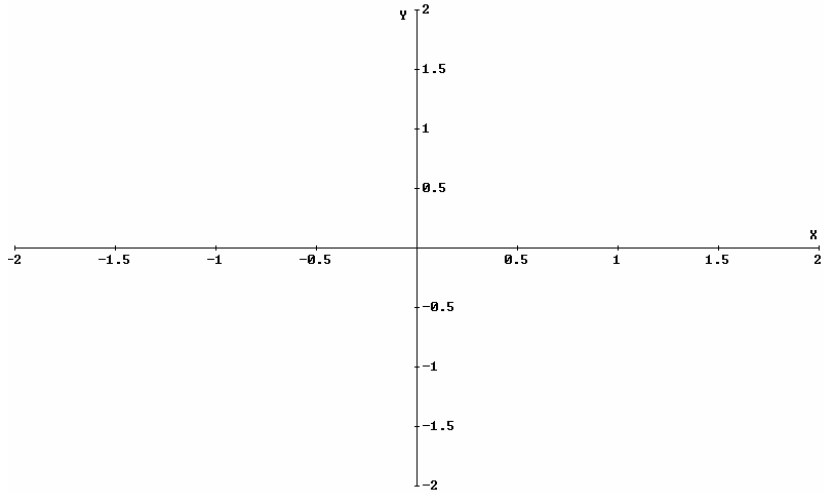
Assignment 25:Vector-Valued Functions, Part I(11.1-3) Name___________________
Please provide a handwritten response.

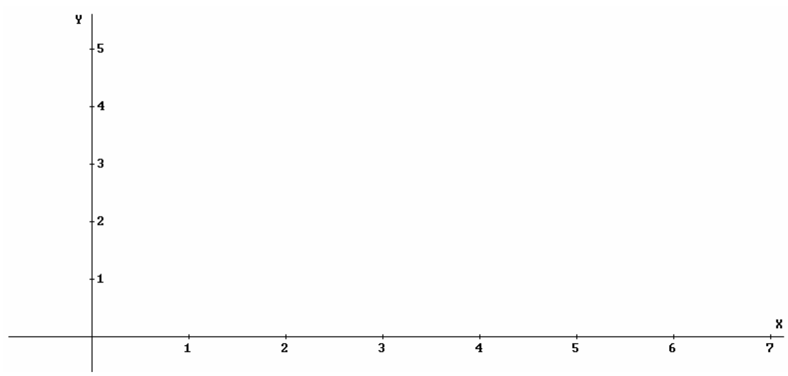
1a. **Author** r(t):=[cos(3t),sin(2t)] to define the vector-valued function $\mathbf{r}(t) = \langle \cos(3t), \sin(2t) \rangle$, $0 \le t \le 2\pi$ and draw the graph of $\mathbf{r}(t)$ in the same manner as we did when plotting parametric plots. Use 0 as the **minimum** and 2pi as the **maximum** plot parameters. Sketch the resulting “Lissajous curve” on the axes at right.



1b. To list the points $\mathbf{r}(0), \mathbf{r}\left(\frac{\pi}{4}\right)$, etc. **Author** vector(r(n pi/4), n, 0, 8) and simplify using =. Mark these coordinates, with their corresponding values of *t*, on the graph, and then draw arrows to show the orientation of the curve.

1c. Thinking of $\mathbf{r}(t)$ as representing the position of a moving point, **Author** v(t):=r'(t) to find the velocity vector $\mathbf{v}(t) = \mathbf{r}'(t)$, then **Author** speed(t):=abs(v(t)) to find the speed.

Simplify all the functions by selecting the right side of each defining equation and clicking =. This will re-define each function to a simplified form. Sketch the graph of *speed*(*t*) over $0 \le t \le 2\pi$ on the axes at right; based on this graph, does the moving point ever stop? Why?



1d. Now **Author** r1(t):=r(t+3sin(t)) to define the reparameterization $\mathbf{r}_1(t) = \mathbf{r}(t + 3\sin(t))$, $0 \le t \le 2\pi$. Plot $\mathbf{r}_1(t)$ over $0 \le t \le 2\pi$. What is the subtle difference between this graph and that in **1a**? Erase the graph and click , select **Plot Mode** to Points and use 400 points. Why are the points grouped together in some places?

1e. Imitate **1c** to plot over $0 \le t \le 2\pi$ the speed of a point moving under $\mathbf{r}_1(t)$, note the approximate values of t where the speed is zero, and then highlight the right portion of the speed function and apply to find more accurate values where it is zero. (What happens when we apply to both sides?) Evaluate $\mathbf{r}_1(t)$ at each value of t to find the coordinates of these points where $\mathbf{r}_1(t)$ "stops" and record the results below.

1f. Suppose we knew only the graph of a vector-valued function; so far, can we say for sure whether there are any points at which the function "stops"?

1g. Finding the points where this curve crosses itself, which amounts to finding pairs of numbers s and t such that $\mathbf{r}(s) = \mathbf{r}(t)$, would be difficult by hand but is much easier in *Derive*. That is, we want to find values of s and t that solves $\begin{Bmatrix} \cos(3s) = \cos(3t) \\ \sin(2s) = \sin(2t) \end{Bmatrix}$. First we must load a utility file by clicking **File→Load→Utility File**, select "Solve.mth" and click **Open**. (You may need to change to *Derive*'s **Math** folder.) Next, **Author** Newtons([cos(3s)-cos(3t),sin(2s)-sin(2t)], [s,t], [0.5,3.6], 10) then simplify the result with to find 10 iterations of Newton's Method with a starting point of $s = 0.5$ and $t = 3.6$. The last row of the table shows the solution (s,t) as an ordered pair; evaluate $\mathbf{r}(t)$ at the given value of t to find the coordinates of the point where the curve crosses itself. Record the result below. Now modify the starting values to find the exact t value and coordinates of the point in the first quadrant where the curve crosses itself, and record the result below. (Hint: To find a starting point, try clicking **Options→Trace Plots** from the 2D-plot Window; the pointer box now follows the graph. The coordinates of the pointer are shown at the bottom and the value of t is found at the very top of the window.)

2a. Redefine $\mathbf{r}(t)$ as $\mathbf{r}(t) = \langle 2\cos(t) + \sin(2t),\ 2\sin(t) + \cos(2t) \rangle$, $0 \le t \le 2\pi$ and sketch the graph on the axes at right.

2b. Find and mark any stationary point of $\mathbf{r}(t)$ as we did in **1e**.

2c. Looking at the curve, would we want to modify our answer to **1f**? Why?