

Assignment 10: Newton's Method (3.1)
Please provide a handwritten response.

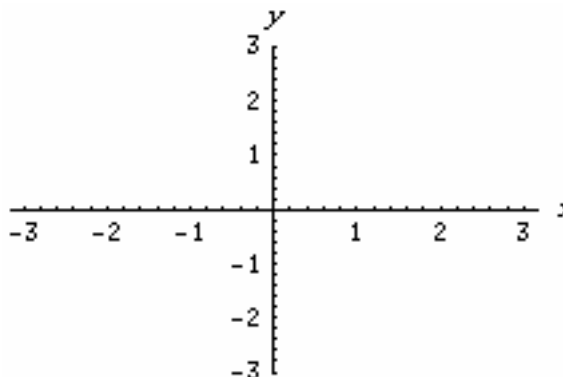
Name _____

1a. We can use Newton's method to find the approximate zeros of $f(x) = x - \frac{7}{4} \sin x + \frac{1}{8}$.

Execute the command

```
f[x_] = x - (7/4) Sin[x] + 1/8
```

and then use the **Plot** command just as you have done before to draw the graph of f over $-3 \leq x \leq 3$. Sketch the result on the axes at right.



1b. How many zeros does f seem to have over $-3 \leq x \leq 3$? Roughly, where they are located?

2a. Suppose that we apply Newton's method to f with $x_0 = -1.3$; what would you expect the successive approximations x_1, x_2, \dots to do? Why?

2b. To apply Formula (1.9) execute $\mathbf{x0} = -1.3$ followed by

$$\mathbf{x1} = \mathbf{x0} - \mathbf{f}[\mathbf{x0}]/\mathbf{f}'[\mathbf{x0}]$$

Enter this value of x_1 in the table at right. Next execute

$$\mathbf{x2} = \mathbf{x1} - \mathbf{f}[\mathbf{x1}]/\mathbf{f}'[\mathbf{x1}]$$

and enter the value of x_2 in the table; proceed in this way until the table is complete. Would you say that Newton's method was successful in this case? What is your approximate value of the zero we are looking for?

n	x_n
0	-1.3
1	
2	
3	
4	
5	

3a. Now we would like to increase the value of x_0 by a small amount to find the next zero of f to the right of the one we just found. We will try $x_0 = -1.0$; execute

```
Clear[x0, x1, x2, x3, x4, x5]
```

followed by $\mathbf{x0} = -1.0$. Then repeat your calculations of $\mathbf{x1}, \mathbf{x2}, \dots$ from Question **2b** and complete the table at right. (It's not necessary to retype these commands — you can simply scroll up in your *Mathematica* window and re-execute the ones you already have.) Did Newton's method succeed in this case?

n	x_n
0	-1.0
1	
2	
3	
4	
5	

3b. Since using $x_0 = -1.0$ did not lead to the zero we were looking for, perhaps we need to increase x_0 a bit more, say to -0.8 . Repeat your steps from part **a** but with $x_0 = -0.8$, and complete the table at right. Did Newton's method lead to a root? Was it the one we were looking for?

n	x_n
0	-0.8
1	
2	
3	
4	
5	

3c. Try it one more time with $x_0 = -0.6$, and complete the table at right. Do you at last get the results you expected?

n	x_n
0	-0.6
1	
2	
3	
4	
5	

3d. What do you think is the most difficult aspect of using Newton's method?

4a. In an earlier assignment we used the **FindRoot** command, which finds zeros of functions numerically. Actually **FindRoot** is based upon Newton's method, which is the reason we must supply an approximate value to **FindRoot** for the zero being found. Execute

FindRoot[f[x], {x, -1.3}]

and record the result below. Does **FindRoot** arrive at the same conclusion we did when $x_0 = -1.3$?

4b. Our attempt to use $x_0 = -1.0$ did not lead to any zero of f at all; execute

FindRoot[f[x], {x, -1.0}]

and record the result below. Did **FindRoot** arrive at a zero of f ? Do you think that **FindRoot** operates purely by Newton's method, or does it have other strategies as well?

5. To see in yet another way the sensitivity of Newton's method to the initial guess, execute **FindRoot[f[x], {x, 0.9625}]** and **FindRoot[f[x], {x, 0.9626}]**, and record the results below. How far apart are the initial guesses? How far apart are the results?