

Assignment 23: Polar Coordinates (9.4–7)
Please provide a handwritten response.

Name _____

1. Define the limaçon $r = f(\theta) = 4\cos\theta - 2$
 by executing

```
f[theta] = 4Cos[theta] - 2
```

and define the ellipse $r = g(\theta) = \frac{4}{2 + \cos\theta}$ (see
 Theorem 7.2) by executing

```
g[theta] = 4/(2 + Cos[theta])
```

Next load in a package by executing

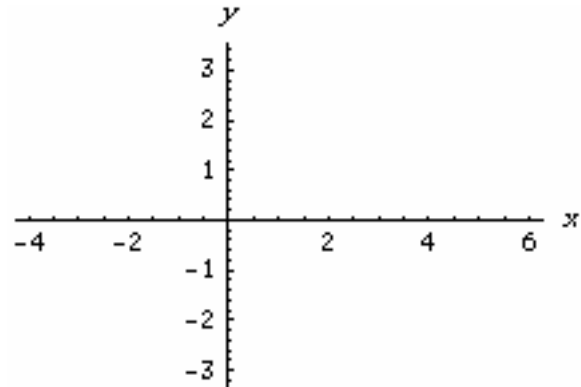
```
Needs["Graphics`Graphics`"]
```

and then execute the following commands to graph the curves above:

```
limacon = PolarPlot[f[theta], {theta, 0, 2Pi}]
```

```
ellipse = PolarPlot[g[theta], {theta, 0, 2Pi}]
```

Use the **Show** command to combine these two graphs and sketch the result on the axes at right. How many points of intersection do these two curves have?



2a. To find the points of intersection we begin by numerically solving the equation
 $f(\theta) = g(\theta)$ for θ using the **NSolve** command; execute

```
s1 = NSolve[f[theta] == g[theta], theta]
```

Mathematica includes results involving the imaginary unit I , which despite their mathematical interest we will ignore throughout this assignment. Record the other results below; how many eligible values of θ did *Mathematica* find? Also execute

```
Solve[f[theta] == g[theta], theta]
```

and tell why we used **NSolve** to find **s1** instead.

2b. Now execute `{f[theta], theta}/.s1` to list the points (in polar coordinates) corresponding to the values of θ found above, and label the eligible ones on the graph above. (It's helpful to remember that 1 radian is $\frac{180}{\pi} \approx 57^\circ$.) Have we found all of the points of intersection yet? Could we have used **g** rather than **f**?

2c. According to Exercise 3, Section 9.5 we should also solve the equation $f(\theta) = -g(\theta + \pi)$ for θ to find points of intersection; execute

```
s2 = NSolve[f[theta] == -g[theta + Pi], theta]
```

and, imitating part **b**, label the resulting eligible points on the graph.

3a. Below write down a sum of definite integrals that gives the area of the region lying both inside the ellipse and inside the smaller loop of the limaçon; also shade this region on the graph.

3b. Use **Integrate** if possible, and **NIntegrate** if necessary, to evaluate these integrals, and record the total area below. Based on the graph, is your answer plausible?

3c. Repeat parts **a** and **b** for the region lying inside the large loop of the limaçon (ignore the small loop) but outside the ellipse. Be sure to keep careful track of what parts of each curve correspond to which values of θ , and be ready to apply **N** to the output of **Integrate** if you don't get a numerical result at first.

4a. Implement the slope formula (5.4), Section 9.5, by executing

```
ls[a_] = (f'[a] Sin[a] + f[a] Cos[a]) / (f'[a] Cos[a] - f[a] Sin[a])
```

where a represents a particular value of θ and **ls** stands for “limaçon slope”. Next execute

```
s3 = NSolve[ls[a] == 0, a]
```

to find the values of θ for which the tangent line to the limaçon are horizontal, and record the result below. Also find the corresponding points on the curve as in part **b** and mark them on the graph.

4b. The graph suggests that the two curves might be orthogonal at two of their intersection points. Modify the first command in part **a** to define a function **es** giving the slope for the ellipse and decide whether this is the case. (Recall that perpendicular lines have negative reciprocal slopes unless they are horizontal and vertical.)