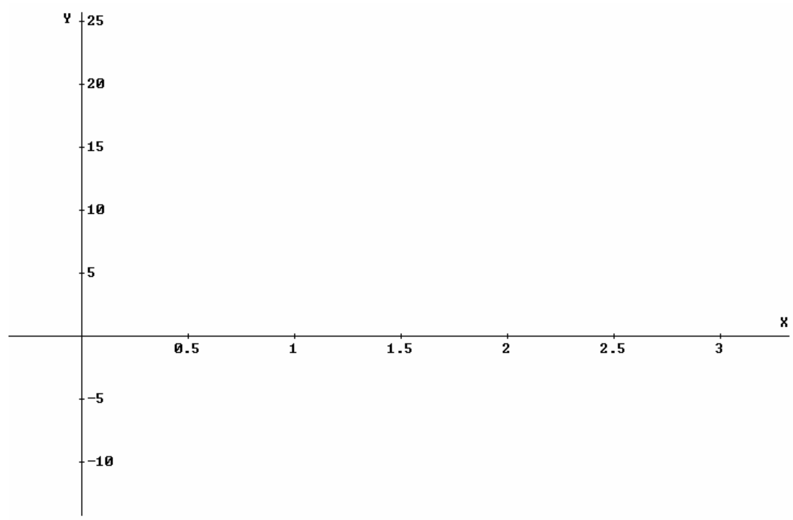
Assignment 8: Derivatives of Explicit Functions (2.1-7)   Name___________________
Please provide a handwritten response.

1a. We know the derivative of $f(x) = 3x^3 + 2x - 1$ is $f'(x) = 9x^2 + 2$. To carry out this calculation in *Derive,* first **Author** f(x):=3x^3+2x-1; then **Author** f '(x). Simplify the result using =. Record the result below; did *Derive* find the derivative correctly?

1b. Try calculating the derivative using ∂ and **Simplify**. Are the results the same?

1c. The slope $m_{\tan}$ of the line tangent to the graph of *f* at, say, *x* = 1 is given by $f'(1)$. **Author** and simplify f '(1) to see that $m_{\tan}$ =11. Also, find *f*(*1*) to see that *y* = 4 when *x* = 1.



Verify that the equation of our tangent line is $y = 11(x-1) + 4 = 11x - 7$.

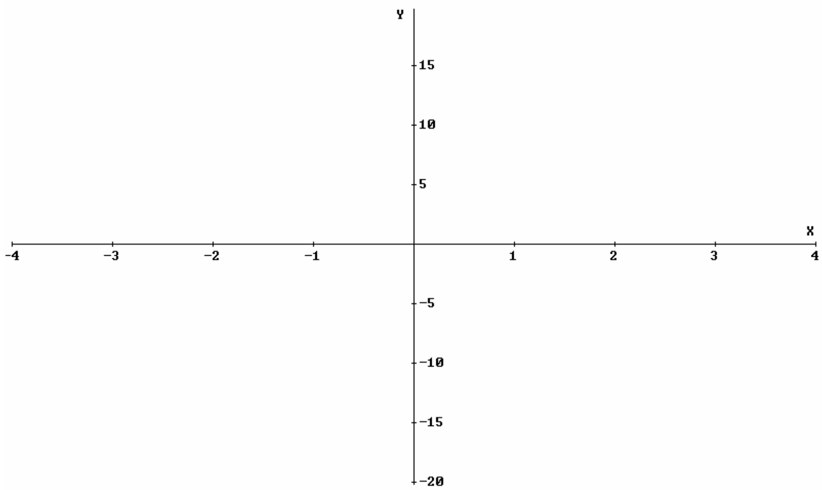
Author t(x):=11x-7 and plot *f*(*x*) and *t*(*x*) together. Zoom to an appropriate view and sketch the result on the axes above. Does the tangent line look as though its slope is 11? Why?

2a. To find the second derivative f'' of *f*, **Author** f "(x). (The double-prime symbol " consists of the <u>single-quote (or apostrophe) twice</u>, not the double-quote once!) Simplify the result and record it below; is it correct? (We can also use ∂, just specify 2 for order.)

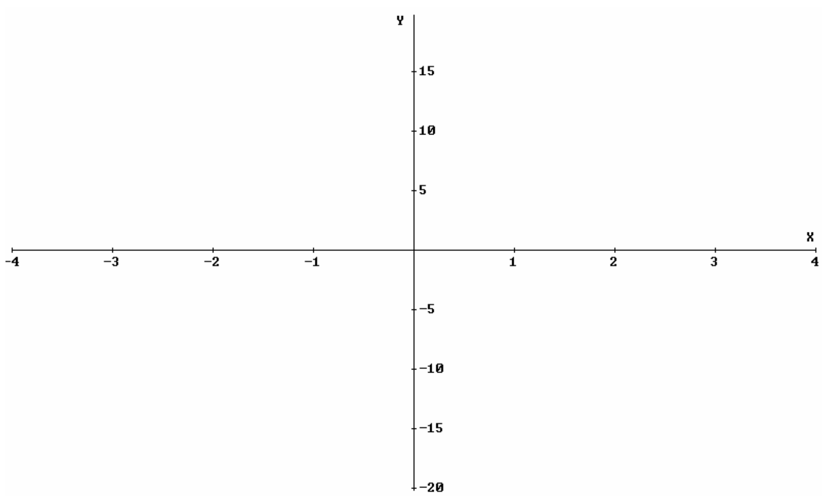
2b. Now, enter $g(x) = \sin\left(\frac{2x}{x+1}\right)$ by **Author**ing g(x):=sin(2x/(x+1)). Find $g''(x)$ by **Author**ing g"(x). Record the simplified result below; is it correct? Would you want to calculate this by hand?!

3a. **Author** f(x):=x^2*exp(sin(x)) to define $f(x) = x^2 e^{\sin(x)}$. Calculate the first derivative of *f*(*x*) by **Author**ing and simplifying f '(x). Record the result below; what rules and formulas presented in the text did *Derive* need to calculate the derivative?

3b. Plot $f'(x)$. Choose an appropriate view and sketch the result on the axes at right.



3c. According to the definition of the derivative; if *h* is a small fixed number, then the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ should be close to $f'(x)$, and so their graphs should lie close together. For the moment, let's choose *h* = 0.5. Keep the graph of $f'(x)$ and use  to switch to the algebra window. Next, **Author** and plot r(x):=(f(x+0.5)-f(x))/0.5 (careful with the parentheses!). Sketch the two plots on the axes at right, using a dotted curve for the graph of $r(x)$.



3d. Change from *h* = 0.5 to *h* = 0.4 in the definition of *r*(*x*) and try **3c** again. Are the two graphs closer together? Can you still tell them apart?

3e. Experiment with smaller and smaller values of *h* until the graphs of $f'(x)$ and *r*(*x*) become indistinguishable on your computer screen. How small does *h* have to be for this to happen?