Assignment 22: Parametric Equations (9.1–3) **Name____________________**
Please provide a handwritten response.

1a. Execute

```
x:=Pi*t-0.6*sin(Pi*t);
```

and

```
y:=2*t+0.4*sin(Pi*t);
```

followed by

```
plot([x,y,t=0..1]);
```

to plot the curve for $\begin{cases} x = \pi t - 0.6\sin \pi t \\ y = 2t + 0.4\sin \pi t \end{cases}$, and sketch the result on the axes at right. (Note that here the *y*–axis is pointing upward as usual, so our skier is skiing uphill!)

1b. Execute `evalf({subs(t=0.5,x),subs(t=0.5,y)});` to find the point on the curve corresponding to $t = \frac{1}{2}$, mark this point on the curve with a large dot and draw the line tangent to the curve there. What do you estimate the slope of this line to be?

1c. Execute

```
dy:=diff(y,t);dx:=diff(x,t);
evalf(subs(t=0.5,dy)/subs(t=0.5,dx));
```

to find this slope exactly, and record the result below.

1d. Execute `Int(sqrt(dx^2+dy^2),t=0..1);` and `evalf(%);` to find the length of the curve in Exercise 16, Section 9.3, and record the result in the table below. Repeat the process for Exercises 13 and 14.

Exercise	Arc length	Time
13		
14		
16		

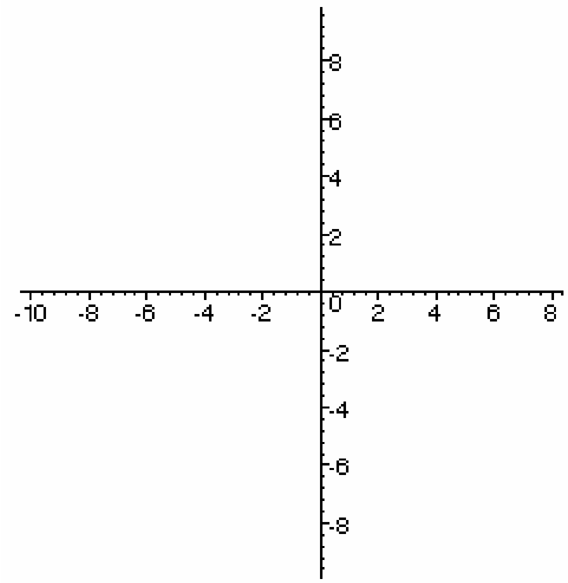
1e. The time needed to ski this curve can be calculated using the formula in Example 3.3 (taking $k = 1$ for convenience); execute `Int(sqrt((dx^2+dy^2)/y),t=0..1); evalf(%);` and record the result in the table.

1f. Now modify the commands in part **a** and re–execute the commands in parts **c, d** and **e** to complete the table for Exercises 13-15 in Section 9.3 as well. Based on these examples, does there seem to be any correlation between the arc length and the time?
1g. In the same way find the arc length and time for the cycloid of Example 3.3; in both categories, where does it rank among the other three curves considered so far?

2a. Define the parametric curve $\begin{cases} x = 8\cos t - 2\cos 4t \\ y = 8\sin t - 2\sin 4t \end{cases}$ in *Maple* as in Question **1a**, and then execute

```
plot([x,y,t=-Pi..Pi],scaling=constrained);
```

to draw this curve over $-\pi \le t \le \pi$; sketch the result on the axes at right. (Also try it without the **`scaling=constrained`** option and tell what this option does.)



2b. Where are the "corner" points of this curve? By Theorem 2.1, at such points both $x'(t)$ and $y'(t)$ must be zero. Execute

```
xlist:={solve(diff(x,t)=0,t)};
ylist:={solve(diff(y,t)=0,t)};
```

to list the values of t for which $x'(t) = 0$ and $y'(t) = 0$. Record the results below.

2c. To find the values of t common to the lists execute the command

```
corners:=xlist intersect ylist;
```

and record the results below. Finally execute

```
seq([subs(t=op(i,corners),x),subs(t=op(i,corners),y)],i=1..2
);
```

and **`evalf(%);`** to find the cöordinates of the corner points. Did *Maple* find all the corners?

2d. ***Maple*** gave only positive answers for the solution of $y'(t) = 0$. Execute the command **`plot(diff(y,t),t=-Pi..Pi);`** to see the negative solutions. Execute **`ylistneg:={seq(-1*op(i,ylist),i=1..4)};`** to multiply the **`ylist`** by -1. Combine these with the original **`ylist`** by executing the command **`ylist:=ylist union ylistneg;`**. Now execute the commands in part **2c**, change **`i=1..2`** to **`i=1..3`** to find the cöordinates of the corner points and label them on the graph.

2e. Find the "corner" point(s) on the curve $\begin{cases} x = 2\cos 2t + \cos 4t \\ y = 2\sin 2t + \sin 4t \end{cases}$. How many are there?