Assignment 28: Partial Derivatives (12.3-7) **Name___________________**
Please provide a handwritten response.

1a. To define the function $f(x,y,z)=e^{2xy}-\frac{z^2}{y}+xz\sin(y)$, **Author** f(x,y,z):=exp(2xy)-z^2/y+xz sin(y) followed by dif(f(x,y,z),y) (In *Derive*, dif refers to the differential operator command). Simplify the partial derivative using ; is this indeed $f_y(x,y,z)$? We could also use to find the partial derivative.

1b. To find the second-order mixed partial derivative $f_{yx}(x,y,z)=\frac{\partial^2 f}{\partial x \partial y}(x,y,z)$ **Author** and simplify dif(dif(f(x,y,z),y),x). Record the result below; is it correct? (Carefully note the order of operations in our command.)

1c. **Author** and simplify dif(f(x,y,z),y,2) to find $f_{yy}(x,y,z)=\frac{\partial^2 f}{\partial y^2}(x,y,z)$. Use three times to substitute in x = -0.2, y = 3, and z = $\sqrt{7}$ to calculate $f_{yy}(-0.2,3,\sqrt{7})$; use then and record all results below.

2a. **Author** f(x,y):=x^3+3xy-y^3 and plot the 3D-plot using followed by again. Use **Set→Plot Range** to change the *x* and *y*-axes range to [-2,2]. Can we clearly see what the surface looks like? Change the viewpoint from which we are looking by clicking and using the up and down arrow keys on the keyboard. Can we now see the critical points of this surface; that is, the local extremum or saddle point(s), if any? We now want to find these points.

2b. To calculate the gradient of *f*, $\nabla f(x,y)$, we **Author** grad(f(x,y)). Simplify and record the result below. (The 0 at the third position represents $f_z(x,y)$ and is included automatically by *Derive*.)

2c. Highlight the first position (f_x) only, click to solve for variable *y*, and simplify the result. Record it below. We just solved for when $f_x=0$ in terms of *x*.

2d. Since critical points occur when both partial derivatives are equal to zero, highlight the gradient's second position (f_y), click [SUB] and substitute in the value of y that we found in **2c**. (Instead of typing the equation we're substituting in, highlight it in the main window, click the right mouse button and use **insert expression** to enter our substitution in the substitution dialog box.) Click **OK**. Now highlight the last expression, click [solve] and **Solve** to find the x-values where both partial derivatives are equal to zero. Record the results below; are there really four values of x that make the partial derivatives equal to zero? Use substitutions to find the y and z-values of the two "real" critical points.

2e. To apply the second derivative test, **Author** fxx(x,y):=dif(f(x,y),x,2) and fyy(x,y):=dif(f(x,y),y,2) followed by the mixed partial fxy(x,y):=dif(dif(f(x,y),x),y). Finally, **Author** discrim(x,y):=fxx(x,y) fyy(x,y)-(fxy(x,y))^2. Simplify the right sides using [=] and use these functions to classify each of the critical points of f; record the results below.

3a. Enter $f(x,y)=\left(x^2-3xy+3y^2+4x\right)e^{-2x^2-(1/2)y^2}+\sin\left(\dfrac{x+y}{100}\right)$ (!!) by carefully **Author**ing f(x,y):=(x^2-3xy+3y^2+4x)exp(-2x^2-1/2y^2)+sin((x+y)/100). Close the previous 3D-Plot window, then plot this function and try to find the number of critical points. How many critical points does f seem to have?

3b. To obtain the contour plot, **Author** and simplify vector(f(x,y)=c, c, -1, 4, 0.1). Plot the level curves in a 2D-Plot window; use these results to list below the rough coordinates of each critical point. Classify the type of each critical point.

3c. Like **2c** and **2d** above, the **Solve** command cannot find the critical points for this function. Here, however, unlike **2c** and **2d**, an exact algebraic value is not possible. We will use Newton's method to help. Click **File→Load→Utility File,** select "Solve.mth" and click **Open**. **Author** newtons([dif(f(x,y),x),dif(f(x,y),y)],[x,y],[-0.5,-0.1]). Highlight the two partial derivatives (only!) and click [=]. With the last expression highlighted, click [≈] to find the best approximation of the critical point near $(-0.5,-0.1)$. Now change the starting point for x and y to find the other critical points as well, and record the results below. How close were the estimates in **3b**?