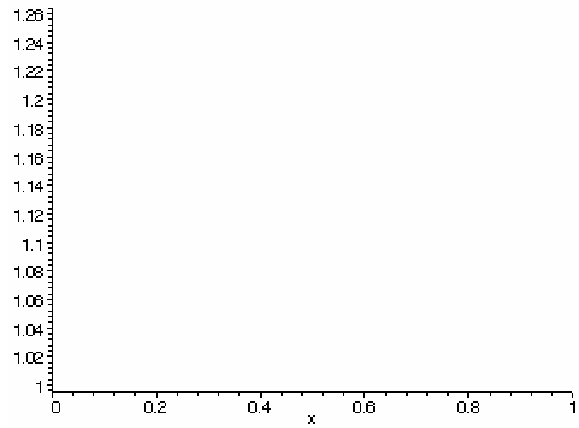
Assignment 13: Numerical Integration (4.7) **Name____________________**
Please provide a handwritten response.

1. To apply the estimation rules of this section to $\int_0^1 \sqrt[3]{x^2+1}\,dx$. Execute

```
f:=x->(x^2+1)^(1/3);
```

to define $f(x)=\sqrt[3]{x^2+1}$ and then use the `plot` command to graph f over $0 \le x \le 1$; sketch the result on the axes at right. Based on this graph, what would be your estimate of $\int_0^1 \sqrt[3]{x^2+1}\,dx$? (Be careful about where the origin is!)



2a. To apply the Midpoint Rule to this integral we must load the student package just as in the preceding assignment. Execute the command **`with(student);`**. We will use values of a=0, b=1, and n=10. Execute the commands **`a:=0;`**, **`b:=1;`**, and **`n:=10;`**.

2b. The midpoint of each interval $[x_{i-1}, x_i]$ is given by $c_i = \frac{x_{i-1}+x_i}{2}$. Find the Midpoint approximation $\sum_{i=1}^{n} f(c_i)\Delta x$ by executing

```
evalf(middlesum(f(x),x=a..b,n));
```

Is this result plausible? Enter it in the table on the next page.

3. To calculate the Trapezoid Rule approximation $\sum_{i=1}^{n} \frac{f(x_{i-1})+f(x_i)}{2}\Delta x$, execute

```
evalf(trapezoid(f(x),x=a..b,n));
```

and enter the result in the table.

4. To calculate the Simpson's Rule approximation, execute

```
evalf(simpson(f(x),x=a..b,n));
```

Enter the result in the table.

n	Midpoint	Trapezoid	Simpson's
10			
20			
50			

5. Execute `n:=20;` to replace n=10 with n=20 and re–execute all of the commands in Questions **2b**–**4** in order. Enter the results in the table. Which of the three approximations did not change when n was increased?

6. Repeat Question **5** but with `n:=50;` instead, and enter the results in the table. Are the three approximations drawing closer together as n increases?

7. *Maple* can accurately calculate difficult definite integrals like the one we are studying. Execute the command

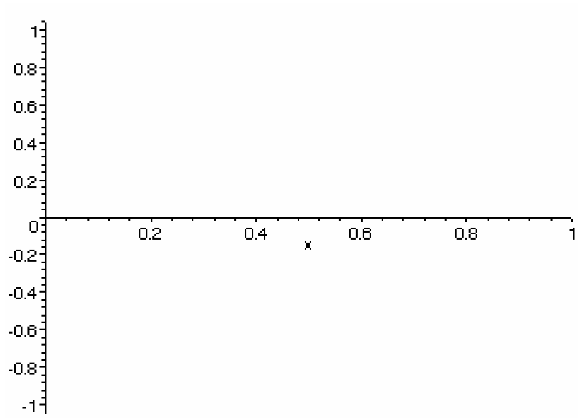
```
evalf(int(f(x),x=0..1));
```

and record the result below. Based on this, which of the three approximation methods applied above was most accurate?

8a. You can almost always take the results of `evalf` to be completely accurate. However, there are some unusual situations that cause trouble for `evalf`. Execute

```
g:=x->sin(1/x);
```

to define $g(x) = \sin\frac{1}{x}$ and then use the `plot` command to draw the graph of g over $[0,1]$, and sketch the result (as best you can!) on the axes at right.



8b. Execute `evalf(int(g(x),x=0..1));` to calculate $\int_0^1 g(x)dx$ and describe what happens below. Do you think the numerical result given is trustworthy?