

Assignment 8: Derivatives of Explicit Functions (2.1–9) Name _____
Please provide a handwritten response.

1a. The derivative of the function $f(x) = 3x^3 + 2x - 1$ is $f'(x) = 9x^2 + 2$. To carry out this calculation in *Maple*, first execute the command

`f := x -> 3*x^3 + 2*x - 1;`

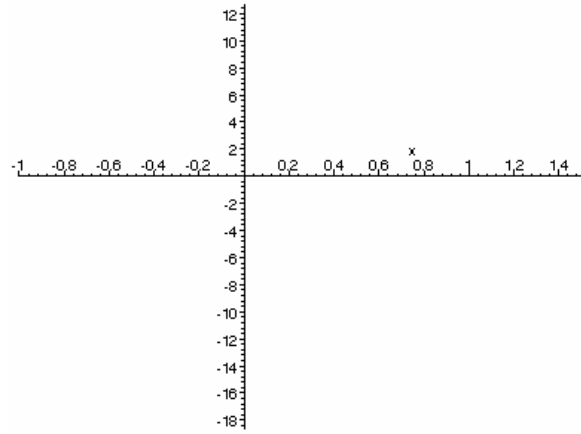
followed by the command `f1 := diff(f(x), x);`. Record the result below; did *Maple* find the derivative correctly?

1b. The slope m_{tan} of the line tangent to the graph of f at, say, $x = 1$ is given by $f'(1)$; execute `subs(x=1, f1)`; to see that $m_{\text{tan}} = 11$ in this case. Also execute `f(1)`; to see that $y = 4$ when $x = 1$. The equation of our tangent line is therefore $y = 11(x - 1) + 4 = 11x - 7$; execute

`t := x -> 11*x - 7;`

followed by

`plot([f(x), t(x)], x=-1..1.5);`



to graph both f and the tangent line together. Sketch the result on the axes at right. Does the tangent line really look as though its slope is 11? Why?

2a. To find the second derivative f'' of f , execute the command `diff(f(x), x, x);`. Record the result below; is it correct?

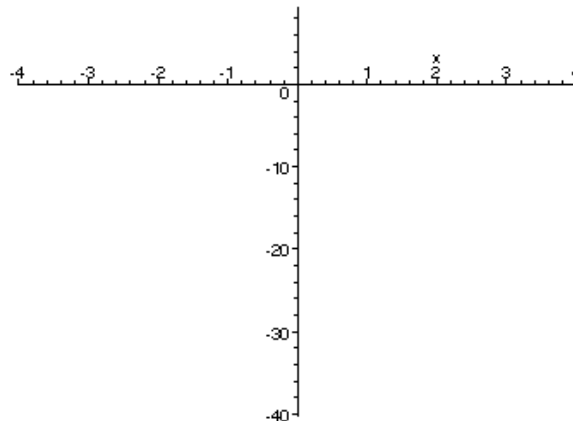
2b. Next execute the commands `g := x -> sin(2*x/(x+1));` and `diff(g(x), x, x);`, and record the result below; would you care to work this out by hand?!

3a. Execute `f := x -> x^2 * exp(sin(x))`; to define the function $f(x) = x^2 e^{\sin x}$, followed by `f1 := diff(f(x), x)`; . Record the result below; what rules and formulas presented in this chapter did *Maple* need to find $f'(x)$?

3b. Execute the command

```
plot(f1, x=-4..4);
```

to plot the derivative $f'(x)$ over $-4 \leq x \leq 4$; sketch the result on the axes at right.



3c. According to the definition of derivative, if h is a small fixed number, then the

difference quotient $\frac{f(x+h) - f(x)}{h}$ should

be close to $f'(x)$, and so their graphs should

lie close together. For the moment let's choose $h = 0.5$; execute

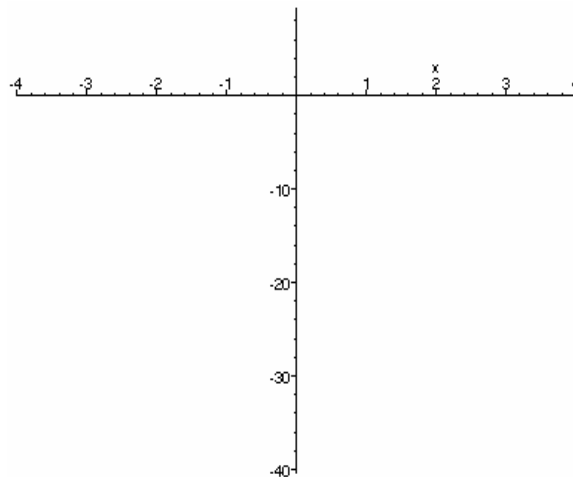
```
r := (f(x+0.5) - f(x)) / 0.5;
```

to define $r(x)$ as the quotient

$\frac{f(x+0.5) - f(x)}{0.5}$, and then execute

```
plot([f1, r], x=-4..4);
```

to graph both $f'(x)$ and $r(x)$ over $-4 \leq x \leq 4$ on the same axes. Sketch the result on the axes at right, using a dotted curve for the graph of $r(x)$.



3d. Change the `0.5` to `0.4` in the definition of $r(x)$ in part **c**, and execute the commands in parts **b** and **c** again. Are the two graphs closer? Can you still tell them apart?

3e. Experiment with smaller and smaller values of h until the graphs of $f'(x)$ and $r(x)$ over $-4 \leq x \leq 4$ become indistinguishable on your computer screen. How small does h have to be for this to happen?