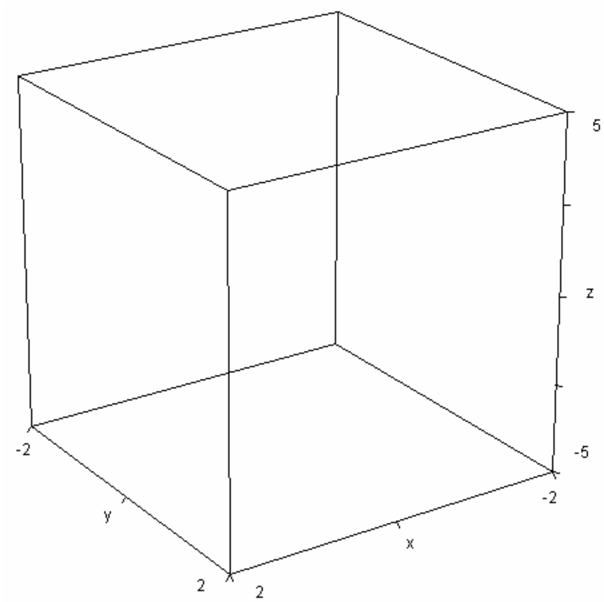
Assignment 30: Triple Integrals (13.4-7) **Name___________________**
Please provide a handwritten response.

1a. We want to calculate the surface area of that portion of the paraboloid $f(x,y) = 4 - x^2 - y^2$ that lies above the triangular region R in the xy-plane with vertices at the points (0,0), (1,1), and (1,0).

Author f(x,y):=4-x^2-y^2 and plot this in a 3D-Plot window. Set the ranges of the *x*- and *y*-axes to [-2,2] and sketch the results in the box at right.



1b. The surface area is given by the following integral

$$\int_0^1 \int_y^1 \sqrt{[f_x(x,y)]^2 + [f_y(x,y)]^2 + 1}\, dxdy\,.$$

First define the integrand by **Author**ing ingr(x,y):=sqrt(dif(f(x,y),x)^2+dif(f(x,y),y)^2+1) and then set up the double-integral by **Author**ing int(int(ingr(x,y),x,y,1),y,0,1). Simplify this double integral by clicking **=**; record the result below.

1c. Discuss the result in **1b**. Was *Derive* successful here? Is this a "real" answer? Explain.

1d. Does converting to polar coordinates help? After converting the limits **Author** int(int(ingr(rcos(t),rsin(t)) r, r, 0, sec(t)),t,0,pi/4) to set-up the integral in polar coordinates. (Note that use of t in place of θ; t is simply more convenient!) Simplify the integral using **=**. Then convert to decimal by clicking **≈**. Record both results below. Do they seem reasonable?

2. To enter the triple integral $\int_0^2 \int_0^{4-2x} \int_0^{4-2x-z} 6xy \; dy\, dz\, dx$ carefully **Author** int(int(int(6xy,y,0,4-2x-z),z,0,4-2x),x,0,2). (Note the order in which everything goes! We could also have used ∫ three times to enter the triple integral much like we did with a double integral.) Simplify the integral by clicking =. Record the result below.

3. The triple integral $\iiint\limits_Q z\, e^{\sqrt{x^2+y^2}}\, dV$ would be written $\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{4-r^2}} r\, z\, e^r \; dz\, dr\, d\theta$ when converted to cylindrical coordinates. Enter and simplify this integral. Record both an exact and a decimal value for the integral.

4a. We want to examine the solid bounded below by $x^2 + y^2 + z^2 = 4z$ and above by $z = \sqrt{x^2 + y^2}$. First, we will attempt to draw the “roof” of the solid. The “roof” is formed by the equation $x^2 + y^2 + z^2 = 4z$. Since *Derive* will only graph explicit 3D-Plots, we must solve this equation for *z*. Highlight the expression and click , select variable *z*, and click **Solve**. Record the results below. Select the largest equation, since we want a “roof”, and plot its graph in a 3D-Plot window. Write a short description of the surface.

4b. In spherical coordinates, the equation $x^2 + y^2 + z^2 = 4z$ is equivalent to $\rho = 4\cos(\phi)$. The equation $z = \sqrt{x^2 + y^2}$ forms the “floor” of the solid. What does this simplify to? How would we describe it?

4c. Set up an integral in spherical coordinates giving the volume of the solid; record it below. Use *Derive* to find both the exact and approximate decimal values. (In *Derive*, we can enter rho for ρ and phi for ϕ.)