Assignment 7: Limits, Part III (1.7)　　　　　　　　Name_____
Please provide a handwritten response.

1a. The function $f(x) = \dfrac{\left(x^3 + 4\right)^2 - x^6}{x^3}$
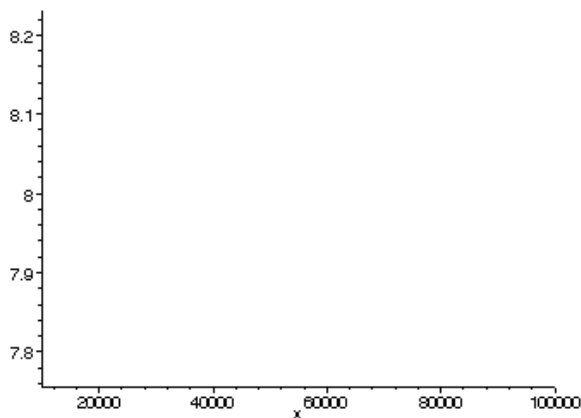
illustrates the dangers of loss of significance
errors. Execute the command

```
f:=x->((x^3+4)^2-x^6)/x^3;
```

to define f and then execute the command

```
plot(f(x),x=10000..100000);
```

to produce a graph. Sketch the result on the
axes at right. Does this graph give any
indication of the value of $\lim\limits_{x\to\infty} f(x)$? Explain.

1b. Next, execute the commands **evalf(f(1000));**,
evalf(f(10000));, etc. to complete the table at right.

1c. Now execute the command

```
limit(f(x),x=infinity);
```

and record the result below. Is it likely that all of these results
are correct? Which ones are not?

x	$f(x)$
1000	
10000	
100000	
1000000	
10000000	

1d. Since f can be rewritten as $f(x) = \dfrac{8x^3 + 16}{x^3}$; execute the

command

```
f:=x->(8*x^3+16)/x^3;
```

and then complete the table at right with this new (but
equivalent) formula for f. Do you think these new results are
more trustworthy?

x	$f(x)$
1000	
10000	
100000	
1000000	
10000000	

2. Scientific notation is used to write very large or very small numbers in a convenient form; for example, .000000000002673 would be written in scientific notation as 2.673×10^{-12}. In *Maple* execute the command `2.673*10^(-12);` and record the result below.
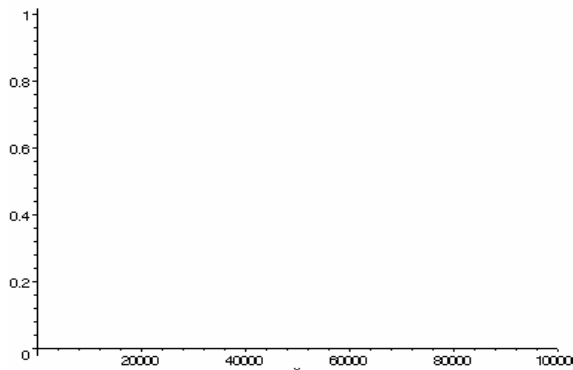
3a. To find a value of x for which loss of significance occurs in $\lim_{x\to\infty}\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$, define $g(x)=\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$ by executing the command

```
g:=x->sqrt(x)*(sqrt(x+4)-sqrt(x+2));
```

Then execute the command

```
plot(g(x),x=0..100000);
```

and sketch the result on the axes at right. Based on this graph, what value would you give for $\lim_{x\to\infty}\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$?

3b. Next, execute the commands `g(1.*10^8);`, `g(1.*10^9);`, etc. to complete the table at right. Where does loss of significance occur?

3c. We can rewrite g to avoid loss of significance; you can check that multiplying $g(x)$ by $\dfrac{\sqrt{x+4}+\sqrt{x+2}}{\sqrt{x+4}+\sqrt{x+2}}$ gives

$\dfrac{2\sqrt{x}}{\sqrt{x+4}+\sqrt{x+2}}$. Enter the command

x	$g(x)$
1×10^8	
1×10^9	
1×10^{10}	
1×10^{11}	
1×10^{12}	

```
g:=x->2*sqrt(x)/(sqrt(x+4)+sqrt(x+2));
```

Then complete the table at right just as in part **b**. Do these results seem more reliable?

3d. Finally, execute the command

```
limit(g(x),x=infinity);
```

and record the result below. Does it seem to be correct?

x	$g(x)$
1×10^8	
1×10^9	
1×10^{10}	
1×10^{11}	
1×10^{12}	

3e. Repeat parts **a** and **b** for $g(x)=x\left(\sqrt{x^3+8}-x^{3/2}\right)$. Is there a loss of significance for this function?