**Assignment 22: Parametric Equations (9.1–3)** **Name____________________**
**Please provide a handwritten response.**

**1a.** Execute

```
x[t_] = Pi t - 0.6Sin[Pi t]
```

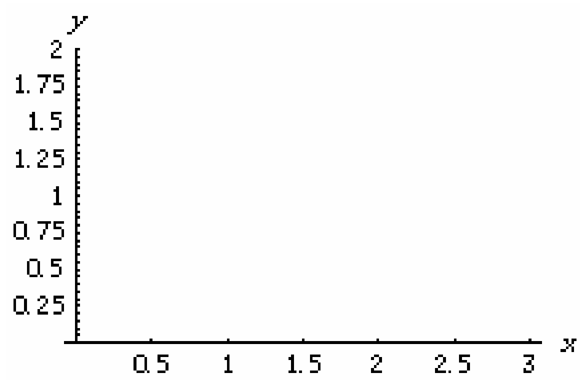
(watch those spaces after the **`Pi`** !) and

```
y[t_] = 2t + 0.4Sin[Pi t]
```

followed by

```
ParametricPlot[{x[t], y[t]}, {t, 0, 1}]
```

to plot the curve in Exercise 16, Section 9.3 and sketch the result on the axes at right. (Note that here the *y*–axis is pointing upward as usual, so our skier is skiing uphill!)




**1b.** Execute

```
{x[1/2], y[1/2]}
```

to find the point on the curve corresponding to $t = \frac{1}{2}$, mark this point on the curve with a large dot and draw the line tangent to the curve there. What do you estimate the slope of this line to be?

**1c.** Execute **`y'[1/2]/x'[1/2]`** to find this slope exactly, and record the result below.

**1d.** Execute **`NIntegrate[Sqrt[x'[t]^2 + y'[t]^2], {t, 0, 1}]`** to find the length of this curve, and record the result in the table on the next page.

**1e.** The time needed to ski this curve can be calculated using Formula (3.2) (taking $k = 1$ for convenience); execute

```
NIntegrate[Sqrt[(x'[t]^2 + y'[t]^2)/y[t]], {t, 0, 1}]
```

and record the result in the table.

**1f.** Now clear **`x`** and **`y`** , modify the commands in part **a** and re–execute the commands in parts **d** and **e** to complete the table for Exercises 13-15, Section 9.3 as well. Based on these examples, does there seem to be any correlation between the arc length and the time?

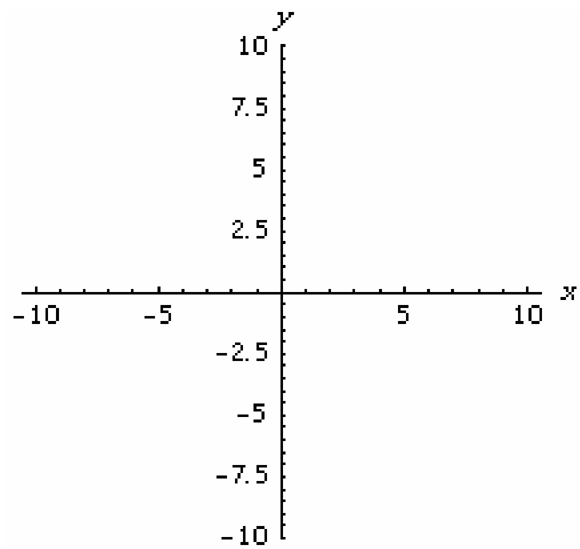
| Exercise | Arc length | Time |
|---|---|---|
| 19 | | |
| 20 | | |
| 21 | | |
| 22 | | |

**1g.** In the same way find the arc length and time for the cycloid of Example 3.3; in both categories, where does it rank among the other four curves considered so far?

**2a.** Clear your variables once again and define the parametric curve $\begin{cases} x = 8\cos t - 2\cos 4t \\ y = 8\sin t - 2\sin 4t \end{cases}$ in *Mathematica* as in Question **1a**, and then execute

```
ParametricPlot[{x[t], y[t]}, {t, 0, 2Pi}, AspectRatio->Automatic]
```

to draw this curve over $0 \le t \le 2\pi$ ; sketch the result on the axes at right. (Also try it without the **`AspectRatio`** option and tell what this option does.)




**2b.** Where are the "corner" points of this curve? By Theorem 2.1, at such points both $x'(t)$ and $y'(t)$ must be zero. Execute

```
xlist = Solve[x'[t] == 0, t]
```

to list the values of *t* for which $x'(t) = 0$ and record the results below. Is the warning message of concern?

**2c.** Likewise execute **`ylist = Solve[y'[t] == 0, t]`** followed by

```
corners = Intersection[xlist, ylist]
```

to find the values of *t* common to the lists, and record the results below. Finally execute **`{x[t], y[t]}/.corners`** to find the cöordinates of the corner points, and label them on the graph.

**2d.** Find the "corner" point(s) on the curve in Exercise 24, Section 9.2. How many are there?