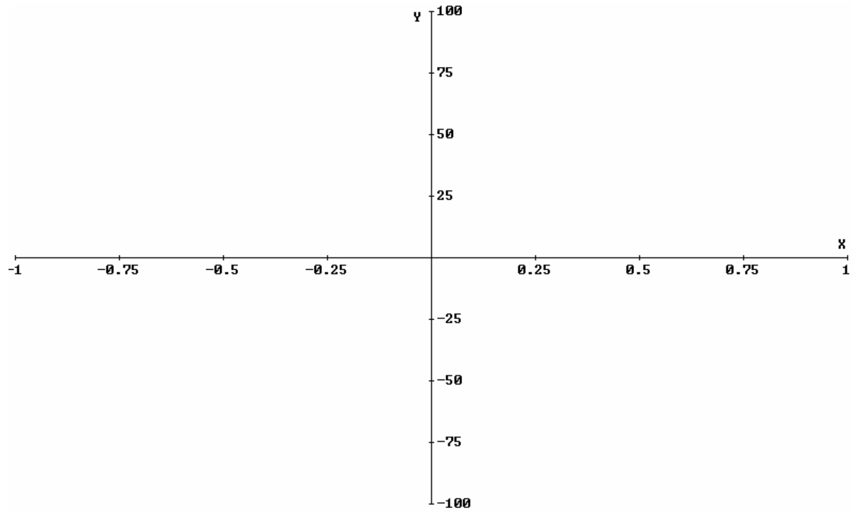
Assignment 19: Improper Integrals (7.7) **Name___________________**
Please provide a handwritten response.

1a. The integrals $\int_{-1}^{1} \frac{1}{x}\,dx$ and $\int_{-1}^{1} \frac{1}{x^2}\,dx$ are both improper and divergent. **Author** and plot the functions $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$ together. Select an appropriate view and sketch the results on the axes at right; label the functions.



1b. To try to evaluate $\int_{-1}^{1} \frac{1}{x}\,dx$, **Author** int(1/x,x,-1,1) and simplify using **=**. (We could have **Author**ed 1/x then used **∫** to set up the definite integral with a **lower limit** of -1 and an **upper limit** of 1.) Does *Derive* give a value for this integral?

1c. Likewise evaluate $\int_{-1}^{1} \frac{1}{x^2}\,dx$ by **Author**ing int(1/x^2,x,-1,1) and using **=** to simplify. Record the result below.

1d. Does *Derive* confirm that each of these integrals is divergent? Explain carefully below why you think *Derive* gives the two different results.

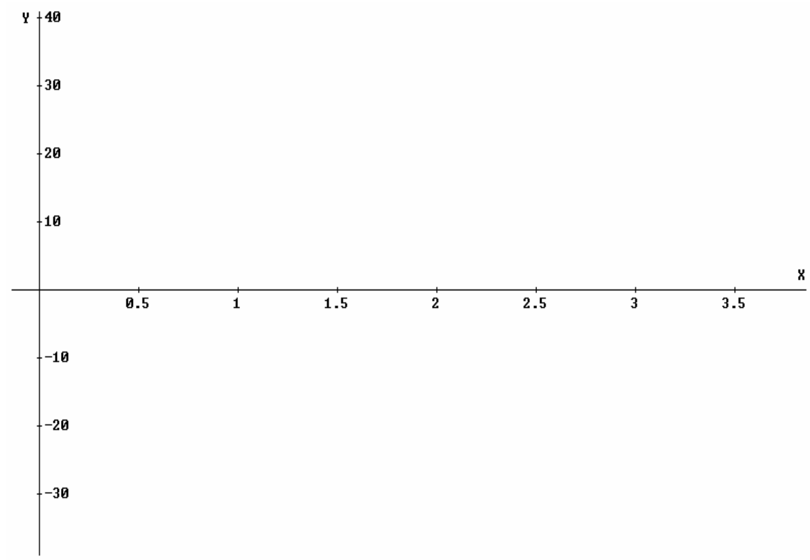
1e. What does *Derive* give for $\int_{-1}^{2} \frac{1}{x}\,dx$? What command was entered? Record *Derive*'s result below; is it correct? Why?

2a. Examine the function $f(x) = \dfrac{1}{\sqrt{1+\cos(x)}}$.

Author 1/sqrt(1+cos(x)) and plot its graph over $0 \le x \le \pi$. Sketch the result on the axes at right. Explain why the integral

$$\int_0^{\pi} \frac{1}{\sqrt{1+\cos(x)}}\,dx \text{ is}$$

improper.



2b. Let *Derive* evaluate the integral by **Author**ing int(1/sqrt(1+cos(x)), x,0,pi). Use ≈ to simplify the result. Carefully note *Derive*'s response then record the result below; does this integral seem to converge? Explain.

2c. What happens if the function is entered as 1/(1+cosx)^(0.5)? Use = to simplify. Did *Derive* give the same answer after integrating? Explain *Derive*'s result.

3a. To evaluate the integral $\int_0^{\infty} xe^{-2x}dx$, **Author** int(xexp(-2x),x,0,inf) and use = to simplify. Record the result below; is it correct?

3b. In the same way, evaluate $\int_0^{\infty} x^2e^{-2x}dx$. Record the answer below and explain how these two answers can be the same.