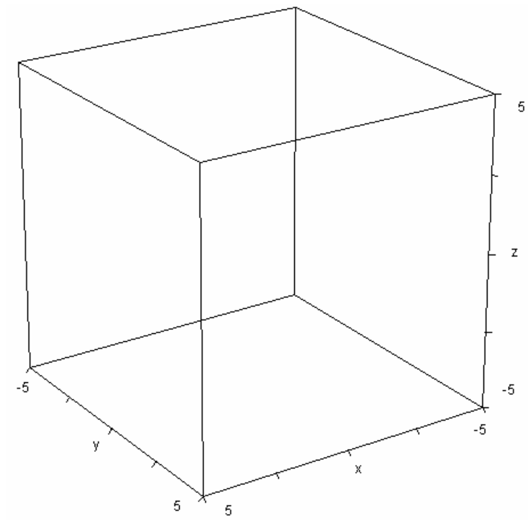
Assignment 32: Vector Fields in Space (14.6-8) **Name____________________**
Please provide a handwritten response.

1a. Set up a double integral and evaluate the surface integral $\iint_S g(x,y,z)dS$ where $g(x,y,z) = y$ and S is the portion of the paraboloid $z = x^2 + y^2$ below $z = 4$. First, **Author** z = x^2+y^2 and plot its graph; sketch the results on the axis at right and also sketch the plane $z = 4$. Now we can "see" the surface of this solid. Notice that the paraboloid resembles a "bowl" and the plane looks like its "lid"!



1b. **Author** g(x,y,z):=y then parameterize the surface using $x = u\cos(v)$, $y = u\sin(v)$, and $z = u^2$ over $0 \le u \le 2$, $0 \le v \le 2\pi$ by **Author**ing r(u,v):=[ucos(v),usin(v),u^2].

1c. Since $dS = \|\mathbf{r}_u \times \mathbf{r}_v\| dA$, it will be useful for us to **Author** norm(u,v):=cross(dif(r(u,v),u),dif(r(u,v),v)) and to set up the integrand by **Author**ing integrand(u,v):=g(r(u,v)sub1,r(u,v)sub2,r(u,v)sub3)abs(norm(u,v)). Finally, set up the double integral with **Author** int(int(integrand(u,v),u,0,2),v,0,2pi) to calculate the surface integral. Simplify the result with = and record it below.

2a. We want to study the flux integral $\iint_S \mathbf{F} \bullet \mathbf{n}\, dS$, where $\mathbf{F}(x,y,z)$ is the vector field $\langle y, -x, 1 \rangle$, S is the same as in Question 1 above and **n** is downward. We can keep our previous definitions of $\mathbf{r}(u,v)$ and $\mathbf{norm}(u,v)$. **Author** f(x,y,z):=[y,-x,1].

2b. Because **n** is a unit normal vector pointing downward, we require the *z*-component to be negative. Why? Would we expect $\iint_S \mathbf{F} \bullet \mathbf{n}\, dS$ to be positive, negative, or zero?

2c. To find **n** we can use $\mathbf{n}(u,v) = \dfrac{-\mathbf{norm}(u,v)}{\|\mathbf{norm}(u,v)\|}$. Explain below why we know that this is the "correct" normal vector and why we have used the negative. **Author** then simplify n(u,v):= - norm(u,v)/abs(norm(u,v)) and record the result below.

2d. The flux integral becomes $\iint_S \mathbf{F} \bullet \mathbf{n}\ dS = \iint_R \mathbf{F} \bullet \mathbf{n} \|\mathbf{norm}\|\ dA$. Why? Before integrating, we will again define the integrand by **Author**ing integrand(u,v):=f(r(u,v)sub1,r(u,v)sub2,r(u,v)sub3).n(u,v) abs(norm(u,v)). Set up the double integral with **Author** int(int(integrand(u,v),u,0,2),v,0,2pi). Simplify this and record the result below.

3a. Use the Divergence Theorem to compute $\iint_{\partial Q} \mathbf{F} \bullet \mathbf{n}\, dS$ where $\mathbf{F}(x, y, z) = \langle x^3, y^3 - z, xy^2 \rangle$ and Q is bounded by $z = x^2 + y^2$ and $z = 4$; note that this is essentially the region we've been working with in Questions 1 and 2 so we can continue to use our previous definitions of $\mathbf{r}(u, v)$ and $\mathbf{norm}(u, v)$.

Author f(x,y,z):=[x^3,y^3-z,xy^2]. Next calculate curl **F** = $\nabla \times \mathbf{F}$ by **Author**ing and simplifying curl(f(x,y,z)); record result below. Now calculate divergence of **F,** denoted div **F**, with div **F** = $\nabla \bullet \mathbf{F}$ by **Author**ing and simplifying div(f(x,y,z); record this result below also. Are the results correct?

3b. Now set up (by hand) an iterated integral giving $\iiint_Q \nabla \bullet \mathbf{F}(x, y, z) dV$ and use *Derive* to evaluate it; record your integral and *Derive*'s results below.

3c. By Stokes' Theorem, $\iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\, dS$ is the same whether S is the "bowl" or the "lid" of ∂Q. Our parameterization in Questions 1 and 2 was for the "bowl", so we'll examine this first. Now, take the curl from **3a** and use  three times to substitute in r(u,v)sub1 for x then r(u,v)sub2 for y and r(u,v)sub3 for z (if needed). Give this last expression a name by **Author**ing delf(u,v):= then inserting the expression using right-click. Simplify and record results below.

Set-up the integral with **Author** int(int(delf(u,v).norm(u,v),u,0,2),v,0,2pi). Simplify and record results below. Now make slight modifications in the above to calculate $\iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\, dS$ for the "lid"; do the two results agree? What are they?