Assignment 28: Partial Derivatives (12.3–7) **Name____________________**
Please provide a handwritten response.

1a. To define the function $f(x,y,z)=e^{2xy}-\frac{z^2}{y}+xz\sin y$ execute

```
f:=exp(2*x*y)-(z^2/y)+x*z*sin(y);
```

followed by `diff(f,y);` to find $f_y(x,y,z)$. Is the result correct?

1b. To find the second–order mixed partial derivative $f_{yx}(x,y,z)=\frac{\partial^2 f}{\partial x \partial y}(x,y,z)$ execute `diff(f,x,y);` and record the result below. (Note the order in which the variables are listed in the command.)

1c. Execute `diff(f,y,y);` to find $f_{yy}(x,y,z)=\frac{\partial^2 f}{\partial y^2}(x,y,z)$, followed by `subs(x=0.2,y=3,z=sqrt(7),%);` and `evalf(%);` to find $f_{yy}\left(-0.2,3,\sqrt{7}\right)$; record the results below.

2a. Execute `f:=(x,y)->x^3+3*x*y-y^3;` followed by

```
plot3d({f(x,y)},x=-0.5..1.5,y=-1.5..0.5,axes=boxed);
```

to draw the graph of $f(x,y)=x^3+3xy-y^3$ over $-0.5\le x\le 1.5$, $-1.5\le y\le 0.5$; is it clear from this plot what critical point(s) *f* has over this range, and of what type?

2b. Modify the preceding command as follows; as usual, no carriage returns!

```
plot3d({f(x,y)},x=-0.5..1.5,y=-1.5..0.5,
    axes=boxed,orientation=[45,90]);
```

Now tell below what you can see regarding critical points. What was the effect of the extra option?

2c. To calculate $\nabla f(x,y)$ execute `with(linalg);` followed by

```
G:=grad(f(x,y),[x,y]);
```

and record the result below.

Now execute the command `G[1];`. What does this command do?

2d. Because the critical points of *f* occur where $\nabla f(x,y) = \mathbf{0}$, execute

```
evalf(solve({G[1]=0,G[2]=0},{x,y}));
```

and record the result below. Are there really three different critical points? Do these points appear consistent with the graph in part **b**? To compute *f(*3,2*)* execute `f(3,2);`. Use this command to find the corresponding *z*–values for your critical points.

2e. To apply the second derivative test execute

```
fxx:=diff(f(x,y),x,x);
fyy:=diff(f(x,y),y,y);
fxy:=diff(f(x,y),x,y);
```

followed by

```
DSC:=fxx*fyy/fxy^2;
```

Execute the command `subs(x=2,y=3,DSC);` to evaluate equation `DSC` at the point (2,3). Use the equation `DSC` and the `subs` command to classify each of the critical points of *g* and record the results below.

3a. Define $f(x,y) = (x^2 - 3xy + 3y^2 + 4x)e^{-2x^2 - \frac{1}{2}y^2} + \sin\left(\frac{x+y}{100}\right)$ (!) by executing

```
f:=(x,y)->(x^2-3*x*y+3*y^2+4*x)*exp(-2*x^2-y^2/2)
        +sin((x+y)/100);
```

Modify the last command in Question **2a** to plot $f(x,y)$ over $-1.5 \le x \le 1.5$, $-3 \le y \le 3$. How many critical points does *f* seem to have over this range?

3b. Execute

```
with(plots); contourplot({f(x,y)},x=-1.5..1.5,y=-3..3);
```

and use your results so far to give below the rough cöordinates of each of the critical points and what type of critical point it is.

3c. Because the `solve` command cannot find the critical points for this function, execute

```
G:=grad(f(x,y),[x,y]);
fsolve({G[1]=0,G[2]=0},{x=-0.5,y=-0.1});
```

to find the exact cöordinates of the critical point near $(-0.5,-0.1)$. Change the starting point for *x* and *y* in `fsolve` to find the other critical points as well, and record the results below.