Assignment 25: Vector–Valued Functions, Part I (11.1–3) Name____________________
Please provide a handwritten response.

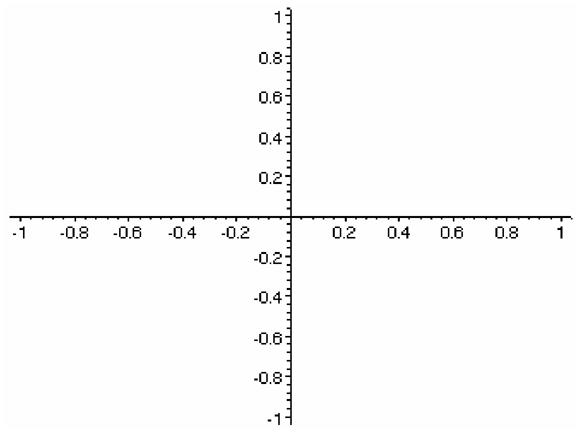
1a. Execute

```
r:=t->[cos(3*t),sin(2*t)];
```

to define the vector–valued function $\mathbf{r}(t)=\langle\cos 3t,\sin 2t\rangle$, $0\le t\le 2\pi$ and draw the graph of $\mathbf{r}(t)$ by executing

```
plot([op(r(t)),t=0..2*Pi]);
```

Sketch the resulting "Lissajous curve" on the axes at right.



1b. To list the points $\mathbf{r}(0)$, $\mathbf{r}\left(\frac{\pi}{4}\right)$, etc. execute

```
evalf(seq(r(n*Pi/4),n=0..8));
```

Mark these cöordinates, with their corresponding values of *t*, on the graph, and then draw arrows to show the orientation of the curve.

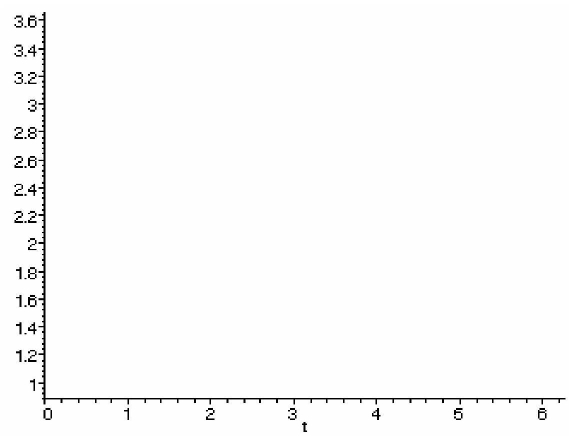
1c. Thinking of $\mathbf{r}(t)$ as representing the position of a moving point, execute

```
v:=diff(r(t),t);
```

to find the velocity vector $\mathbf{v}(t)=\mathbf{r}'(t)$, followed by

```
with(linalg);
speed:=norm(v,2);
```

to find the speed $\|\mathbf{v}(t)\|=\sqrt{\mathbf{v}(t)\cdot\mathbf{v}(t)}$. Sketch the graph of $\|\mathbf{v}(t)\|$ over $0\le t\le 2\pi$ on the axes at right; based on this graph, does the moving point ever stop?



1d. Now execute `r1:=r(t+3*sin(t));` to define the reparameterization $\mathbf{r}_1(t)=\mathbf{r}(t+3\sin t)$, $0\le t\le 2\pi$ and execute `plot([op(r1),t=0..2*Pi]);` to draw the graph of $\mathbf{r}_1(t)$ over $0\le t\le 2\pi$. What is the subtle difference between this graph and that in part **a**? Draw the graph of $\mathbf{r}_1(t)$ also over $0\le t\le 2\pi-0.05$; what light does this shed?

1e. Execute `vr1:=diff(r1,t);` to find $vr1(t) = r1'(t)$. Imitate part **c** to plot over $0 \le t \le 2\pi$ the speed of a point moving under $\mathbf{r}_1(t)$, note the approximate values of t where the speed is zero, and apply `fsolve` to `speed=0` to find more accurate values. Then use `subs` and `r1` to find the cöordinates of these points where $\mathbf{r}_1(t)$ "stops", and record the results below. (You may have to use `evalf` to convert to a decimal.)

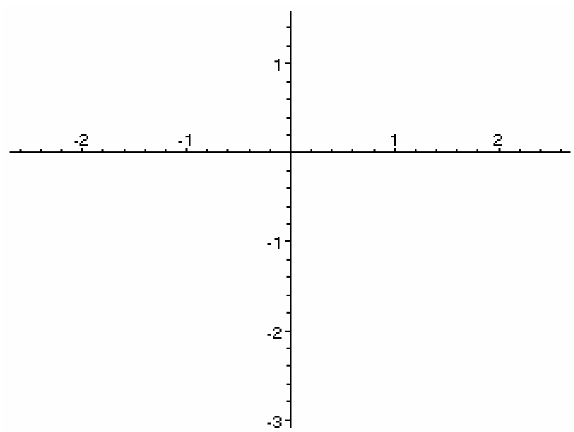
1f. Suppose you knew only the graph of a vector–valued function; so far, can we say for sure whether there are any points at which the function "stops"?

1g. Finding the points where this curve crosses itself, which amounts to finding pairs of numbers s and t such that $\mathbf{r}(s) = \mathbf{r}(t)$, would be difficult symbolically but is easy using `fsolve` provided suitable starting values for s and t are given. For example, execute

```
fsolve({op(1,r(t))=op(1,r(s)),op(2,r(t))=op(2,r(s))},
              {s,t},{s=0..1,t=4..5});
```

to start near $s = 0.6$ and $t = 4$. Evaluate `r(t)` at the values and record the result below.

2a. Clear variables and redefine `r(t)` as $\mathbf{r}(t) = \langle 2\cos t + \sin 2t, 2\sin t + \cos 2t \rangle$, $0 \le t \le 2\pi$ and sketch the graph of $\mathbf{r}(t)$ on the axes at right.



2b. Find and mark on the graph any stationary points of $\mathbf{r}(t)$, as above.

2c. Execute `r1:=r(t+sin(t));` to define the vector–valued function $\mathbf{r}_1(t) = \mathbf{r}(t + \sin t)$, $0 \le t \le 2\pi$. Execute `plot([op(r1),t=0..2*Pi]);` to draw the graph of $\mathbf{r}_1(t)$ over $0 \le t \le 2\pi$. Check for stationary points; how do the results compare with part **b**?

2d. Repeat part **c** with $\mathbf{r}_1(t)$ redefined by $\mathbf{r}_2(t) = \mathbf{r}(t^2)$, $0 \le t \le \sqrt{2\pi}$.

2e. Would you now modify your answer to Question **1f**? How?