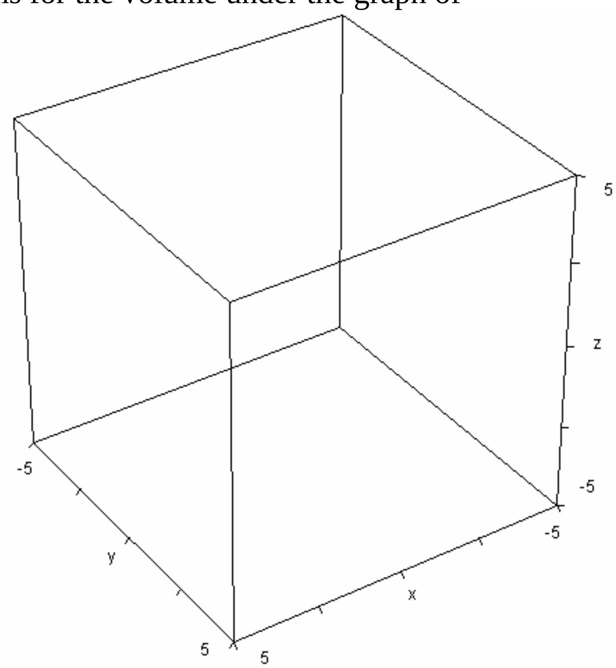
Assignment 29: Double Integrals (13.1-3) **Name___________________**
Please provide a handwritten response.

1. To evaluate the double integral $\int_0^1 \int_0^{y^2} \frac{3}{4+y^3}\,dxdy$, **Author** f(x,y):=3/(4+y^3) followed by int(int(f(x,y),x,0,y^2),y,0,1). Simplify with = then obtain a numerical result using ≈; record both results below. Are they correct? (We could also use ∫ to create the "inside" integral then use it again to integrate the first integral, which would result in a double integral!)

2a. We want to calculate the Riemann sums for the volume under the graph of $f(x,y) = x^2 \sin\left(\frac{\pi y}{6}\right)$ over the rectangle $R = \{(x,y) \mid 0 \le x \le 6,\ 0 \le y \le 6\}$. **Author** f(x,y):=x^2sin(pi y/6). It is nice to see the graph, so highlight the function and click to open a 3D-Plot window. Click and change the eye position to x = 10 and y = 15. Click to see the graph; sketch the result in the box at right. Do you see the region we want the volume of?



2b. We can find Riemann sums for the volume using not only 4 or 9 squares, but also 16, 25, etc. If we partition R into n^2 squares, then $\Delta A_i = \frac{36}{n^2}$ for all i. It will be more convenient here to label the center of each square as (u_i, v_j), $1 \le i, j \le n$.

Specifically, $u_i = \frac{3}{n} + (i-1)\frac{6}{n}$, $1 \le i \le n$ and $v_j = \frac{3}{n} + (j-1)\frac{6}{n}$, $1 \le j \le n$.

Thus, $V \approx \sum_{j=1}^{n}\sum_{i=1}^{n} f(u_i, v_j)\Delta A_i = \frac{36}{n^2}\sum_{j=1}^{n}\sum_{i=1}^{n} u_i^2 \sin\left(\frac{\pi v_j}{6}\right)$. To carry this out in *Derive*, we will set up several functions. **Author** n:=2 then **Author** u(i):=3/n+(i-1)6/n and **Author** v(j):=3/n+(j-1)6/n.

Finally, **Author** 36/n^2 sum(sum(f(u(i),v(j)),i,1,n),j,1,n). Our value $n = 2$ corresponds to $2^2 = 4$ squares total in the partition. Simplify this result with ≈ and record it below. Does this volume seem reasonable?

2c. Now increase n to 3 (corresponding to $3^2 = 9$ squares) by **Author**ing n:=3. Highlight the double-summation from above and click ≈ to approximate the Riemann sum using the newest value of n. Record the result below; is it more or less accurate than the result in **2b** above?

2d. Repeat this process to fill out the table at right. Do the results seem to converge to the true volume? What does the true volume appear to be?

n	$\sum_{j=1}^{n}\sum_{i=1}^{n} f(u_i, v_j)\Delta A_i$
2	
3	
10	
50	

2e. How large must n be to make the Riemann sum less than 275.03?

2f. The exact value of the volume is given by $\int_0^6\int_0^6 f(x,y)\,dx\,dy$, which we can find by using ∫ twice or by **Author**ing int(int(f(x,y),x,0,6),y,0,6). Find the exact value then a decimal value; record the results below. Do they agree with our answer in **2d**?

3a. To study $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} \sin\left(x^2+y^2\right) dy\,dx$ we first **Author** f(x,y):=sin(x^2+y^2); then plot this in a new 3D-Plot window with the axis range set to [-2,2] (Use **Set→Plot Range**). How would you describe the resulting surface? What part of it corresponds to the given integral?

3b. Enter the integral using either the int command or ∫ twice. Try to simplify with ≈ and =. (Click the **Abort** button to stop, if needed.) What happened?

3c. Now **Author** int(int(rf(rcos(t),rsin(t)),t,0,pi),r,0,2) to transform the integral to polar coordinates. Simplify with = to get an exact value, then use ≈ to obtain a decimal. Record both results below; which integral (**3b** or **3c**) was easier for *Derive*?