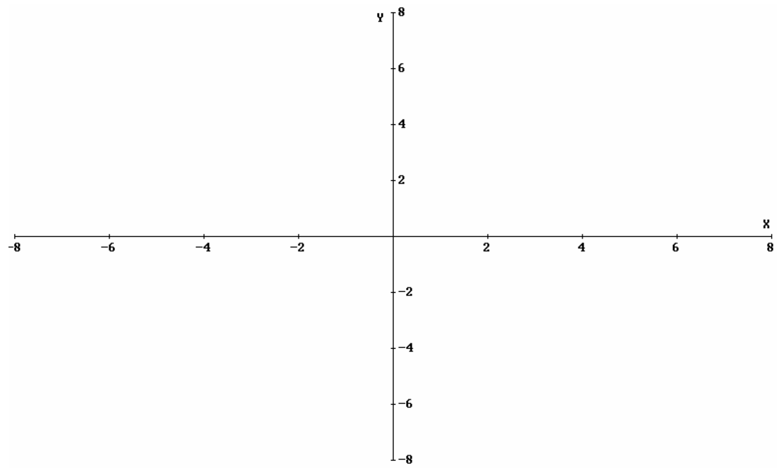
Assignment 23: Polar Coordinates (9.4-7) **Name**____________________
Please provide a handwritten response.

1. Define the limaçon $r = f(\theta) = 4\cos(\theta) - 2$ by **Author**ing f(theta):=4cos(theta)-2 and define the ellipse $r = g(\theta) = \dfrac{4}{2+\cos(\theta)}$ by **Author**ing g(theta):=4/(2+cos(theta)). (We could also select θ from the list found in the **Author** expression box.)

Click to switch to a 2D-Plot Window then select **Set→Coordinate System** and select **Polar**. Then click to return to the algebra window, highlight $f(\theta)$, return to the plot window and click .
Specify a **minimum value** of 0 and a **maximum value** of 2π. In a similar manner, plot $g(\theta)$. Zoom to an appropriate view and sketch the result on the axes at right. How many points of intersection do these two curves have?



2a. To find the points of intersection we begin by solving the equation $f(\theta) = g(\theta)$ for θ. **Author** f(theta)=g(theta) and use to algebraically solve this equation. What happened? Use **Declare→Simplification Settings** and set **Trig Powers** to **Cosines** then try to solve the equation again. (We have to "help" *Derive* convert the sines into cosines so it can solve the equation!) Use to obtain numerical results instead of algebraic ones. *Derive*'s results will include the imaginary number *i*, which, despite its mathematical interest we will ignore throughout this assignment. Record the other results below; call them a, b, c, …. How many eligible values of θ did *Derive* find?

2b. Now **Author** vector([f(theta),theta],theta,[a,b,c,…]) and to list the points (in polar coordinates) corresponding to the values of θ found above, and label the eligible ones on the graph above. (Note: Replace a,b,c.. with the values from **2a**. Also, remember that θ is in radians.) Could we have used g(theta) rather than f(theta) above? Have we found all of the points of intersection yet?

2c. We should also solve the equation $f(\theta) = -g(\theta + \pi)$ for θ to find points of intersection. **Author** f(theta)= -g(theta+pi) and solve using and as in **2a**; record the results below then find the coordinates as in **2b**. Label the eligible points on the graph in Question **1**.

3a. Below write down a sum of definite integrals that gives the area of the region lying both inside the ellipse and inside the smaller loop of the limaçon; also shade this region on the graph.

3b. Use the integration command, int or , along with to numerically evaluate these integrals and record the total area below. Based on the graph, is your answer plausible?

3c. Repeat parts **a** and **b** for the region lying inside the large loop of the limaçon (ignore the small loop) but outside the ellipse. Be sure to keep careful track of what parts of each curve correspond to which values of θ. (Think hard!)

4a. Implement the slope formula for f by carefully **Author**ing
ls(a) := (f '(a)sin(a)+f(a)cos(a))/(f '(a)cos(a)-f(a)sin(a)) where *a* represents a particular value of θ and ls stands for "limaçon slope". Next highlight just the right portion of the last expression and simplify with . Then use then to find the values of θ for which the tangent lines to the limaçon are horizontal, and record the result below. Also, find the corresponding points on the curve as in **2b** and mark them on the graph.

4b. The graph suggests that the two curves might be orthogonal at two of their intersection points. Modify the first commands in **4a** to define a function "es" giving the slope for the ellipse and decide whether this is the case. (Recall that perpendicular lines have negative reciprocal slopes unless they are horizontal and vertical. Also, simplify the right portions of the expressions before asking Derive to evaluate the slopes.)