Assignment 12: Integration and Riemann Sums (4.1–4)   Name____________________
Please provide a handwritten response.

1a. The **`int`** command is used to find both indefinite and definite integrals. Execute

```
f:=x->4*x-2*sqrt(x);
```

followed by

```
int(f(x),x);
```

to find the indefinite integral $\int \left(4x - 2\sqrt{x}\right)dx$ and record the result below. Is the answer correct? Note *Maple* omits the arbitrary constant "$+c$".

1b. Next execute **`f:=x->2*x^3/(x^4+1);`** and

```
F:=int(f(x),x);
```

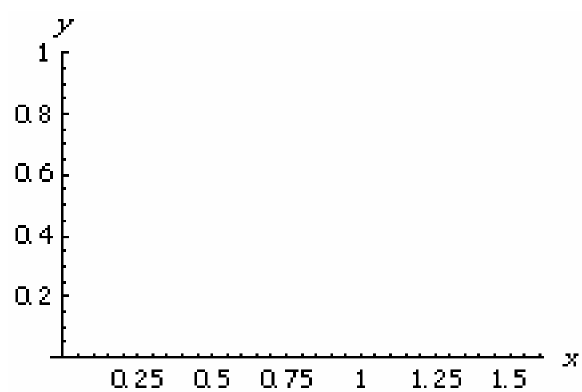
to calculate $F(x) = \int \frac{2x^3}{x^4+1}dx$; record the result below.

1c. By definition of antiderivative, what should $F'(x)$ be? Execute **`diff(F,x);`** and record the result below; is it correct?

2a. To approximate the area under the graph of $f(x) = \cos x$ on the interval $\left[0, \frac{\pi}{2}\right]$ execute

```
f:=x->cos(x);
```

and then use the **`plot`** command to graph f over $[0, \pi/2]$. (Remember that π is denoted by **`Pi`** in *Maple*.) Sketch the result on the axes at right.



2b. Use $n = 50$ rectangles in our approximation; moreover, in this case, our endpoints a and b are given by $a = 0$ and $b = \pi / 2$. Execute in order the commands **`a:=0;`** , **`b:=Pi/2;`** , **`n:=50;`** and **`deltax:=evalf((b-a)/n);`**. What value for Δx did *Maple* give? Is this correct?

2c. The Riemann sum $\sum_{i=1}^{n} f(x_i)\Delta x$ for right–hand evaluation can be found using the **`rightsum`** command. To use this command, we must load the student package; execute

```
with(student);
evalf(rightsum(f(x),x=a..b,n));
```

and record the result below. Is this a plausible approximation to the area?

2d. The Riemann sum for left–hand evaluation is $\sum_{i=1}^{n} f(x_{i-1})\Delta x$. Execute

```
evalf(leftsum(f(x),x=a..b,n));
```

and record the result below. Is your answer greater or less than your result in part **d**? Why should this be so?

2e. Likewise, the Riemann sum for midpoint evaluation is $\sum_{i=1}^{n} f\left[\frac{1}{2}(x_{i-1}+x_i)\right]\Delta x$. Execute

```
evalf(middlesum(f(x),x=a..b,n));
```

and record the result below.

2f. Execute **`n:=100;`** in order to replace n=50 by n=100 and re–execute all of the commands in Question **2**. Do the three approximations in parts **c**–**e** become more spread out or closer together? Is this what you would expect?

3. The exact value of the area we approximated in Question **2** is given by $\int_0^{\pi/2} \cos x \, dx$. The **`int`** command can also find such definite integrals: Execute

```
int(f(x),x=0..Pi/2);
```

and record the result below. Based on the evidence we have already gathered, is this answer plausible?