing

```
Integrate[delF[u, v].norm[u, v], {u, 0, 2}, {v, 0, 2Pi}]
```

Now make slight modifications in the above to calculate $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\,dS$ for the "lid"; do the two results agree? What are they?