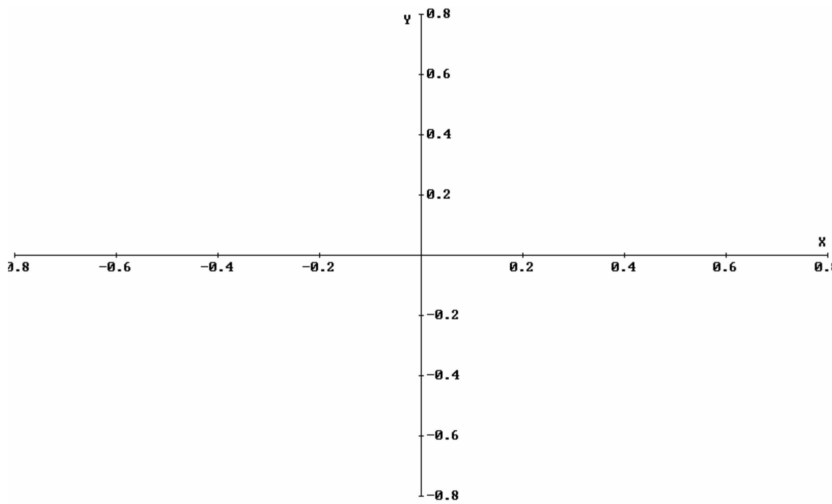
Assignment 26:Vector-Valued Functions, Part II(11.1-4) Name__________________
Please provide a handwritten response.

1a. To plot the Cornu spiral execute the following commands; the output refers to the "Fresnel integrals" of applied mathematics.

Author f(t):=int(cos(pi u^2/2),u,0,t) and g(t):=int(sin(pi u^2/2),u,0,t) then create a vector-valued function using **Author** r(t):=[f(t),g(t)]. Plot this curve over $-\pi \le t \le \pi$, zoom to an appropriate view, and sketch the result on the axis at right.



1b. Apply either ∫ or the int command to sqrt(f '(t)^2 + g'(t)^2) to find the arc length of the curve from t = 0 to t = c. Simplify the result using = and record it below. What does this say about this parameterization of this curve?

1c. Create a function called *ut* to calculate the unit tangent vector, **T**(t), of **r**(t); **Author** ut(t):=r'(t)/abs(r'(t)). Highlight the expression on the right of the definition and click = to redefine **ut**(t) in a different form. (*Derive* simply works "better" if we help it simplify things!) Find the unit tangent vector at the point corresponding to $t = \pi/3$ by **Author**ing ut(pi/3); use = to simplify and record the result below.

1d. We will use the formula $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$ to find the curvature κ of this curve at $t = c$; **Author** and simplify abs(ut'(c))/abs(r'(c)); record the result below. What does this say about the curve and its curvature; what happens as t increases?

2a. Find the unit tangent and principal unit normal vectors to the curve determined by $\mathbf{r}(t) = \langle \sin(2t), \cos(2t), t \rangle$ at the point corresponding to $t = \dfrac{\pi}{3}$. It is sometimes helpful to visualize these curves. Click to switch to a 3D-Plot window; click again to plot the curve. *Derive* automatically selects the interval for *t*. (Is this good?)

Author ut(t):=r'(t)/abs(r'(t)) and simplify the right side as in **1c**. To calculate the principal unit normal vector, **Author** un(t):=ut'(t)/abs(ut'(t)) and simplify the right side. To calculate the vectors we are looking for, **Author** and simplify ut(pi/3) and un(pi/3). Record the results below.

2b. What can we say about the curvature of $\mathbf{r}(t)$? This time, we will calculate curvature using $\kappa = \dfrac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$. We begin by **Author**ing k(t):=abs(cross(r'(t),r''(t)))/(abs(r'(t)))^3; simplify the right side of this function. What does this result mean? Look at the curve again and explain this result.

3. Let $\mathbf{r}(t) = \langle \cos(2t), t, \sin(2t) \rangle$. Find the unit tangent and principal unit normal vectors at the points $t = 0$ and $t = \dfrac{\pi}{2}$; find the curvature at $t = \dfrac{\pi}{2}$. Record and label all results.

4a. Suppose $\boldsymbol{r}(t) = \langle \cos(t), \ln(t), \sin(t) \rangle$. Find the unit tangent and the principal unit normal vectors to this curve. Simplify and record the results below.

4b. Calculate the curvature using the formula from **2b** above. Now use *Derive* to find the value(s) of *t* for which $\kappa(t)$ is greatest, and record below the corresponding points on the curve.