Assignment 32: Vector Fields in Space (14.6–8) **Name____________________**
Please provide a handwritten response.

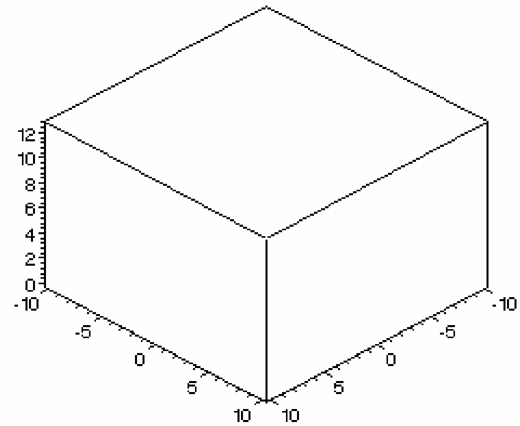
1a. To graph the surface S defined parametrically by $x = u\cos v, y = u\sin v, z = v$ over $0 \le u \le 10$, $0 \le v \le 4\pi$ execute

```
r:=(u,v)->[u*cos(v),u*sin(v),v];
```

followed by

```
plot3d(r(u,v),u=0..10,v=0..4*Pi,
          axes=boxed);
```

Sketch the result in the box at right and describe the surface.



1b. We want to study the flux integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z)$ is the vector field $\langle y,-x,1\rangle$. Execute

```
with(plots);
```

and then draw **F** using the commands

```
F:=(x,y,z)->[y,-x,1];
fieldplot3d(F(x,y,z),x=-10..10,y=-10..10,z=0..10,
        axes=boxed,color=black);
```

Suppose $\mathbf{F}(x,y,z)$ were the velocity vector of the wind at the point (x,y,z) and you released a leaf into this wind; in your own words, where would it go?

1c. Taking the unit normal **n** to have positive z–component, would we expect $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ to be positive, negative or zero? Why?

1d. To find a normal vector $\mathbf{r}_u \times \mathbf{r}_v$ execute

```
with(linalg);
N:=crossprod(diff(r(u,v),u),diff(r(u,v),v));
```

and explain how we know that this is the "correct" normal vector.

1e. To find the integrand $\mathbf{F}\cdot\mathbf{n}dS$ execute

```
assume(u,real);assume(v,real);Fn:=dotprod(F(op(r(u,v))),N);
```

followed by

```
with(student); Doubleint(Fn,u=0..10,v=0..4*Pi); value(%);
```

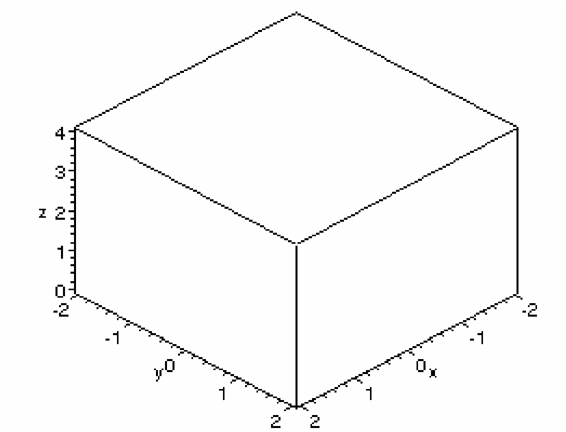
and record the result below. Were your expectations borne out?

2a. To graph the region Q bounded by $z = x^2 + y^2$ and $z = 4$ using cylindrical cöordinates execute

```
unassign('r');
cylinderplot({[r,t,r^2],[r,t,4]},r=0..2,t=0..2*Pi,axes=boxed);
```

Sketch the result in the box at right. Next redefine **F** to be the vector field $\mathbf{F}(x, y, z) = \langle x^3, y^3 - z, xy^2 \rangle$.

Execute
`div:=diverge(F(x,y,z),[x,y,z]);`
and `crl:=curl(F(x,y,z),[x,y,z]);`
to find $\nabla\cdot\mathbf{F}$ and $\nabla\times\mathbf{F}$, and record the results below.



2b. Now set up an iterated integral giving $\iiint_Q \nabla\cdot\mathbf{F}(x, y, z)dV$ and use *Maple* to evaluate it; record your answers below.

2c. By Stokes' Theorem, $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS$ is the same whether S is the "bowl" or the "lid" of ∂Q. Parameterize the "bowl" using $\mathbf{r}(u,v) = \langle u\cos v, u\sin v, u^2 \rangle$, $0 \le u \le 2$, $0 \le v \le 2\pi$ and use Question **1d** to find $\mathbf{r}_u \times \mathbf{r}_v$. Execute

```
defF:=subs(x=r(u,v)[1],y=r(u,v)[2],z=r(u,v)[3],op(crl));
```

and then find $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS$ for the "bowl" by executing

```
Doubleint(dotprod(defF,N),u=0..2,v=0..2*Pi);value(%);
```

Now make slight modifications in the above to calculate $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS$ for the "lid"; do the two results agree? What are they?