Assignment 30: Triple Integrals (13.4–7) **Name**_____
Please provide a handwritten response.

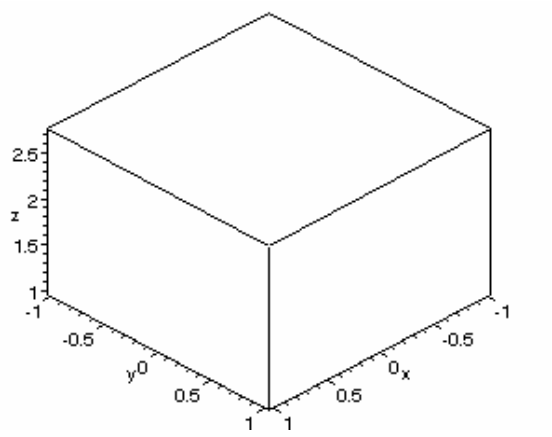
1a. To graph the portion of $z = f(x, y) = e^{x^2+y^2}$ inside $x^2 + y^2 = 1$, execute

```
f:=(x,y)->exp(x^2+y^2);
```

and then use **plot3d** as we have before to draw the graph of f over $-1 \le x \le 1$, $-1 \le y \le 1$. Next execute **with(plots);** and

```
cylinderplot([r,t,f(r*cos(t),
    r*sin(t))],t=0..2*Pi,
     r=0..1,axes=boxed);
```

to draw the graph of $z = f(r\cos\theta, r\sin\theta)$ in cylindrical cöordinates over $r \le 1$, $0 \le \theta \le 2\pi$, and sketch the result in the box at right. Which method gave the better graph? Why?



1b. The surface area is given by

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{\left[f_x(x,y)\right]^2 + \left[f_y(x,y)\right]^2 + 1}\,dy\,dx \ .$$

First define the integrand above by executing

```
F:=sqrt(diff(f(x,y),x)^2+diff(f(x,y),y)^2+1);
```

and then try to find the surface area by executing

```
with(student);
Doubleint(F,y=-sqrt(1-x^2)..sqrt(1-x^2),x=-1..1);
value(%);
```

Were **Doubleint** and **value** successful here?

1c. Does converting to polar cöordinates help? Execute

```
G:=simplify(subs(x=r*cos(t),y=r*sin(t),F));
Doubleint(G,r=0..1,t=0..2*Pi);
evalf(value(%));
```

and record the result below.

2. To evaluate the triple integral $\int_0^2 \int_0^{4-2x} \int_0^{4-2x-z} 6xy\,dydzdx$, execute

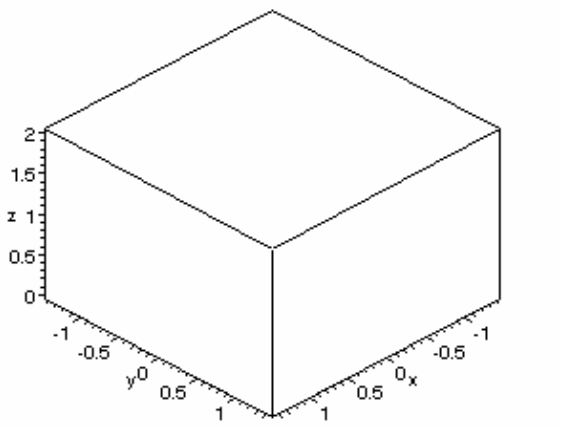
```
Tripleint(6*x*y,y=0..4-2*x-z,z=0..4-2*x,x=0..2);
value(%);
evalf(%);
```

Do you get the correct value?

3a. To draw a picture of the region Q between $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{4 - x^2 - y^2}$ execute

```
cylinderplot({[r,t,r],
[r,t,sqrt(4-r^2)]},r=0..sqrt(2),
t=0..2*Pi,axes=boxed);
```

to define the two plots over the same ranges for r and θ. Sketch the result in the box at right. How would you describe in words the shape of this region?



3b. The triple integral $\iiint_Q ze^{\sqrt{x^2+y^2}}\,dV$ would be written $\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{4-r^2}} rze^r\,dzdrd\theta$ when converted to cylindrical cöordinates. Use *Maple* to compute this integral.

4a. We can draw the "roof" of the solid using another command from the package that was loaded in Question **1a**. Because $z = \rho\cos\phi$ in spherical cöordinates, the equation $x^2 + y^2 + z^2 = 4z$ is equivalent to $\rho = 4\cos\phi$; execute (still using **t** for θ)

```
sphereplot(4*cos(phi),t=0..2*Pi,phi=0..Pi/4,axes=boxed);
```

and describe the result below.

4b. How would you describe the "floor" of the solid?

4c. Set up an integral giving the volume of the solid, and then evaluate it, recording the value below; use **rho** or some other convenient label to represent ρ in *Maple*.