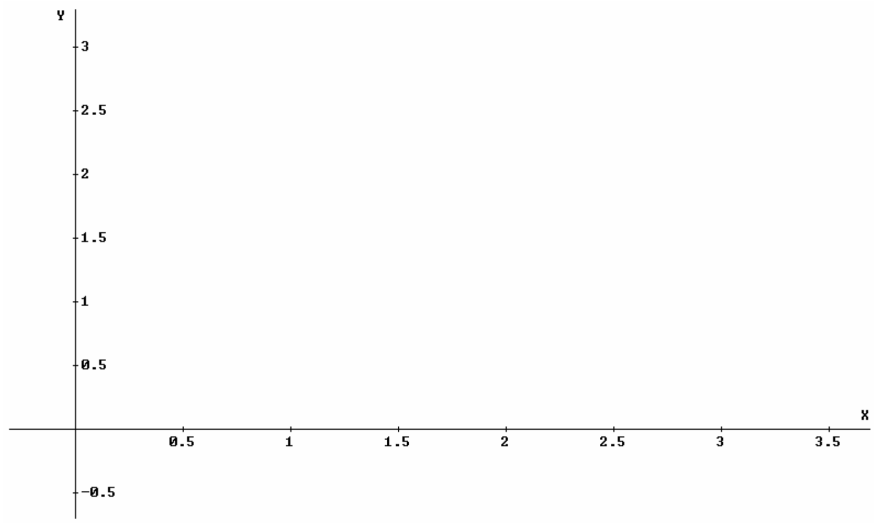
**Assignment 22: Parametric Equations (9.1-3)** **Name**___________________
**Please provide a handwritten response.**

**1a**. **Author** x(t):= pi*t - 0.6 sin(pi t) and y(t):= 2t + 0.4 sin(pi t). Next, **Author** [x(t),y(t)] and click to open a 2D-Plot Window; click and specify a minimum value of **0** and a maximum value of **1**. Sketch the result on the axes at right.




**1b**. **Author** [x(1/2),y(1/2)] to find the point on the curve corresponding to $t = 1/2$. Use = to simplify the result. Mark this point on the curve with a large dot and draw the line tangent to the curve there. What do you estimate the slope of this line to be?

**1c**. **Author** and = y'(1/2)/x'(1/2) to find this slope exactly, and record the result below.

**1d**. Use *Derive* to calculate the arc length $\int_0^1 \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$ by **Author**ing int(sqrt(x'(t)^2+y'(t)^2),t,0,1) and using ≈ to simplify the result. Record the result in the table on the next page.

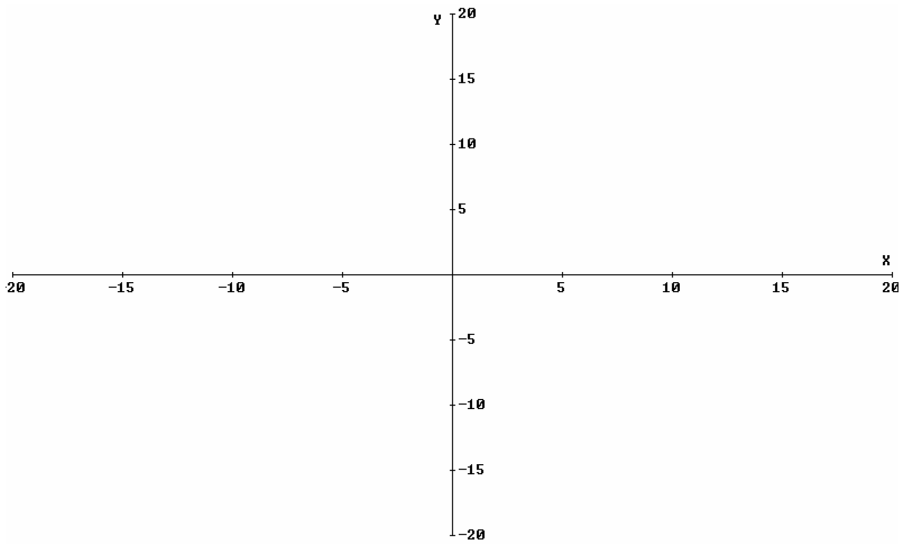
**1e**. Suppose a skier is skiing along the curve. The time needed to ski the curve can be calculated using Time $= \int_0^1 k\sqrt{\frac{(x'(t))^2 + (y'(t))^2}{y(t)}}\, dt$. Now, taking $k = 1$ for convenience, **Author** int(sqrt((x'(t)^2+y'(t)^2)/y(t)),t,0,1) and use ≈ to simplify the result; record the result in the table.

**1f**. Now modify the commands in part **a** and re-execute the commands in parts **d** and **e** to complete the table for the given paths, as well; in all cases, *t* ranges from 0 to 1. Based on these examples, does there seem to be any correlation between the arc length and the time?

| Path | Arc Length | Time |
|---|---|---|
| $x = \pi t - 0.6\sin(\pi t),\ y = 2t + 0.4\sin(\pi t)$ | | |
| $x = \pi t, y = 2\sqrt{t}$ | | |
| $x = \pi t, y = 2\sqrt[4]{t}$ | | |
| $x = -\frac{1}{2}\pi\left(\cos(\pi t) - 1\right),\ y = 2t + \frac{7}{10}\sin(\pi t)$ | | |

**1g**. In the same way, find the arc length and time for the cycloid $x = \pi t - \sin(\pi t)$, $y = 1 - \cos(\pi t)$, $0 \le t \le 1$. In both categories, where does it rank among the other four curves considered so far?

**2a**. Define the parametric curve $\begin{cases} x(t) = 8\cos(t) - 2\cos(4t) \\ y(t) = 8\sin(t) - 2\sin(4t) \end{cases}$ in *Derive* as we did in Question **1**. Plot the curve over $0 \le t \le 2\pi$ and sketch the result on the axes at right.




**2b**. Where are the "corner" points of this curve? At such points both $x'(t)$ and $y'(t)$ must be zero. **Author** x'(t)=0 and use (to solve) then **Simplify** to list the values of *t* for which $x'(t) = 0$ and record the result below.

**2c**. Likewise, **Author** y'(t)=0 and solve this equation. Record below the values of *t* <u>common</u> to the lists; call these values a, b,and c. Now **Author** vector([x(t),y(t)],t,[a,b,c]) and simplify with = to find the coordinates of the corner points. (The vector command "plugs" in the different values of *t* in one step.) Label these points on the graph above.

**2d**. Find the corner point(s) on the curve $x = 2\cos(2t) + \cos(4t),\ y = 2\sin(2t) + \sin(4t)$. How many are there?