

Assignment 23: Polar Coordinates (9.4–7)
Please provide a handwritten response.

Name _____

1. Define the limaçon $r = f(\theta) = 4\cos\theta - 2$ by executing

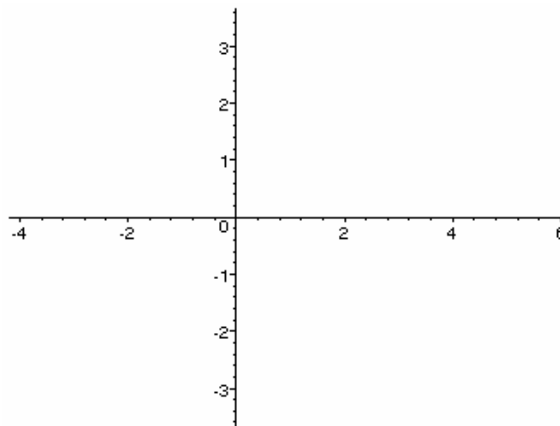
```
f:=q->4*cos(q)-2;
```

and define the ellipse $r = g(\theta) = \frac{4}{2 + \cos\theta}$ (Theorem 7.2) by executing

```
g:=q->4/(2+cos(q));
```

Next load in a package by executing

```
with(plots);
```



Execute the following command to graph the curves above:

```
polarplot({f(q),g(q)},q=0..2*Pi);
```

and sketch the result on the axes at right. How many points of intersection do these two curves have?

2a. To find the points of intersection we begin by numerically solving the equation $f(\theta) = g(\theta)$ for θ using the `evalf(solve)` command; execute

```
evalf(solve(f(q)=g(q),q));
```

Maple includes results involving the imaginary unit i , which despite their *Maple* interest we will ignore throughout this assignment. Record the other results below; how many eligible values of θ did *Maple* find? Also execute

```
s1:=fsolve(f(q)=g(q),q);
```

and tell why we used `fsolve` to find `s1` instead.

2b. Now execute `[g(s1),s1]`; to list the point (in polar coordinates) corresponding to the value of θ found above. Execute the command `s2:=fsolve(f(q)=g(q),q=-s1)`; to find the point that is symmetric to `s1`. Now execute `[g(s2),s2]`; and label the points on the graph above. (It's helpful to remember that 1 radian is $\frac{180}{\pi} \approx 57^\circ$.) Have we found all of the points of intersection yet? Could we have used `f` rather than `g`?

2c. We should also solve the equation $f(\theta) = -g(\theta + \pi)$ for θ to find points of intersection; execute

```
s3:=fsolve(f(q)=-g(q+Pi),q);
```

and, imitating part **b**, find point **s4** and label the points on the graph.

3a. Below write down a sum of definite integrals that gives the area of the region lying both inside the ellipse and outside of the limaçon; also shade this region on the graph.

3b. Use **int** if possible, and **evalf** if necessary, to evaluate these integrals, and record the total area below. Based on the graph, is your answer plausible?

3c. Repeat parts **a** and **b** for the region lying inside the large loop of the limaçon (ignore the small loop) but outside the ellipse. Be sure to keep careful track of what parts of each curve correspond to which values of θ , and be ready to apply **evalf** to the output of **int** if you don't get a numerical result at first.

4a. Implement the slope formula for f by executing

```
df:=q->D(f)(q);
```

```
ls:=(df(a)*sin(a)+f(a)*cos(a))/(df(a)*cos(a)-f(a)*sin(a));
```

where a represents a particular value of θ and **ls** stands for "limaçon slope". Next execute

```
h1:=evalf(solve(ls=0,a));
```

to find the values of θ for which the tangent line to the limaçon are horizontal, and record the result below. Also find the corresponding points on the curve as in part **b** and mark them on the graph.

4b. The graph suggests that the two curves might be orthogonal at two of their intersection points. Modify the first command in part **a** to define a function **es** giving the slope for the ellipse and decide whether this is the case. (Recall that perpendicular lines have negative reciprocal slopes unless they are horizontal and vertical.)