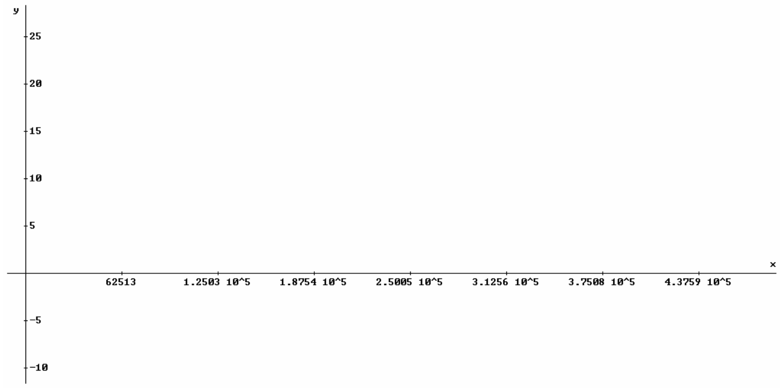
Assignment 7: Limits, Part III (1.6) **Name:__________________**
Please provide a handwritten response.

1a. Use the function $f(x) = \dfrac{\left(x^3 + 4\right)^2 - x^6}{x^3}$ to illustrate the danger of loss of significance errors. **Author** $f(x)$ and plot its graph. Set an x range from -100 to 500000 and select an appropriate view. Sketch the results on the axes at right. Does the graph give any indication of the value of $\lim_{x\to\infty} f(x)$? Explain.



1b. Next, try to find $\lim_{x\to -\infty} x\left(\sqrt{x^2+4}+x\right)$ using tables. **Author** the function as $h(x)$ and use ≈ to find $h(-1000)$, $h(-10000)$, etc. to complete the table at right. Are these results correct? Is the limit zero? Explain.

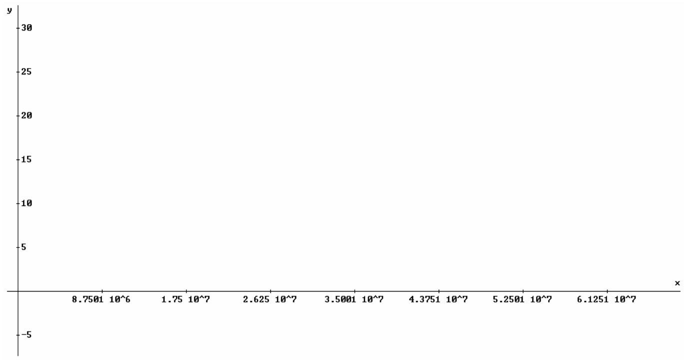
x	$h(x)$
-1000	
-10000	
-100000	
-1000000	
-10000000	

1c. Again calculate $h(-1000)$, $h(-10000)$, etc. to complete the table at right; instead of using ≈ to simplify, highlight each expression and click **Simplify** → **Approximate**, enter 15 digits of precision, then click **Approximate**. This temporarily sets output to 15 digits of precision. Record results to as many digits as given by *Derive*. What does the limit appear to be?

x	$h(x)$
-1000	
-10000	
-100000	
-1000000	
-10000000	

1d. Note that $h(x)$ can be written as $h(x) = \dfrac{4x}{\sqrt{x^2+4}-x}$. (How?) Repeat steps **a** through **c** with this new (but equivalent) formula. Do the results differ? What do you think $\lim_{x\to -\infty} x\left(\sqrt{x^2+4}+x\right)$ is? Why?

2. Scientific notation is used to write very large or very small numbers in a convenient form; for example, 0.000000000002673 would be written in scientific notation as $2.673 \text{x} 10^{-12}$. In *Derive*, **Author** 2.673*10^(-12) and record the result below.



3a. We seek a value of *x* for which loss of significance occurs in $\lim_{x\to\infty} \dfrac{5}{x\left(\sqrt{x^2+1}-x\right)}$.

Author and plot g(x):=5/(x(sqrt(x^2+1)-x)). Set an *x*-range from -1000 to 7*10^7 and a *y*-range from –10 to 30. Sketch the plot on the axes at right.

3b. Next, **Author** g(1000), g(10000) etc. Use ≈ to simplify the functions and enter the results in the left table below. Do these results agree with the graph? Now, simplify the functions using **Simplify → Approximate** and enter 20 as the digits of precision. Then click **Approximate**. Enter the “new” results in the table at right.

x	*g*(*x*)
1000	
10000	
100000	
1000000	
10000000	

x	*g*(*x*)
1000	
10000	
100000	
1000000	
10000000	

3c. We can rewrite *g*(*x*) by rationalizing the denominator. Verify that by multiplying by $\dfrac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}$ we can write *g*(*x*) as $g(x) = \dfrac{5\left(\sqrt{x^2+1}+x\right)}{x}$.

Author this new function and repeat part **3b**. Do the different approximations now agree? Enter the results in the table below.

x	*g*(*x*) using ≈	*g*(*x*) using Simplify→Approximate method
100		
1000		
10000		
100000		
1000000		