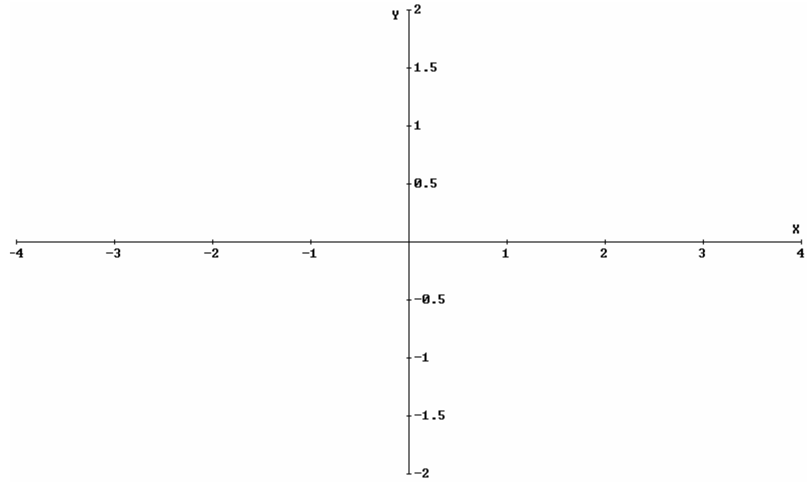
Assignment 21: Fourier Series (8.9) **Name___________________**
Please provide a handwritten response.

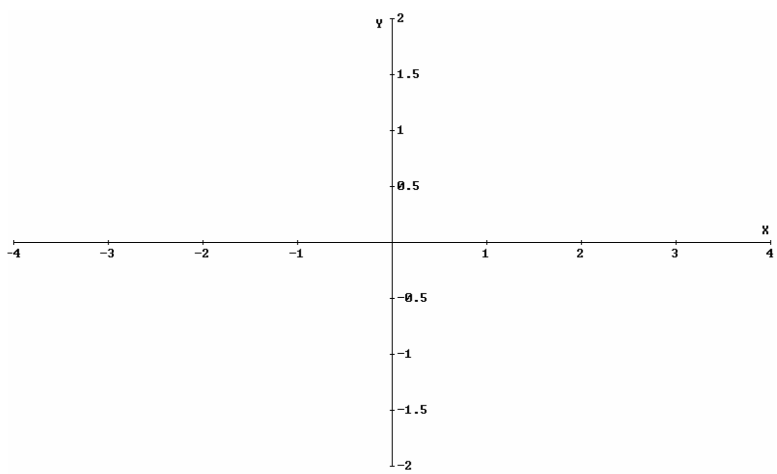
1. **Author** f(x):=sign(x) and use *Derive* to graph the function over $-\pi \leq x \leq \pi$. Sketch the result on the axes at right.



2a. We can find the Fourier coefficients of *f* in at least two different ways in *Derive*. To apply the Euler-Fourier formulas directly, **Author** asubzero:=1/pi int(f(x),x,-pi,pi) then **Author** a(k):=1/pi int(f(x)cos(kx),x,-pi,pi) and b(k):=1/pi int(f(x)sin(kx),x,-pi,pi). Use = to simplify all three results. Record all results below and explain why the first two results came out as they did.

2b. Now construct the partial sum *f5*(*x*) of the Fourier series of *f* by **Author**ing f5(x):=asubzero/2 + sum(a(k)cos(kx) + b(k)sin(kx), k, 1, 5). Simplify the result using = and record it below. Also graph *f5*(*x*) and sketch the result on the axes from **1** above.

2c. To measure how well this partial sum approximates *f*, **Author** err(x):=f(x)-f5(x) then simplify the result with =. Select **Window→New 2D-plot Window** from the main menu then click to plot the graph of *err*(*x*) by itself. Zoom to an appropriate view and sketch the result on the axes at right. Roughly, what is the largest value, positive or negative, of the error in this approximation? (Remember to use only $-\pi \leq x \leq \pi$.)



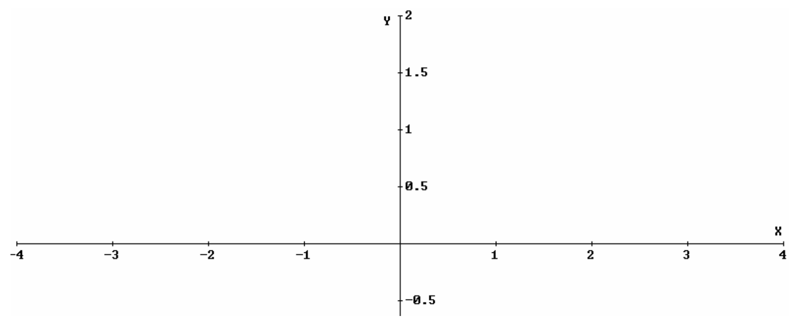
2d. Repeat parts **b** and **c** with **5** replaced by **50** and explain below why we might naturally expect our answer about the error in part **c** to become smaller. Does it?

2e. Experiment with still larger values of *n*, as computer memory allows; are you able to find a partial sum of the Fourier series of *f* for which the maximum error in the approximation over $-\pi \le x \le \pi$ is smaller than our results so far?

2f. The Gibbs phenomenon is the tendency of approximating partial sums of Fourier series to badly undershoot or overshoot the limit function near jump discontinuities (places where the limit function changes rapidly.) How does this apply to our results in parts **c** - **e**?

3a. Click **Help→Index** then type floor. Highlight floor and click **Display**. Scroll down until you find this command. What does it do? Record an explanation below then close the help window and return to the algebra window.

Author g(x):= x - floor(x) and graph its plot over $-2 \le x \le 2$. (You may need to delete old plots.) Zoom to an appropriate view and sketch the results on the axes at right.



3b. The period of this function is not 2π; what is it? We could again use the Euler-Fourier formulas to find the Fourier series of this function, but we will explore *Derive*'s built-in method of calculating Fourier series. First, return to the algebra window and click **File→Load→Utility File** and select "Int_apps.mth" from the list of files. Click **Open** to load this file. (You may need to change to *Derive*'s **Math** folder to find the file.)

We can think of *g* as being a periodic function with period 1 over $0 \le x \le 1$. **Author** fourier(g(x),x,0,1,4) and simplify the result with **=**. Why are there no cosine terms in the result? Plot this partial sum on the axes in **3a**.

4. What is the coefficient of $\cos\left(\frac{5\pi x}{3}\right)$ in the Fourier expansion of $f(x) = x^2$ on the interval [-3,3]? Use at least six terms in the partial sum.