Assignment 24: Vectors (10.1-5) **Name___________________**
Please provide a handwritten response.

1a. In *Derive*, vectors (like many other things) are represented as lists; for example, **Author** a:=[3,-2] and b:=[4,1] to represent the vectors $\mathbf{a} = \langle 3,-2 \rangle$ and $\mathbf{b} = \langle 4,1 \rangle$. Then **Author** 5a - 3b to calculate 5**a** - 3**b.** Simplify with = and record the result below; is it correct?

1b. **Author** and simplify a.b to calculate the dot product $\mathbf{a} \bullet \mathbf{b}$ of **a** and **b** and record the result below. (The . character is just the normal period.) Is the result correct?

1c. In *Derive*, we can find the magnitude of the vector $\|\boldsymbol{a}\|$ using the formula $\|\boldsymbol{a}\| = \sqrt{\mathbf{a} \bullet \mathbf{a}}$; **Author** and simplify sqrt(a.a); record the result below. Also, try **Author**ing abs(a) and simplify the result. Are the results the same? (This is another way to calculate the magnitude of a vector.)

2a. Similarly, three-dimensional vectors are represented by lists of length three; **Author** c:=[4,-1,7] and d:=[3,3,-5] to represent the vectors $\mathbf{c} = \langle 4,-1,7 \rangle$ and $\mathbf{d} = \langle 3,3,-5 \rangle$. Then **Author** and simplify 4c+d and (c+2d).c and record the results below. (Clearly denote the answers!)

2b. **Author** and simplify a.c and describe the result below; what is the problem with this operation?

2c. The cross product is represented in *Derive* by the **cross** command; **Author** cross(c,d) then use = to simplify the result and record it below. Are **c** and **d** parallel?

3a. **Author** a:=[-1,0,2] and b:=[2,-3,1] to define vectors parallel to the two lines given by $x = 3 - t, y = 4, z = -2 + 2t$ and $x = 1 + 2s, y = 7 - 3s, z = -3 + s$. Now find the dot and cross products of **a** and **b**; record below. Are the lines parallel, perpendicular or neither?

3b. Find the angle between the lines $x = 4 - 2t, y = 3t, z = -1 + 2t$ and $x = 4 + s, y = -2s, z = -1 + 3s$. Note that in *Derive* $\cos^{-1}(x)$ is denoted acos(x) and expresses the answer in radians and in exact form. Dividing by "deg" will convert the output to degree measure.

4a. Define **a** and **b** to be vectors parallel to the two lines $x = 3 + t, y = 3 + 3t, z = 4 - t$ and $x = 2 - s, y = 1 - 2s, z = 6 + 2s$. Use the cross product to show that the two lines are not parallel. Thus, they either intersect or are skew. Record the cross product below.

4b. The solve command can detect whether the lines meet, because this is equivalent to finding whether a system of three linear equations in the two unknowns *s* and *t* has a solution. **Author** solve([3 + t = 2 - s, 3 + 3t = 1 - 2s, 4 - t = 6 + 2s], [s, t]) then simplify with **=**; record the result below. To what point in space do these values of *s* and *t* correspond?

4c. Determine if the lines $x = 1 - 2t, y = 2t, z = 5 - t$ and $x = 3 + 2s, y = -2, z = 3 + 2s$ are parallel, skew, or intersect.

5a. The solve command is also very useful for finding parametric equations for the line of intersection, if any, of two given planes; here the situation is reversed and we are solving a system of two linear equations in the three unknowns *x*, *y*, and *z*. To find the intersection of the planes $3x + y - z = 2$ and $2x - 3y + z = -1$, **Author** solve([3x + y - z = 2, 2x - 3y + z = -1], [x, y, z]) then use **=** to simplify the result; record this below as a set of parametric equations. (*Derive* uses @*n* as a parameter where *n* is an integer. We should use *t* as the parameter, however.)

5b. Find the line of intersection of the planes $x - 2y + z = 2$ and $x + 3y - 2z = 0$; record your conclusions below.

6a. Confirm that the lines $x = 4 + t, y = 2, z = 3 + 2t$ and $x = 2 + 2s, y = 2s, z = -1 + 4s$ intersect and find the point of intersection.

6b. Use the cross product to find a normal vector for the plane containing the lines given in **6a**; record the result below. Now write down an equation for this plane.