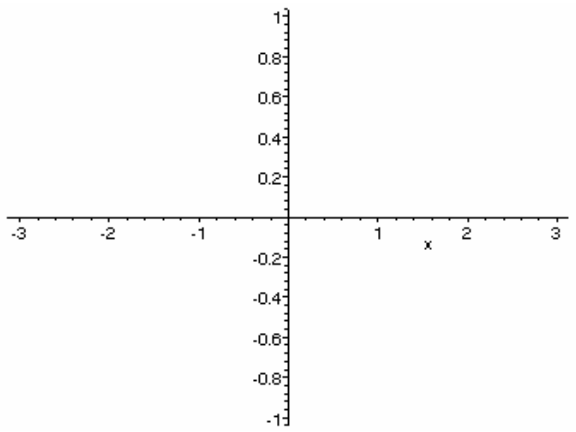
Assignment 21: Fourier Series (8.9) **Name**_____
Please provide a handwritten response.

1. Execute **`?signum;`**. Describe *signum(x)* below. Then execute **`f:=x->signum(x);`** to define *f*, and use the **`plot`** command to sketch the graph of *f* over $-\pi \le x \le \pi$; sketch the result on the axes at right.



2a. We can find the Fourier coefficients of *f* in Maple. To apply the Euler–Fourier formulas directly, execute the following commands, noting the use of **`*`** between **`k`** and **`x`** to indicate multiplication:

```
a0:=(1/Pi)*int(f(x),x=-Pi..Pi);
a:=k->(1/Pi)*int(f(x)*cos(k*x),x=-Pi..Pi);
b:=k->(1/Pi)*int(f(x)*sin(k*x),x=-Pi..Pi);
```

Record this last result below, and explain why the first two results came out as they did.

2b. Now construct the partial sum **`F5`** of the Fourier series of *f* by executing

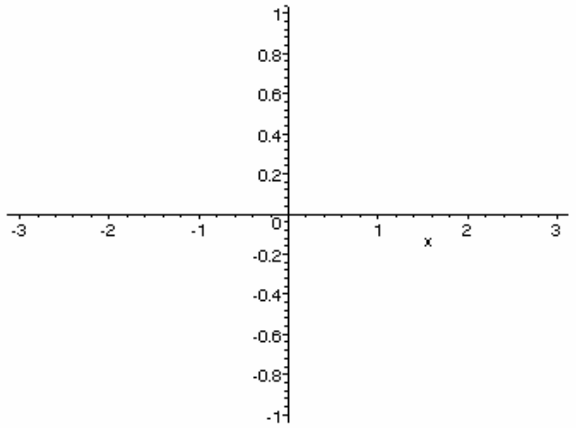
```
F5:=a0/2+sum(a(k)*cos(k*x)+b(k)*sin(k*x),k=1..5);
```

Record the result below. Execute **`plot([f(x),F5],x=-Pi..Pi);`** to graph **`f`** and **`F5`** together over $-\pi \le x \le \pi$ and sketch the result on your graph above.

2c. To measure how well this partial sum approximates *f* execute

```
plot(f(x)-F5,x=-Pi..Pi);
```

and sketch the result on the axes at right. Roughly, what is the largest value, positive or negative, of the error in this approximation?



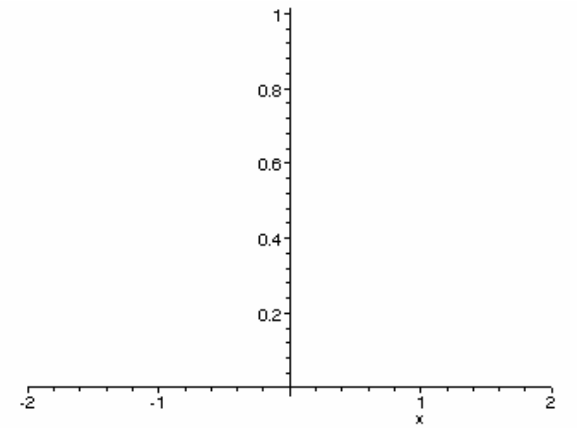
2d. Repeat parts **b** and **c** with **`5`** replaced by **`50`** and explain below why we might naturally expect our answer about the error in part **c** to become smaller. Does it?

2e. Experiment with still larger values of *n*, as computer memory allows; are you able to find a partial sum of the Fourier series of *f* for which the maximum error in the approximation over $-\pi \le x \le \pi$ is smaller than your results so far? (When *n* is large it will be helpful to attach a colon to the end of the command in part **b** to suppress the output on the screen.)

3a. Execute **`?floor;`** and describe *floor(x)* below. Then execute

```
g:=x->x-floor(x);
```

and graph *g* over $-2 \le x \le 2$; sketch the result on the axes at right. Do the vertical lines have any significance?



3b. The period *T* of this function is not 2π; what is it? Try to use the Euler–Fourier formulas to modify the first command in Question **2a** to define **`a0`** for *g*. Was this successful? Why?

3c. Actually *Maple* has the capability to build a procedure to find the **`N`** partial sums of a Fourier series for a function **`f`**, centered at zero with period **`T`**. Execute the following commands. (Don't worry when you get the message "`Warning, premature end of input`." This message will disappear when you execute the last line.)

```
FT:=proc(f,T,N)L:=T/2;a0:=1/L*int(f,x=-L..L);
    ak:=1/L*int(f*cos(k*Pi*x/L),x=-L..L);
    bk:=1/L*int(f*sin(k*Pi*x/L),x=-L..L);
a0/2+sum(ak*cos(k*Pi*x/L)+bk*sin(k*Pi*x/L),k=1..N);end;
```

To find the 5th partial sum for *g(x)*, execute the command **`FT(x-floor(x),1,5);`** Why is it that the constant term is nonzero but there are no cosine terms in the result?

4. Execute **`FT(x^2,6,5);`** to find the Fourier expansion of $f(x) = x^2$ on [-3,3]. What is the coefficient of $\cos\left(\frac{5\pi x}{3}\right)$?