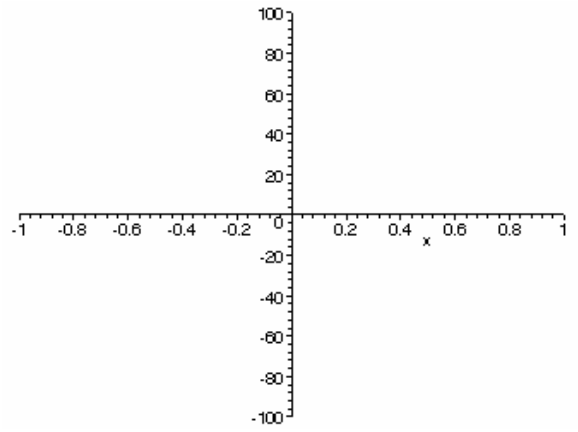
Assignment 19: Improper Integrals (6.6) **Name**_____

Please provide a handwritten response.

1a. The integrals $\int_{-1}^{1} \frac{1}{x}\,dx$ and $\int_{-1}^{1} \frac{1}{x^2}\,dx$ are both improper and divergent. Execute

```
plot([1/x,1/x^2],x=-1..1,-100..100);
```

to sketch the functions $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$ over $-1 \le x \le 1$ and sketch the results on the axes at right, labeling the graphs.



1b. To try to evaluate $\int_{-1}^{1} \frac{1}{x}\,dx$, execute

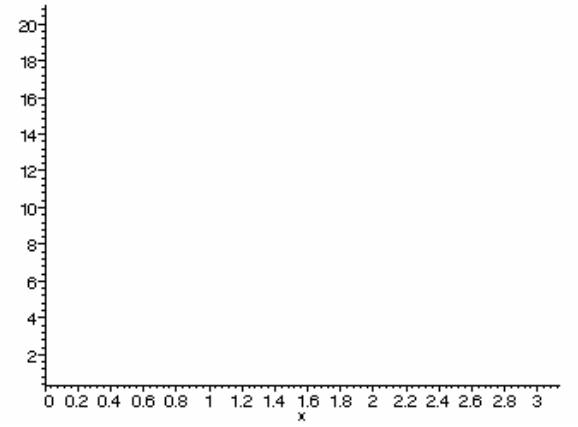
```
int(1/x,x=-1..1);
```

and record the result below. Does *Maple* give a value for this integral? Execute **`evalf(%);`** to determine if *Maple* can find a numeric answer.

1c. Likewise evaluate $\int_{-1}^{1} \frac{1}{x^2}\,dx$ by executing **`int(1/x^2,x=-1..1);`**, and record the result below.

1d. Does *Maple* confirm that each of these integrals is divergent? Explain carefully below why *Maple* nevertheless gives very different results for them.

2a. Use the **`plot`** command to sketch the graph of $f(x) = \frac{1}{\sqrt{1+\cos x}}$ over $0 \le x \le \pi$ on the axes at right, and explain why the integral $\int_0^{\pi} \frac{1}{\sqrt{1+\cos x}}\,dx$ is improper.



2b. Execute the command

```
int(1/sqrt(1+cos(x)),x=0..Pi);
```

and record the result below; does this integral converge?

2c. Repeat part **a** but with the command

```
int(1/(1+cos(x))^.5,x=0..Pi);
```

and record the result below; does this integral converge?

2d. Repeat part **a** but with the command

```
int(1/(1+cos(x))^(1/2),x=0..Pi);
```

and record the result below; so, does this integral converge?

3a. To evaluate the integral $\int_0^\infty xe^{-2x}dx$ execute

```
int(x*exp(-2*x),x=0..infinity);
```

and record the result below.

3b. In the same way evaluate $\int_0^\infty x^2e^{-2x}dx$. Execute the command

```
plot([x*exp(-2*x),x^2*exp(-2*x)],x=0..infinity);
```

and explain how these two integrals could be the same.

4a. To evaluate the integral $\int_{-\infty}^\infty \frac{1}{1+x^2}dx$ execute

```
int(1/(1+x^2),x=-infinity..infinity);
```

and record the result below.

4b. In the same way evaluate $\int_{-\infty}^\infty \frac{1}{2+x^2}dx$ and $\int_{-\infty}^\infty \frac{1}{3+x^2}dx$. What conclusions can you draw?