Assignment 31: Vector Fields in the Plane (14.1–4) **Name____________________**
Please provide a handwritten response.

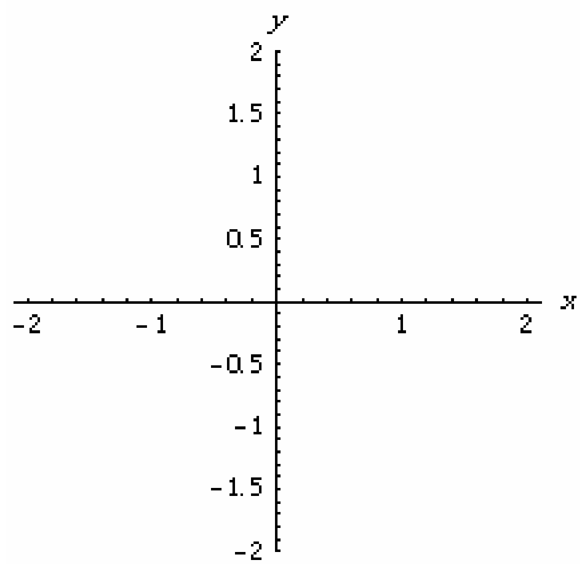
1a. Recall from Assignment 17 that vector fields can be drawn in *Mathematica*. Execute

```
Needs["Graphics`PlotField`"]
```

and then draw the vector field in Exercise 6, Section 14.1 by executing

```
vf = PlotVectorField[{-1, y^2}, {x,-2,2}, {y,-2,2}, Axes->True,
        ScaleFunction->(.25&), ScaleFactor->None]
```

Sketch the result on the axes at right.



1b. The flow lines for this vector field are solutions of the separable differential equation $\frac{dy}{dx} = -y^2$; to graph them using the method of Assignment 16, first execute

```
G[y_] = Integrate[-1/y^2, y]
```

and

```
H[x_] = Integrate[1, x]
```

followed by the commands

```
gensoln = G[y] == H[x] + c
f[x_]=Solve[gensoln,y][[1,1,2]]
```

(The `[[1,1,2]]` extracts the portion of the solution that `Plot` needs.) Then execute

```
flow = Plot[Evaluate[Table[f[x], {c, -3, 3}]], {x, -2, 2},
        PlotRange->{-2, 2}, PlotStyle->Hue[0]]
```

to graph several flow lines simultaneously in red. Finally execute

```
Show[vf, flow, AspectRatio->Automatic]
```

to draw the vector field and flow lines together, and sketch the flow lines on your graph above. Are the vertical lines flow lines too?

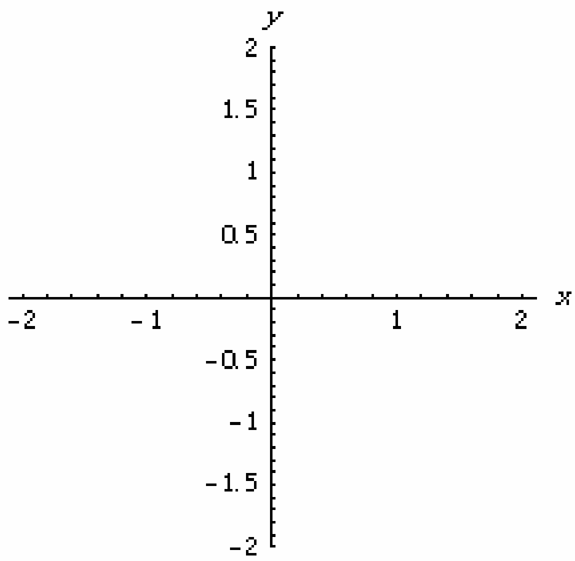
2. To draw the gradient field corresponding to $f(x,y) = y \sin x$ in Exercise 18, Section 14.1 clear `f` and execute `f[x_, y_] = y Sin[x]` followed by

```
gf = PlotGradientField[f[x, y],{x, -2, 2},{y, -2, 2}, Axes->True,
        ScaleFunction->(.25&), ScaleFactor->None]
```

Next, to draw the level curves of $f(x,y)$ in red, execute

```
lc = ContourPlot[f[x, y], {x, -2, 2}, {y, -2, 2}, ContourShading-
        >False, ContourStyle->Hue[0]]
```

followed by `Show[gf, lc]` , and sketch the result on the axes at right. What general connection between level curves and the gradient vector field does this graph bring out?



3a. To study the line integral

$$\oint_C \left(x^2 - y\right)dx + y^2 dy$$

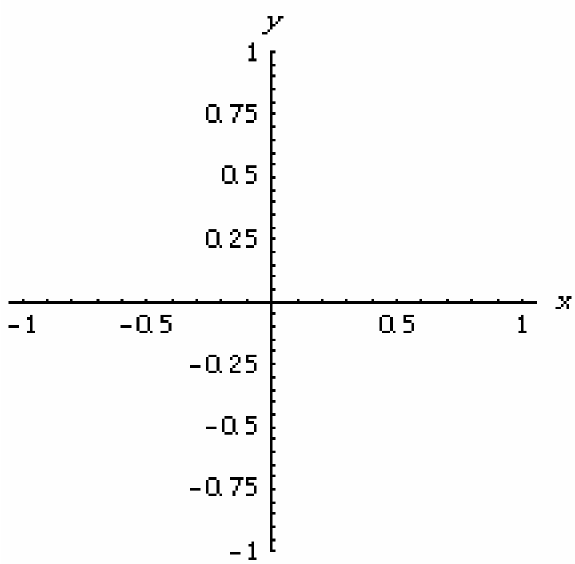
in Exercise 1, Section 14.4 execute the commands `m[x_,y_] = x^2-y` , `n[x_,y_] = y^2` and `F[x_,y_] = {m[x,y], n[x,y]}` to define $M(x,y) = x^2 - y$, $N(x,y) = y^2$ and $\mathbf{F}(x,y) = \langle x^2 - y, y^2 \rangle$. Next clear `vf` and draw the vector field for $\mathbf{F}$ by executing

```
vf = PlotVectorField[F[x,
y],{x, -1, 1},{y, -1, 1},Axes->True, ScaleFunction->(.15&),
                        ScaleFactor->None]
```

Sketch the result on the axes given. Now execute `r[t_] = {Cos[t], Sin[t]}` to parameterize the curve C by $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$, $0 \le t \le 2\pi$ and then execute

```
crv = ParametricPlot[Evaluate[r[t]],{t,0,2Pi},PlotStyle->Hue[0]]
```

followed by `Show[vf, crv]` ; add the result to the axes at right.



3b. The commands `r[t][[1]]` and `r[t][[2]]` give the first and second components of $\mathbf{r}(t)$. (Try them). Thus

```
F[r[t][[1]], r[t][[2]]].r'[t]
```

gives $\mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)$, to which we can then apply `Integrate` over $0 \le t \le 2\pi$ to find $\oint_C \mathbf{F} \cdot d\mathbf{r}$. Record the result of this below and tell whether Green's theorem gives the same result.

3c. Suppose the integration in part **b** were taken over $\frac{3\pi}{4} \le t \le \pi$ instead; would the graph lead you to expect a positive or negative result? Why? What result does *Mathematica* give? Repeat for $\frac{3\pi}{2} \le t \le 2\pi$.