Assignment 26: Vector–Valued Functions, Part II (11.1–4) Name____________________
Please provide a handwritten response.

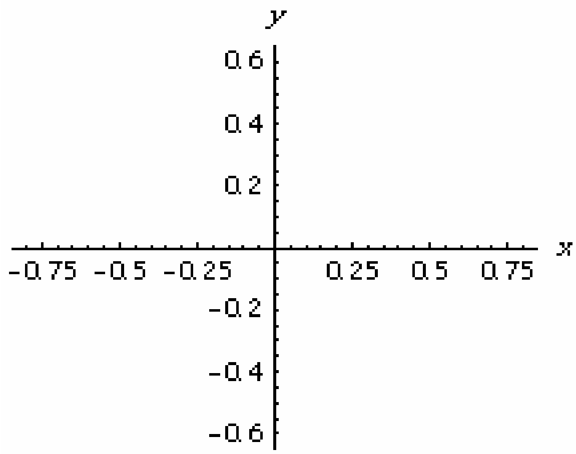
1a. To plot the Cornu spiral of Exploratory Exercise 1, Section 11.4 execute the following commands; the output refers to the "Fresnel integrals" of applied mathematics.

```
f[t_] = Integrate[Cos[Pi u^2/2], {u, 0, t}]
g[t_] = Integrate[Sin[Pi u^2/2], {u, 0, t}]
```

Then execute `r[t_] = {f[t], g[t]}` followed by

```
ParametricPlot[Evaluate[r[t]], {t, -Pi, Pi}]
```

and sketch the result on the axes at right.



1b. Apply the `Integrate` command to

```
Sqrt[f'[t]^2 + g'[t]^2]
```

to find the arc length of the curve from $t = 0$ to $t = c$, and record the result below. What does this say about this parameterization of this curve?

1c. We will use formula (4.6), Section 11.4 to find the curvature of this curve at $t = c$; execute

```
T[t_] = r'[t]/Sqrt[r'[t].r'[t]]
```

to find the unit tangent vector $\mathbf{T}(t)$, followed by

```
Sqrt[T'[c].T'[c]]/Sqrt[r'[c].r'[c]]
```

Does the result tell you much? Execute `Simplify[%]` and record the result below. What does this say about the curve?

2a. Vector–valued functions in space are graphed using the `ParametricPlot3D` command; for example, to draw the graph of $\mathbf{r}(t) = \langle \cos t, \ln t, \sin t \rangle$ (denoted $\mathbf{f}_1(t)$ in Example 1.5) over $0.1 \le t \le 8\pi$, clear `r` and execute

```
r[t_] = {Cos[t], Log[t], Sin[t]}
```

followed by

```
ParametricPlot3D[Evaluate[r[t]], {t, 0.1, 8Pi}]
```

According to the Example, the result should look like Graph B; does it?

2b. Now execute the following modification of the preceding command:

```
ParametricPlot3D[Evaluate[r[t]],{t,0.1,8Pi},ViewPoint->{3,2,2}]
```

Does the result look more like Graph B? The **`ViewPoint`** option specifies the "vantage point" in R^3 from which the graph is viewed; the three–dimensional graphs in the text are drawn as though the viewer were suspended in the first octant (at the point $(3,2,2)$, roughly), whereas *Mathematica* by default takes $(1.3,-2.4,2)$ — not in the first octant — as the viewpoint.

2c. Now clear **`r`** , define **`r[t]`** according to Exercise 25, Section 11.1 and execute the command in part **b**, with **`0.1`** replaced by **`0`** . How would you describe the result? Do you think this is an accurate graph?

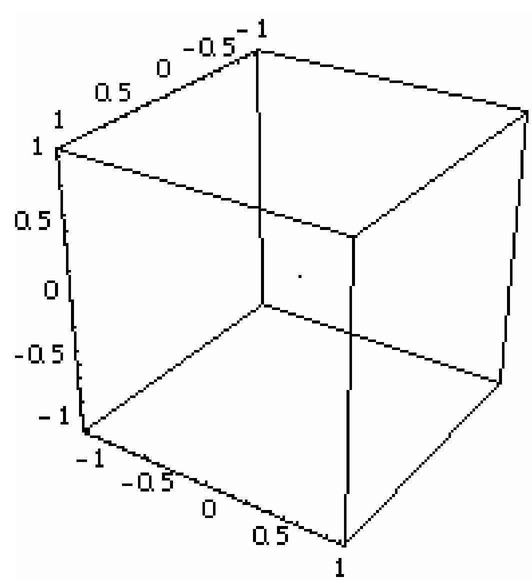
2d. Now execute the following command, as usual without inserting any carriage returns:

```
ParametricPlot3D[Evaluate[r[t]],{t,0,8Pi},ViewPoint->{3,2,2}, PlotPoints->100]
```

Does this result look any better? Execute this command again with **`100`** replaced by **`500`** ; do you think this is the "real thing"?

2e. Why was the graph in part **c** so poor? How did the **`PlotPoints`** option improve it?

3a. Sketch in the box at right the curve in Exercise 33, Section 11.1.



3b. At what point(s) on this curve do you think the curvature is greatest? Execute

```
Cross[r'[t], r''[t]]
```

followed by

```
k[t_]=Sqrt[%.%/(r'[t].r'[t])^3]
```

to define the curvature κ using Theorem 4.1. Now use *Mathematica* to find the value(s) of t for which **`k[t]`** is greatest, and record below the corresponding points on the curve. Was your conjecture correct?