Assignment 26: Vector–Valued Functions, Part II (11.1–4) Name_____
Please provide a handwritten response.

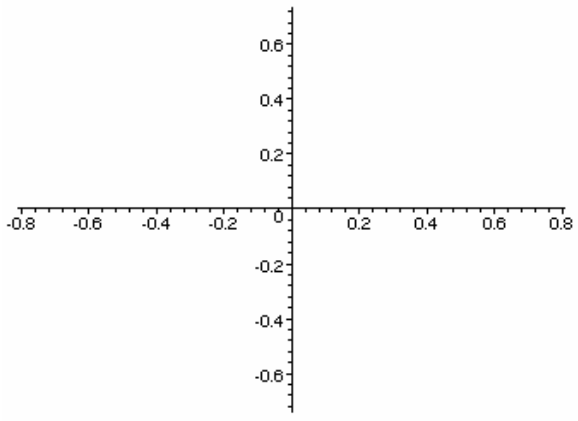
1a. To plot the Cornu spiral execute the following commands; the output refers to the "Fresnel integrals" of applied mathematics.

```
f:=int(cos(Pi*u^2/2),u=0..t);
g:=int(sin(Pi*u^2/2),u=0..t);
```

Then execute `r:=[f,g];` followed by

```
plot([op(r),t=-Pi..Pi]);
```

and sketch the result on the axes at right.



1b. Apply the `int` command to

```
sqrt(diff(f,t)^2+diff(g,t)^2)
```

to find the arc length of the curve from $t = 0$ to $t = c$, and record the result below. What does this say about this parameterization of this curve?

1c. To find the curvature of this curve at $t = c$; execute

```
with(linalg);dr:=diff(r,t);
T:=dr/norm(dr,2);
```

to find the unit tangent vector $\mathbf{T}(t)$, followed by

```
dT:=diff(T,t);
norm(subs(t=c,dT),2)/norm(subs(t=c,dr),2);
```

Does the result tell you much? Execute `simplify(%);`. Assume that all the absolute values are positive to further reduce the expression and record the result below. What does this say about the curve?

2a. Vector–valued functions in space are graphed using the `spacecurve` command; for example, to draw the graph of $\mathbf{r}(t) = \langle \cos t, \ln t, \sin t \rangle$ (denoted $\mathbf{f}_1(t)$ in Example 1.5 over $0.1 \le t \le 8\pi$, execute

```
r:=[cos(t),ln(t),sin(t)];
```

followed by

```
with(plots);spacecurve([op(r)],t=0.1..8*Pi,axes=boxed);
```

According to the Example, the result should look like Graph B; does it?

2b. Now execute the following modification of the preceding command:

```
spacecurve([op(r)],t=0.1..8*Pi,axes=boxed,numpoints=100);
```

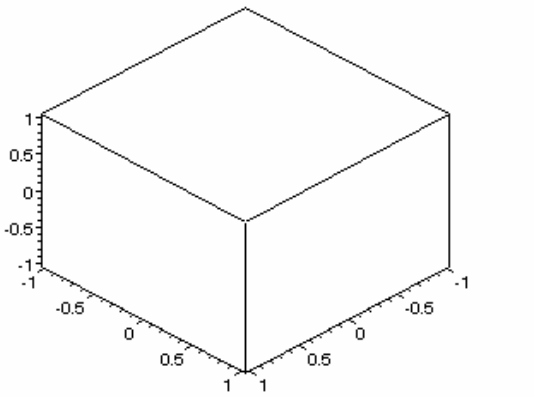
Does the result look more like Graph B? The **`numpoints`** option specifies the number of points that *Maple* computes to draw the graph. The default is 50. Click on the plot to see and try some of the options for the plot. You can select from the buttons or the menu.

2c. Now define **`r(t)`** by $r(t) = \langle \cos 5t, \sin t, \sin 6t \rangle$ and execute the command in part **b**, with **`0.1`** replaced by **`0`**. How would you describe the result? Do you think this is an accurate graph?

2d. Execute this command again with **`100`** replaced by **`500`**; do you think this is the "real thing"?

2e. Why was the graph in part **c** so poor? How did the **`numpoints`** option improve it?

3a. Sketch in the box at right the curve $r(t) = \langle \cos t, \sin t, \cos 2t \rangle,\ 0 \le t \le 2\pi$.



3b. At what point(s) on this curve do you think the curvature is greatest? Execute

```
crossprod(diff(r,t),diff(r,t,t));
```

followed by

```
k:=norm(%,2)/norm(diff(r,t),2)^3;
```

to define the curvature κ using Theorem 4.1.
Now use *Maple* to find the value(s) of *t* for which **`k`** is greatest, and record below the corresponding points on the curve. Was your conjecture correct?