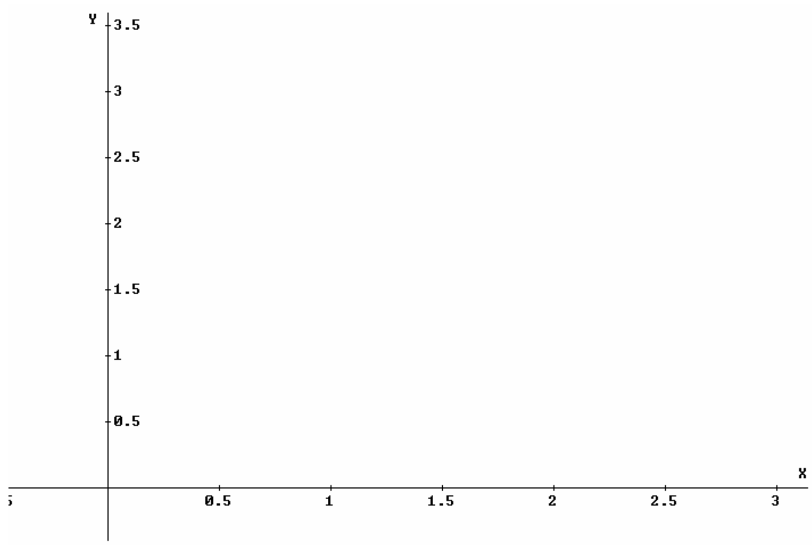
Assignment 17: Euler's Method (6.6) **Name___________________**
Please provide a handwritten response.

1a. To apply Euler's method to the differential function $y' = \sin(y) - x^2$, first define $f(x, y) = \sin(y) - x^2$ by **Author**ing f(x,y):=sin(y)-x^2. In *Derive* functions of two or more variables are handled similarly to functions of one variable; for example, **Author** and use = to simplify f(-3,pi/2) to find $f\left(-3, \frac{\pi}{2}\right)$. Record the result below; is it correct?

1b. To draw a direction field, we must first load a utility package. Click **File→Load→Utility File**. Select the file "Ode_appr.mth" and click **Open**. (You may need to change to *Derive*'s **Math** folder to find the file.) *Derive* then loads the file into memory. If we wanted to see the contents of the file, we could have clicked **File→Load→Math File** then selected the file. The contents of the file would then appear as expressions in the current window. Here, however, we do not need to see all the contents since we'll be using only a small part of the file.

Author direction_field(f(x,y),x,0,2,6,y,1,3,6) then simplify using ≈. Click to switch to a 2D-Plot window then click **Options → Display → Points** and specify **Connect** yes and **Size** small; click **OK**. Now click to see the direction field. Sketch the result on the axes at right.



In using this command, we have entered $f(x, y)$ as being equal to y', we have specified x to vary from 0 to 2 in 6 steps and for y to vary from 1 to 3 in 6 steps. Experiment by increasing the number of steps to, say, 12. What happened?

The direction field suggests the "flow pattern" for the family of solution curves of the differential equation $y' = f(x, y)$. Recall from the last assignment how we plotted these curves after finding the general solution of the differential equation. In this assignment, however, we have plotted a direction field to *suggest* the family of solutions without actually finding a solution!

2a. Now we will use Euler's method to approximate the solutions of the initial value problem $y' = \sin(y) - x^2$ with $y(0) = 2$. (Recall this means *y* = 2 when *x* = 0.)

We will start our solutions at $(0,2)$ with a step size of *h* = 0.1. To approximate $y(2)$ will take 20 steps starting from $x_0 = 0$. (Why?) We will need to use a command from the utility file we loaded to find the approximations.

The command we'll use takes the form euler_ode(r,x,y, x_0, y_0,h,n) where $r = y'$, *x* is the independent variable, *y* is the dependent variable, (x_0, y_0) is the starting point, *h* is the step size, and *n* is the number of steps.

Author euler_ode(f(x,y),x,y,0,2,0.1,20) and use ≈ to simplify the result. We should see a vertical list of ordered pairs with *x* values on the left and *y* values on the right. The first point will be the initial point $(0,2)$. The next point should be $(0.1,\ 2.090929742)$ which represents a second point on the solution curve to the initial value problem. What are $y(1)$ and $y(2)$ according to this approximation?

2b. Highlight the vertical list of ordered pairs from **2a**, switch to the plot window, and graph the ordered pairs. Sketch and label the curve on the axes in **1b**. This curve represents Euler's approximation of the particular solution of the differential equation. Does the equation follow the direction field found earlier?

2c. Suppose we wanted to decrease the step size. Will this increase the accuracy of the approximation? Lets again use Euler's method as above, but use a step size of *h* = 0.05. To get to the point *x* = 2, we will need 40 steps. (Why?)

We could **Author** euler_ode(f(x,y),x,y,0,2,0.05,40) by typing it or by inserting the previous command in the **Author** expression box. Using either method, enter this command and simplify using ≈.

Using this new approximation, again estimate the values of $y(1)$ and $y(2)$.

2d. Plot the ordered pairs from the new approximation. Sketch and label the result on the axes in **1b**. Does this approximate solution curve again follow the direction field?