

Assignment 27: Functions of Two Variables (12.1–2)
Please provide a handwritten response.

Name _____

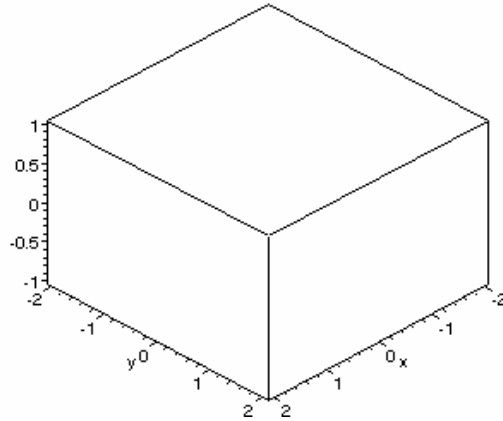
1a. To graph the function $f(x, y) = \sin(y - x^2)$ execute

```
f := (x, y) -> sin(y - x^2);
```

followed by

```
plot3d({f(x, y)}, x=-2..2, y=-2..2, axes=boxed);
```

Sketch the result in the box at right; rather than try to copy every line drawn by *Maple*, just use general outlines and shading to give the overall shape.

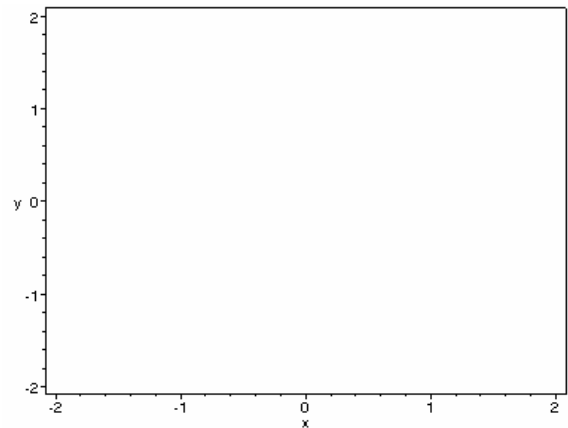


1b. Graph f over a wider range and describe the general appearance of the resulting surface.

1c. To draw a contour plot of f execute

```
with(plots);  
contourplot(f(x, y), x=-2..2,  
            y=-2..2, axes=boxed);
```

and sketch the result in the frame at right.



1d. Now execute the command

```
densityplot({f(x, y)}, y=-2..2, x=-2..2, axes=none);
```

how is the result more, and less, accurate than the preceding result?

2a. The fact that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2} = 0$ and that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$ does not exist can be detected using contour plots.

Execute

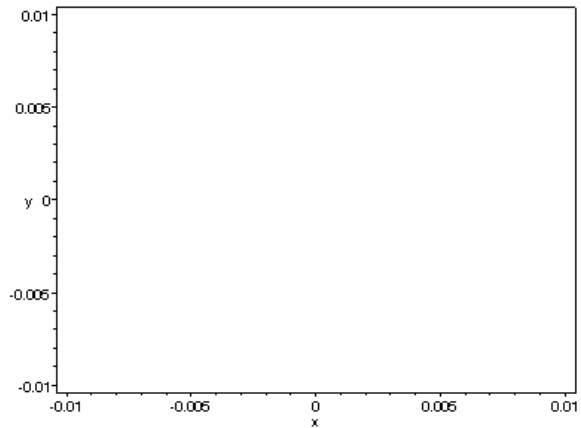
```
f := (x, y) -> x^2*y / (x^2+y^2);
```

followed by

```
contourplot(f(x, y),
x=-0.01..0.01, y=-0.01..0.01,
axes=boxed);
```

and sketch the result in the frame at right.

Execute this command again with `.01` replaced throughout by `.001`; does the pattern seem to change?



2b. How do these graphs support the conclusion that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2}$ exists?

2c. Now execute

```
f := (x, y) -> x^2 / (x^2+y^2);
```

followed by the `contourplot` command in part **a**, and sketch the result in the frame at right. Again, repeat this command with `.01` replaced throughout by `.001`; does the pattern seem to change here?

2d. How do these graphs support the conclusion that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$ does not exist?

2e. Based on contour plots, do you think that $\lim_{(x,y) \rightarrow (0,0)} \frac{x \sin y}{x^2 + y^2}$ exists? Explain your answer.