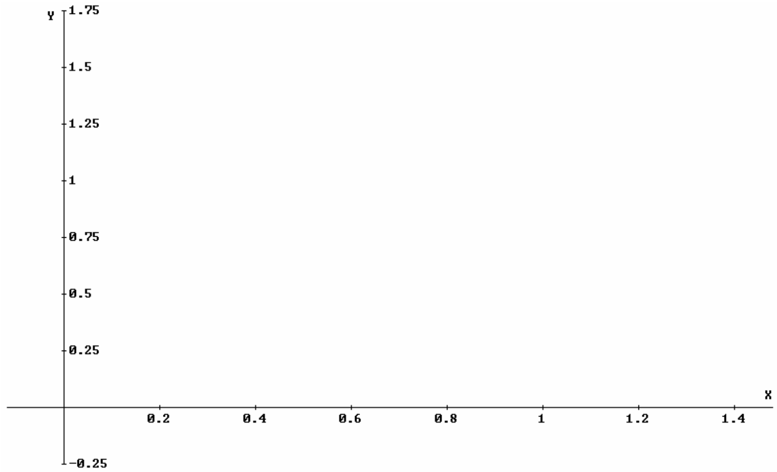
Assignment 13: Numerical Integration (4.7) **Name___________________**
Please provide a handwritten response.

1. We wish to approximate $\int_0^1 \sqrt[3]{x^2+1}\,dx$ using the numerical techniques discussed in our text. **Author** and plot f(x):=(x^2+1)^(1/3). Zoom to an appropriate view and sketch the results on the axes at right. Based on this graph, what would be your estimate of $\int_0^1 \sqrt[3]{x^2+1}\,dx$?



2a. To apply the Midpoint Rule to this integral, we must first define $c_i = a + i\Delta x$ just as we did in the preceding assignment. **Author** one at a time the following: a:=0, b:=1, n:=10, deltax:=(b-a)/n and c(i):=a+i*deltax. Simplify c_i using ≈ and record the result below.

2b. The midpoint of each interval $[c_{i-1}, c_i]$ is $\frac{c_{i-1}+c_i}{2}$ and the height of each rectangle is given by $f\left(\frac{c_{i-1}+c_i}{2}\right)$. The Midpoint approximation to the definite integral is given by $\sum_{i=1}^{n} f\left(\frac{c_{i-1}+c_i}{2}\right)\Delta x$. Enter this by carefully **Author**ing mr:=sum(f((c(i-1)+c(i))/2)*deltax,i,1,n). Use ≈ to simplify. Is this result plausible? If so, enter it in the table on the next page.

3. To calculate the Trapezoid Rule approximation $\sum_{i=1}^{n} \frac{f(c_{i-1})+f(c_i)}{2}\Delta x$, carefully **Author** tr:=sum((f(c(i-1))+f(c(i)))/2*deltax,i,1,n). Use ≈ to simplify and enter the result in the table.

4. Simpson's Rule approximation can be calculated using $\frac{\text{tr}}{3}+\frac{2\text{mr}}{3}$. **Author** and simplify SR:=tr/3 + 2mr/3 and record the result in the table.

n	Midpoint	Trapezoid	Simpson's
10			
20			
50			

5. Calculate the three approximations again using $n = 20$. **Author** n:=20, use the mouse pointer to highlight the definition of mr, then click ≈ to find the approximation using 20 rectangles. Likewise, calculate the new approximations for tr and SR; enter the results in the table. Did the approximations change when n was increased? Why?

6. Repeat question 5 but use $n = 50$. Are the three approximations drawing closer together as n increases? Why?

7. Let's use *Derive* to calculate $\int_0^1 \sqrt[3]{x^2+1}\,dx$ automatically with great accuracy. We use the int command, or ∫, to enter this definite integral. For example, **Author** inf(f(x),x,0,1). Try to find the exact value of this integral using =. Record the result below? Is this what we wanted? Next, try to approximate the result by clicking **Simplify** → **Approximate**, specify 10 digits of precision, and click **Approximate**. Record this result below. How does this compare to the previous approximations? Which of the three approximations is most accurate, assuming *Derive*'s result is "exact."

8a. One can almost always take the results of **Simplify** → **Approximate** using a definite integral to be completely accurate. However, there are some unusual situations. Sketch $\sin(1/x)$ at right then **Author** the definite integral $\int_{0.001}^{1} \sin\left(\frac{1}{x}\right)dx$. Use S**implify** → **Approximate** and describe what happens. Is the numerical result given trustworthy? Try the other methods described in this assignment. Do the approximations agree?