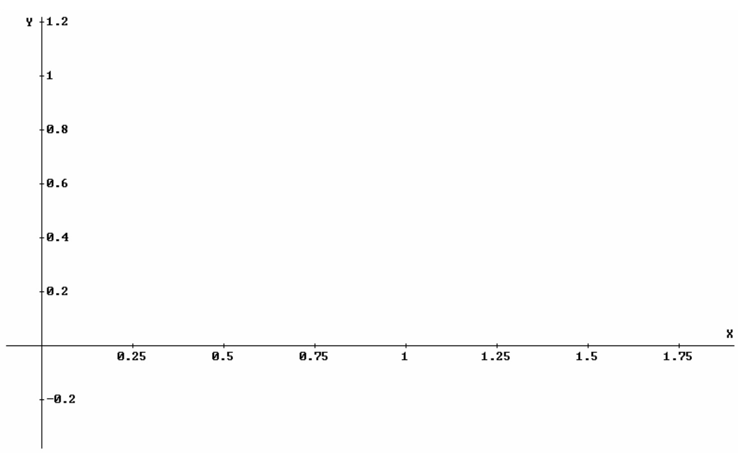
Assignment 12: Integration and Riemann Sums (4.1-4)   Name___________________
Please provide a handwritten response

1a. The int command, or ∫, is used to find both indefinite and definite integrals. **Author** f(x):=4x-2sqrt(x) then **Author** int(f(x),x) to find the indefinite integral $\int\left(4x-2\sqrt{x}\right)dx$. Use = to simplify the integral and record the result below. (We could also use ∫ and specify indefinite integral, with variable x and constant zero.)

1b. Next **Author** f(x):=2x^3/(x^4+1) and g(x):=int(f(x),x) to calculate $\mathrm{g}(x)=\int\frac{2x^3}{x^4+1}dx$. Again use = to simplify and record result below.

1c. By definition of antiderivative, what should $g'(x)$ be? **Author** and simplify g'(x) then record result below; is it correct?

2a. We now want to approximate the area under the graph of $f(x)=\cos(x)$ on the interval $\left[0,\frac{\pi}{2}\right]$. **Author** this function and plot its graph. Zoom to an appropriate view and sketch the result on the axes at right.



2b. We want to approximate this area using n = 50 rectangles; moreover, in this case, our endpoints a and b are given by a = 0 and $b=\frac{\pi}{2}$. **Author** (one at a time) the following constants: a:=0, b:=pi/2, n:=50, and deltax:=(b-a)/n. Use ≈ to approximate the value of Δx. Record the value *Derive* gives below. Is this correct?

2c. It will be convenient to enter $c_i = a + i\Delta x$ as a separate *Derive* function. **Author** c(i):=a+i*deltax then simplify using = or ≈. Record result below.

2d. The Riemann sum $\sum_{i=1}^{n} f(c_i)\Delta x$ for right-hand evaluation can be found using either the sum command or Σ. **Author** sum(f(c(i))*deltax,i,1,n). Simplify this using ≈ and record the result below. Is this a plausible approximation to the area?

2e. The Riemann sum for left-hand evaluation is $\sum_{i=1}^{n} f(c_{i-1})\Delta x$.

Author sum(f(c(i-1))*deltax,i,1,n) and use ≈ to simplify. Record the result below. Is the answer greater or less than the result in part d? Why should this be so?

2f. Likewise, the Riemann sum for midpoint evaluation is $\sum_{i=1}^{n} f\left[\frac{1}{2}(c_{i-1} + c_i)\right]\Delta x$.

Author sum(f(1/2(c(i-1)+c(i)))*deltax,i,1,n). (Be very careful with the parenthesis!) Use ≈ to simplify and record the result below. How does this answer relate to those from parts **d** and **e**?

2g. Would we get more accurate results if we used 100 rectangles instead of just 50? Let's try it. **Author** n:=100 and simplify the three sums again. Since we do not want to retype each sum, use the mouse pointer to select the right-hand sum. Then click ≈ to approximate the right-hand sum. Since we have changed the value of *n*, we should get a slightly different approximation this time. Record the new approximation below. Next, highlight each of the other sums and record their new approximations below. (Clearly label them.) Do the three new approximations become more spread out or closer together? Is this what you would expect?

3. The exact value of the area in Question 2 is given by $\int_0^{\pi/2} \cos(x)\,dx$. The int or ∫ can be used to find such definite integrals. **Author** and ≈ int(f(x),x,0,pi/2). Record result below. Based on the evidence we have already gathered, is this answer plausible?