Assignment 17: Euler's Method (7.3) **Name____________________**
Please provide a handwritten response.

1a. To apply Euler's method to the differential equation $y' = \sin y - x^2$, first define $f(x,y) = \sin y - x^2$ by executing

```
f:=(x,y)->sin(y)-x^2;
```

In *Maple* functions of two or more variables are handled similarly to functions of one variable; for example, execute **`f(-3,Pi/2);`** to find $f\left(-3, \frac{\pi}{2}\right)$ and record the result below; is it correct?

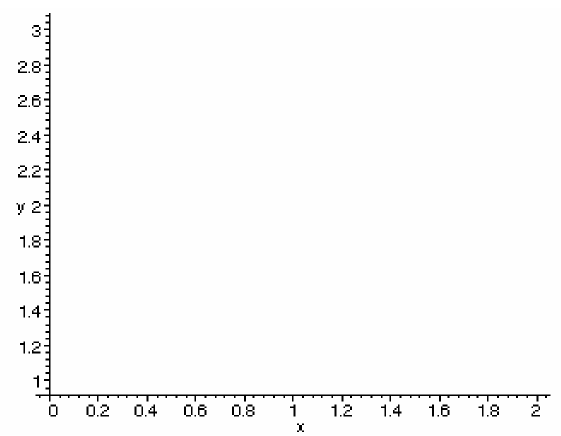
1b. To draw a direction field using *Maple* we must first load in a package; execute

```
with(plots);
```

and

```
fieldplot([1,f(x,y)],x=0..2,y=1..3);
```

Roughly sketch the result on the axes at right. (The **`fieldplot`** command draws at the point (x,y) an arrow in the plane whose slope is $\frac{f(x,y)}{1}$, or $f(x,y)$; following the arrows therefore leads to a curve which is a solution to the differential equation.)



1c. We will store the steps of Euler's method in what *Maple* calls a "list"; at each step we add one more ordered pair to our list using the **`op`** command. For example, execute (using square brackets, not parentheses!)

```
sample:=[[-2,3]];
```

representing a list consisting of the one ordered pair $(-2,3)$, and then execute

```
sample:=[op(sample),[-3,Pi/2]];
```

What did this do to **`sample`**?

1d. Execute **`[3,-2]+[4,7];`** and record the result below; what do you think happened here? (This "list addition" will come in handy in part **g**.)

1e. At each step of Euler's method we must evaluate $f(x,y)$ at the last ordered pair in our list, in order to compute the next ordered pair. This will take just a bit of fancy stuff: Execute **`sample[2];`** and tell below what you think the **`[]`** command does to a list.

1f. Because *Maple* will not do much with **`f(sample[2]);`** (try it), we need specify the x and y values to "apply" **`f`** to the two numbers in **`sample[2]`**. Earlier we calculated $f\left(-3,\frac{\pi}{2}\right)$; now execute **`f(sample[2,1],sample[2,2]);`** and record the result below. Is it correct?

1g. Now we can go ahead with the Exercise. We will begin our list with the initial condition $y(0)=2$ given in the Exercise. To approximate $y(2)$ we will take 20 steps starting from $x_0=0$. Using the **`proc`** command we now can generate our list of Euler steps all at once; execute

```
P:=proc(n,h)m:=[[0,2.0]];
f:=(x,y)->sin(y)-x^2;
for i from 1 to n do m:=[op(m),m[i]+[h,h*f(m[i,1],m[i,2])]];
od;end;
```

(Careful! You must enter m as **`m:=[[0,2.0]];`**, using the decimal on the 2, to generate a list in decimal form.) Now execute the command **`euler1:=P(20,0.1);`** to set the step size h = 0.1 and compute the first 20 iterations. What are $y(1)$ and $y(2)$ according to this approximation?

1h. The command **`plot`** will plot the points of our list. Execute

```
plot([euler1]);
```

1i. Execute **`euler2:=P(40,0.05);`** to set the step size to h = 0.05 and compute the first 40 iterations. Tell below the values of $y(1)$ and $y(2)$ according to this approximation.

1j. Now execute

```
plot([euler2]);
```

followed by **`plot([euler1,euler2]);`** to combine the graphs, and sketch the result on your direction field in part **b**, labeling both curves clearly.