Assignment 20: Infinite Series (8.2-7) **Name**____________________

Please provide a handwritten response.

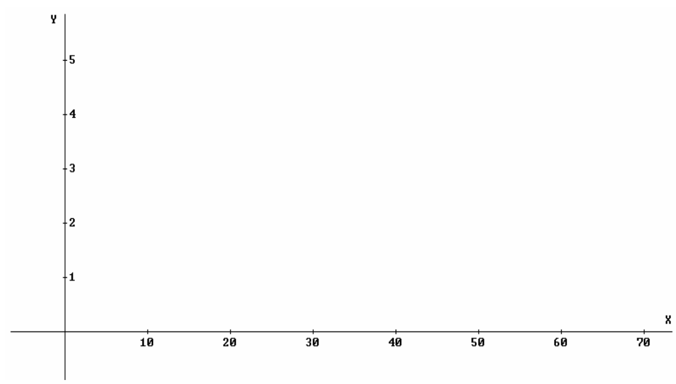
1a. To find the partial sum S_{10} of the infinite series $\sum_{k=1}^{\infty} \frac{1}{k^{0.9}}$ **Author** sum(1/k^0.9,k,1,10) and simplify the result using ≈. Record the result in the table. (We could use =, but the result is lengthy!) By changing 10 to 100, etc. complete the second column of the table. Round to five decimals.

n	$\sum_{k=1}^{n} \frac{1}{k^{0.9}}$	$\sum_{k=1}^{n} \frac{5}{k^{1.1}}$
10		
100		
1000		
10000		
100000		

1b. Likewise modify the command in **1a** to find the partial sums of the infinite series $\sum_{k=1}^{\infty} \frac{5}{k^{1.1}}$ and complete the third column. Notice that in each row, the entry in the second column is smaller than that in the third; can this be the case for all *n*? Why?

1c. Add one more row to the bottom of the table corresponding to $n = 10^8$ and fill it in; are the results consistent with the answer to **1b**?

2a. The text explains that the harmonic series $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges. To get an idea of how quickly or slowly it does so, **Author** s(n):=sum(1/k,k,1,n) then **Author** vector([n,s(n)],n,1,50) to construct a list of ordered pairs $\left(n, \sum_{k=1}^{n} \frac{1}{k}\right)$ where the "*y*-value" is the n^{th} partial sum of the harmonic series. Click = to simplify the vector then click to open a 2D-Plot window. Select **Options→Display →Points** and select **Connect** no and **Size** large. Click **OK** then to plot the ordered pairs from the vector to "see" how the partial sums increase. Sketch the result on the axes above.

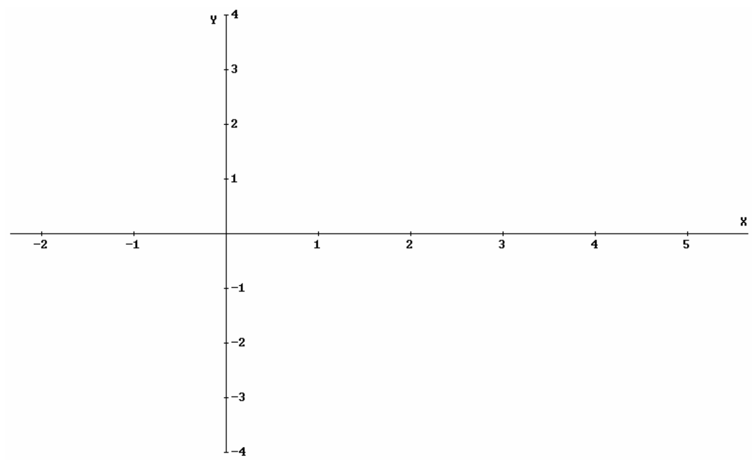


2b. Repeat the vector from **2a** but use 500 in place of 50 and use ≈ to simplify it; *Derive* may need several seconds to find the results. Are the partial sums quickly approaching ∞ ? Why?

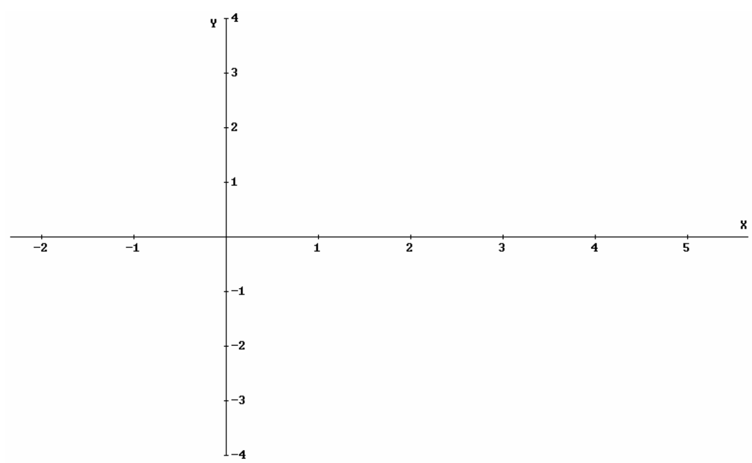
3a. To find the Taylor polynomial with $c = \dfrac{\pi}{2}$ and n = 4 for cos(x), **Author** Taylor(cos(x),x,pi/2,4) OR **Author** cos(x) and click **Calculus→Taylor Series** and fill in the appropriate boxes. Simplify the result using =. Record the result below. (We could use **Simplify→Expand** to make it "look" like a polynomial if we wish.)

3b. Let's give the polynomial a name. **Author** tp(x):= then insert the above polynomial. Enter the result below.

3c. Now plot the cosine function <u>and</u> *tp*(*x*) over $-\pi \le x \le 2\pi$. Sketch the result on the axes at right, labeling the graphs. On roughly what interval are the two graphs indistinguishable on the computer screen?



3d. Change the 4 in **3a** to 8 and repeat the commands in **3a** - **3c** once again (erase the previous graphs first). For the new Taylor polynomial, sketch its graph with labeling on the graph above, and answer the question in **3c** again.



3e. To measure the error in this Taylor approximation, **Author** tp(x)-cos(x) and plot its graph. (Erase the old graphs first!) Sketch the result on the axes at right. On $-\pi \le x \le 2\pi$, how large (positive or negative) does the error become and for what value(s) of *x* is the error greatest?

3f. By increasing *n* still further while keeping everything else the same, can we reduce the maximum error in **3e** to less than 0.1? Experiment to find how large a value of *n* is needed.

3g. Try to answer **3f** with cos(*x*) changed to $\tan^{-1}(x)$ (denoted atan(x)), *c* to 0 and the interval to $-1.5 \le x \le 1.5$. Can you find *n* large enough? Why?