

### Assignment 15: Probability (5.7)

Name \_\_\_\_\_

Please provide a handwritten response.

Suppose we were to toss eight coins together. How many heads would we get? Using *Derive*'s **random** command, we can simulate the repeated tossing of eight coins together and keep track of the number of heads appearing in each toss.

**1a.** Before we start using the **random** command, we need to initialize *Derive*'s random number generator. To do this, **Author** and **Simplify** `random(0)`. We can now simulate tossing one coin by **Authoring** `random(2)`. Simplify the result with `=`. **Random(2)** randomly generates either 0 or 1, each with probability 1/2; we interpret "1" as representing heads and "0" as representing tails. Simplify `random(2)` nine more times and record the results below. (Each time we simplify `random(2)`, *Derive* is "tossing" the coin.)

**1b.** By using the **vector** command, we can generate any number of throws at one time. For example, we can simulate tossing eight coins at once; **Author** `vector(random(2),i,1,8)`. Simplify the result using `=`; we see the outcome of each of the eight coins. Simplify the command two more times. (*Derive* tossed eight coins a total of three times!) Record the results below; how many 1's ("heads") appeared in each toss? (Note: The counter *i* is used here simply to help the vector command keep track of when to start and stop.)

**1c.** We are really interested only in the number of heads appearing in each toss of the eight coins. Because the tails are represented by "0" and the heads by "1", a convenient way to count the heads is simply to toss a coin eight times and add the results. **Author** `sum(random(2),i,1,8)` then use `=` three times and record the result below. How large could this number be? How small?

**1d.** Now we are ready to simulate tossing the eight coins together twenty-five times. **Author** `vector(sum(random(2), i, 1, 8), k, 1, 25)` and use `=` to simplify. Record the result below. This shows the number of heads that occurred during each of the twenty-five tosses of eight coins.

**1e.** We now want to create a distribution for the number of heads we found. Enter in the table on the next page the number of occurrences in part **d** of each possible outcome, as a fraction of the total of twenty-five tosses. A sample is provided, but your specific numbers will probably be different.

# of heads	0	1	2	3	4	5	6	7	8
Occurrences	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$
(Sample)	$\frac{0}{25}$	$\frac{1}{25}$	$\frac{4}{25}$	$\frac{3}{25}$	$\frac{9}{25}$	$\frac{5}{25}$	$\frac{2}{25}$	$\frac{0}{25}$	$\frac{1}{25}$

2. The normal distribution, also called normal curve or bell curve, involves two parameters,  $m$  = mean and  $s$  = standard deviation. Suppose we let  $x$  represent the height of a randomly selected 19 year-old female. We will assume that  $x$  is normally distributed. This means the probability of  $x$  follows a probability density function of

$$\frac{1}{s\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2s^2}}.$$

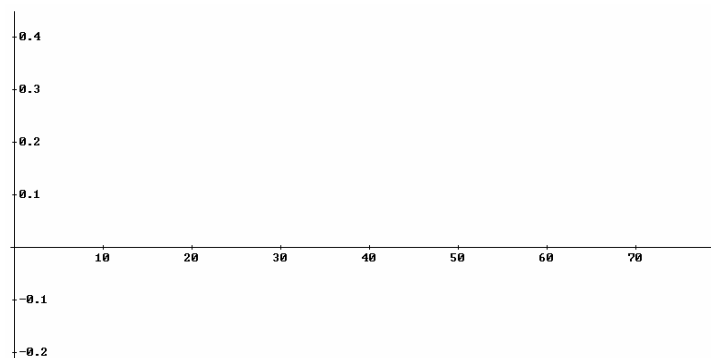
Enter this equation into *Derive* as a function by **Authoring**

$f(x,m,s):=1/(s*\text{sqrt}(2\pi))*\exp(-(x-m)^2/(2s^2))$ . We can use this probability density function to calculate the probability that  $x$  is between  $a$  and  $b$ , denoted  $P(a \leq x \leq b)$ . We

calculate this using  $P(a \leq x \leq b) = \int_a^b f(x,m,s)dx$ .

3a. For example, suppose we know the population of 19 year-old females in the world has a mean height of 63 inches with a standard deviation of 4.3 inches.

**Author**  $f(x,63,4.3)$  then plot the function. Zoom to an appropriate view and sketch the results on the axes at right. Is the result a bell curve? The highest point represents the



height of “most” 19 year-old females. What is this height? Does this seem reasonable?

3b. What is the probability that  $x$  will fall between 55 and 65 inches? Should this probability be low? Do most 19 year-old female students fall in this range (look at the graph)? To calculate this probability, we need to integrate  $f(x,63,4.3)$  from  $x = 55$  to  $x = 65$ . Using the graph above, shade the region represented by the integration. Set up and record the integral below. What is the probability? Does this seem reasonable?

3c. What is the probability that a randomly selected 19 year-old female will be at least 6 feet tall? Calculate this probability using  $\infty$  as the upper boundary and record the result below. Why do we use  $\infty$ ?