Assignment 28: Partial Derivatives (12.3–7) **Name____________________**
Please provide a handwritten response.

1a. To define the function $f(x, y, z,) = e^{2xy} - \frac{z^2}{y} + xz\sin y$ of Exercise 14, Section 12.3 execute

```
f[x_, y_, z_] = Exp[2x y] - (z^2/y) + x z Sin[y]
```

(spaces and all!), followed by `D[f[x, y, z], y]` to find $f_y(x, y, z)$. Is the result correct?

1b. To find the second–order mixed partial derivative $f_{yx}(x, y, z) = \frac{\partial^2 f}{\partial x \partial y}(x, y, z)$ execute `D[f[x, y, z], x, y]` and record the result below. (Note the order in which the variables are listed in the command.)

1c. Execute `D[f[x, y, z], y, y]` to find $f_{yy}(x, y, z) = \frac{\partial^2 f}{\partial y^2}(x, y, z)$, followed by `%/.{x->-0.2, y->3, z->Sqrt[7]}` to find $f_{yy}\left(-0.2, 3, \sqrt{7}\right)$; record the results below.

2a. Clear `f` and execute `f[x_, y_] = x^3 + 3x y - y^3` followed by

```
Plot3D[f[x, y], {x,-0.5,1.5}, {y,-1.5,0.5}, ViewPoint->{3,2,2}]
```

to draw the graph of $f(x, y) = x^3 + 3xy - y^3$ over $-0.5 \le x \le 1.5$, $-1.5 \le y \le 0.5$; is it clear from this plot what critical point(s) *f* has over this range, and of what type?

2b. Modify the preceding command as follows; as usual, no carriage returns!

```
Plot3D[f[x, y], {x,-0.5,1.5}, {y,-1.5,0.5}, ViewPoint->{3,2,2}, BoxRatios->Automatic]
```

Now tell below what you can see regarding critical points. What was the effect of the extra option?

2c. To calculate $\nabla f(x, y)$ execute `Needs["Calculus`VectorAnalysis`"]` followed by `SetCoordinates[Cartesian[x, y, z]]` to specify the cöordinate system and variables we intend to use. Execute `Grad[f[x, y]]` and record the result below. (The 0 at the third position represents $f_z(x, y)$ and is included automatically by `Grad`.)

2d. Because the critical points of f occur where $\nabla f(x,y) = \mathbf{0}$, execute

```
Solve[Grad[f[x, y]] == {0, 0, 0}, {x, y}]
```

and record the result below. Are there really four different critical points? Do these points appear consistent with the graph in part **b**? Execute `f[x, y]/.%` to find the corresponding z–values.

2e. To apply the second derivative test execute `fxx[x_, y_] = D[f[x, y], x, x]`, `fyy[x_, y_] = D[f[x, y], y, y]`, and `fxy[x_, y_] = D[f[x, y], y, x]` followed by

```
discrim[x_, y_] = fxx[x, y] fyy[x, y] - fxy[x, y]^2
```

Use these functions to classify each of the critical points of f and record the results below.

3a. Clear `f` and define $f(x,y) = (x^2 - 3xy + 3y^2 + 4x)e^{-2x^2 - \frac{1}{2}y^2} + \sin\left(\frac{x+y}{100}\right)$ (!) by executing

```
f[x_, y_]=(x^2-3x y+3y^2+4x)Exp[-2x^2-(1/2)y^2] + Sin[(x+y)/100]
```

Modify the last command in Question **2a** to plot $f(x,y)$ over $-1.5 \le x \le 1.5$, $-3 \le y \le 3$. How many critical points does f seem to have over this range?

3b. Execute

```
ContourPlot[f[x, y], {x, -1.5, 1.5}, {y, 3, 3}, PlotPoints->50]
```

and use your results so far to give below the rough cöordinates of each of the critical points and what type of critical point it is.

3c. Because the `Solve` command cannot find the critical points for this function, execute

```
FindRoot[Grad[f[x, y]]=={0, 0, 0}, {x, -0.5}, {y, -0.1}, {t, 1}]
```

to find the exact cöordinates of the critical point near $(-0.5, -0.1)$. (The "dummy" variable `t` is needed to give `FindRoot` the same number of unknowns as equations; you can otherwise ignore it in the input and output.) Change the starting point for x and y to find the other critical points as well, and record the results below.