Assignment 20: Infinite Series (8.2–7) **Name____________________**
Please provide a handwritten response.

To increase the number of decimal places displayed for this assignment, first execute the command `Digits:=20;`.

n	$S_n = \sum_{k=1}^{n} \frac{1}{k^{0.9}}$	$S_n = \sum_{k=1}^{n} \frac{5}{k^{1.1}}$
10		
50		
100		
500		
1000		

1a. To find the partial sum S_{10} of the infinite series $\sum_{k=1}^{\infty} \frac{1}{k^{0.9}}$ execute

```
sum(1/k^0.9,k=1..10);
```

and record the result in the table. By changing the `10` to `50`, etc. complete the second column of the table.

1b. Likewise modify the command in part **a** to find the partial sums of the infinite series $\sum_{k=1}^{\infty} \frac{5}{k^{1.1}}$ and complete the third column. Notice that in each row, the entry in the second column is smaller than that in the third; can this be the case for all *n*? Why?

1c. Add one more row to the bottom of the table corresponding to *n* = 5000 and fill it in; are the results consistent with your answer to part **b**? Execute the command `restart;` to reset *Maple* to its default values.

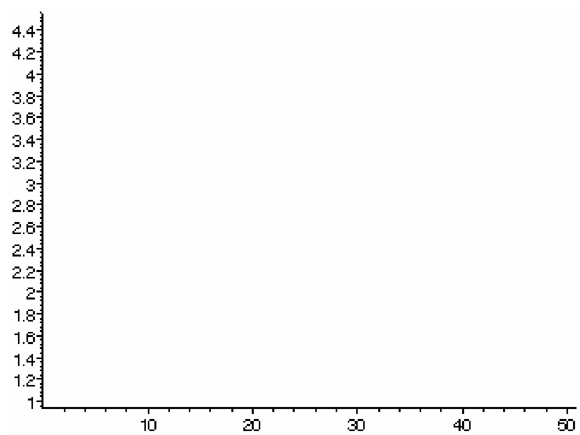
2a. The text explains that the harmonic series $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges. To get an idea of how quickly or slowly it does so, execute

```
s:=n->sum(1./k,k=1..n);
```

followed by

```
psums:=[seq([n,s(n)],n=1..50)]:
```

to construct a list called `psums` of ordered pairs $\left(n, \sum_{k=1}^{n} \frac{1}{k}\right)$ where the "*y*–value" is the *n*th partial sum of the harmonic series. Then execute `plot([psums]);` and roughly sketch the result on the axes at right.



2b. Repeat the last two commands in part **a** with `50` replaced by `500`; would you say that the partial sums are approaching ∞ quickly?

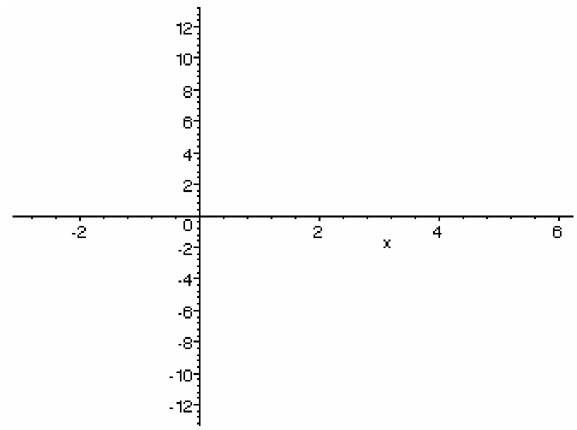
3a. To find the Taylor polynomial with $c = \dfrac{\pi}{2}$ and $n = 4$ for $\cos x$ execute
`taylor(cos(x),x=Pi/2,4);` and record the result below; what do you think the final term means?

3b. We can remove this final term using the `convert` command; execute
`tp:=convert(%,polynom);` and enter the result below.

3c. Now plot the cosine function and the Taylor polynomial over $-\pi \le x \le 2\pi$ by executing

```
plot([cos(x),tp],x=-Pi..2*Pi);
```

Sketch the result on the axes at right, labeling the graphs; on roughly what interval are the two graphs indistinguishable on your computer screen?

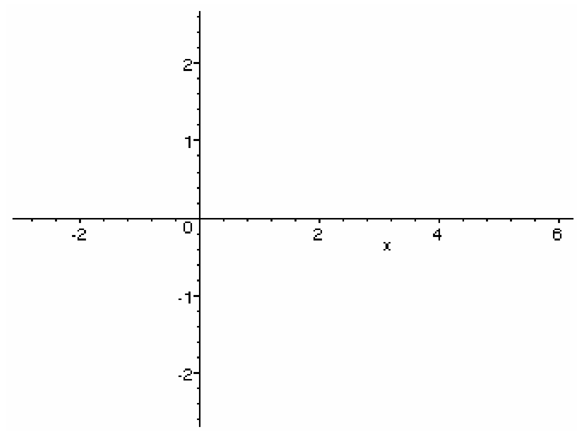


3d. Change the **`4`** in part **a** to **`8`** and then execute the commands in parts **a**–**c** once again. Sketch the graph of the new Taylor polynomial, label it, and answer the question in part **c** again.

3e. To measure the error in this Taylor approximation, execute

```
plot(cos(x)-tp,x=-Pi..2*Pi);
```

and sketch the result on the axes at right. How large (positive or negative) does the error become, and for what value(s) of *x* is the error greatest?



3f. By increasing *n* still further while keeping everything else the same, can we reduce the maximum error in part **e** to less than 0.1? Experiment to find how large a value of *n* is needed.

3g. Try to answer part **f** with $\cos x$ changed to $\tan^{-1} x$ (denoted **`arctan(x)`**), *c* to 0 and the interval to $-1.5 \le x \le 1.5$. Can you find *n* large enough? Why?