Assignment 5: Limits, Part I (1.2)   Name_____
Please provide a handwritten response.

1a. Many ordinary limits can be found in *Maple* using the **limit** command. For example, to evaluate the limit in Example 2.2, execute the command

$$\texttt{limit((3*x+9)/(x\^2-9),x=-3);}$$

and record the result below; is your answer the same as that in the text?

1b. Example 2.4 suggests that $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$; execute the command

$$\texttt{limit(sin(x)/x,x=0);}$$

and record the result below. Does *Maple*'s result support the conjecture made in the text?

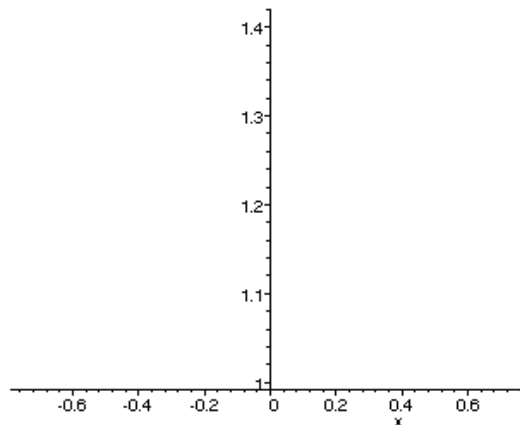**2a.** To evaluate $\lim\limits_{x\to 0}\dfrac{\tan x}{\sin x}$ graphically, first execute the command

$$\texttt{f:=x->tan(x)/sin(x);}$$

followed by

$$\texttt{plot(f(x),x=-Pi/4..Pi/4);}$$

and sketch the result on the axes at right. What value for $\lim\limits_{x\to 0}\dfrac{\tan x}{\sin x}$ does this graph suggest?

2b. To evaluate $\lim\limits_{x\to 0}\dfrac{\tan x}{\sin x}$ numerically, execute the commands

$\texttt{f(0.1);, f(0.01);}$, etc. and complete the table at right.

What value for $\lim\limits_{x\to 0}\dfrac{\tan x}{\sin x}$ does the table suggest?

x	$f(x)$
0.1	
0.01	
0.001	
–0.1	
–0.01	
–0.001	

2c. Finally, execute the command

$$\texttt{limit(f(x),x=0);}$$

and record the result below; did all three approaches lead you to the same conclusion?

3a. The example $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{x^2}$ shows that round–off error can cause very misleading computed results. Execute the command

$$\texttt{f:=x->(cos(x)-1)/x\^{}2;}$$

to define $f(x) = \dfrac{\cos x - 1}{x^2}$ and then use this *Maple* function to complete the table at right. (Be sure to count the zeros!) Then execute the command $\texttt{limit(f(x),x=0);}$ and record the result below.

x	$f(x)$
0.1	
0.0001	
0.0000001	
0.00000001	
0.000000001	

3b. Do you think that all of *Maple*'s results in part **a** are correct? If not, then which one(s) do you think are wrong, and why?

4a. To find one–sided limits we give the direction in the $\texttt{limit}$ command; inserting $\texttt{left}$ gives the limit from the left, and inserting $\texttt{right}$ gives the limit from the right. For example, the function $g(x) = \dfrac{x}{|x|}$ would be written in *Maple* using the $\texttt{abs}$ function by executing the following command:
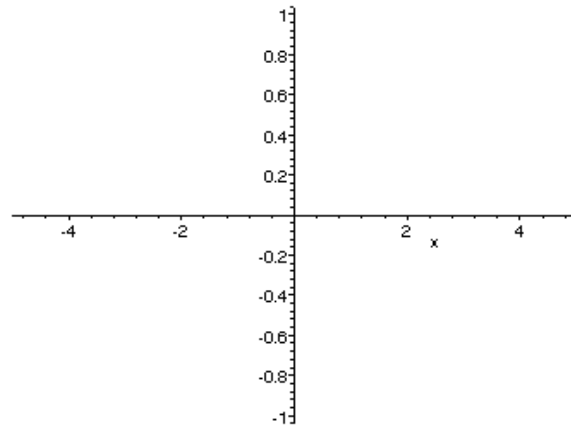
$$\texttt{g:=x->x/abs(x);}$$

Next execute the command

$$\texttt{plot(g(x),x=-5..5);}$$

and sketch the result on the axes at right. Now execute the command

$$\texttt{limit(g(x),x=0,left);}$$

to find $\lim\limits_{x \to 0^-} g(x)$, and record the result below.

4b. Now execute the command $\texttt{limit(g(x),x=0,right);}$ to find $\lim\limits_{x \to 0^+} g(x)$, and record the result below.

4c. Finally execute $\texttt{limit(g(x),x=0);}$; would we have expected this result? Why?