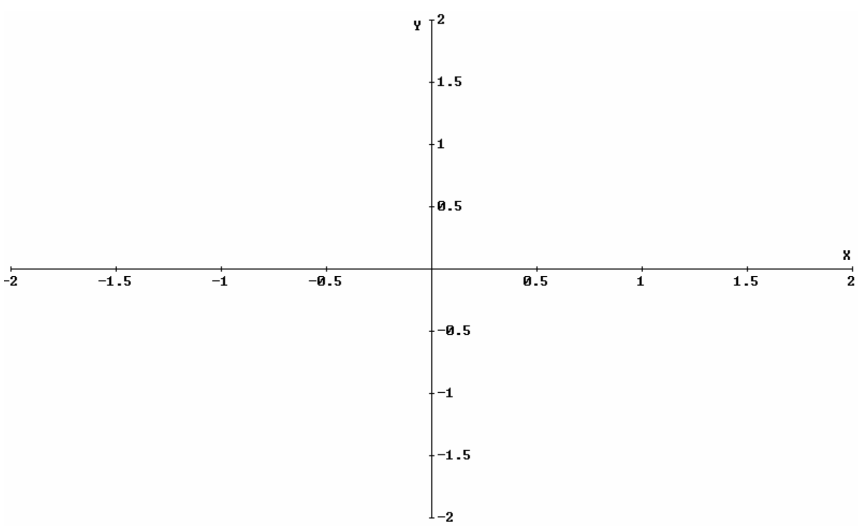
Assignment 31: Vector Fields in the Plane (14.1-4) **Name___________________**
Please provide a handwritten response.

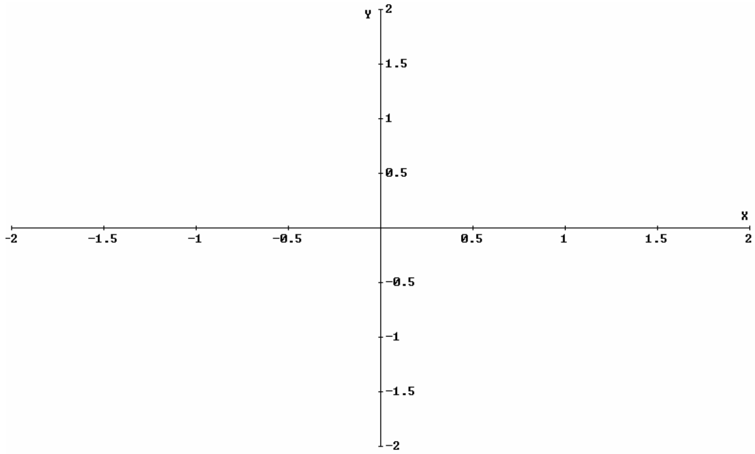
1a. Recall from Assignment 17, that vector fields can be drawn in *Derive*. For example, we will draw the vector field $\boldsymbol{F}(x, y) = \left\langle -1, y^2 \right\rangle$. First, load ode_appr.mth; click **File→Load→Utility File**, then select "ode_appr.mth" from *Derive*'s **Math** folder and click **Open**. Remember that *Derive*'s command direction_field requires the function to be entered as a differential equation of the form $\frac{dy}{dx} = f(x, y)$. The vector field $\boldsymbol{F}(x, y) = \left\langle -1, y^2 \right\rangle$ corresponds to the solutions of the differential equation $\frac{dy}{dx} = -y^2$. **Author** f(x,y):=-y^2 and then **Author** direction_field(f(x,y), x, -2, 2, 20, y, -2, 2, 20). Click to simplify the results; then to open a 2D-Plot window. Click **Options→Display→Points** and specify **Connect** yes and **Size** small; click **OK**. Now click to see the vector field; sketch the result on the axes at right.



1b. To graph the flow lines using the method of Assignment 16, we will solve for the general solution of the above differential equation. First **Author** g(y):=int(-1/y^2,y) followed by h(x):=int(1,x). Enter the general solution using **Author** g(y) = h(x) + c and clicking = to simplify. Finally, we want to construct several flow lines corresponding to different values of *c*. **Author** "vector(" then highlight the general solution's expression and insert it by right clicking; finish the vector with ", c, -2, 2, 1)". Simplify the result using = then plot these on the same graph as the vector field above. Sketch the flow lines on the graph above.

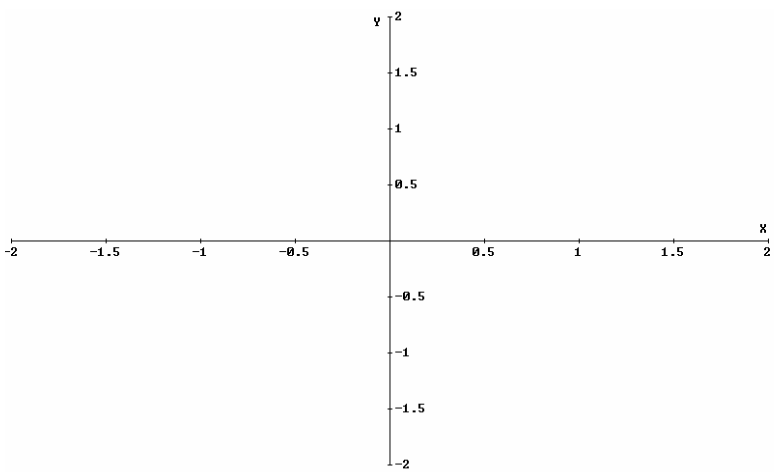
2. An important type of vector field that we already have some experience with is the gradient field, where the vector field is the gradient of some scalar function. To draw the gradient field corresponding to $f(x, y) = y\sin(x)$, we first find the gradient of *f*. Here, $\nabla f(x, y) = \langle y\cos(x), \sin(x)\rangle = \mathbf{F}(x, y)$. Now, as above, the vector field (or gradient field) corresponds to the solutions of the differential equation $\frac{dy}{dx} = \frac{\sin(x)}{y\cos(x)}$. As shown above in **1a**, **Author** direction_field(sin(x)/(ycos(x)), x, -2, 2, 30, y, -2, 2, 30) then simplify using .

Plot the gradient field using the steps in **1a**. (Delete all previous graphs!) Sketch the results on the axes at right.



Next, draw the level curves of $f(x,y) = y\sin(x)$. **Author** f(x,y):=ysin(x) then **Author** vector(f(x,y)=c,c,-2,2,0.1). Sketch the result on the axes above with the gradient field. What general connection between level curves and the gradient vector field does this graph bring out?

3a. To find the line integral $\oint_C (x^2 - y)dx + y^2 dy$ where *C* is the circle $x^2 + y^2 = 1$ oriented counterclockwise, **Author** the following functions: m(x,y):=x^2-y, n(x,y):=y^2, and f(x,y):= [m(x,y), n(x,y)] to define $M(x,y) = x^2 - y$, $N(x,y) = y^2$ and $\mathbf{F}(x,y) = \left\langle x^2 - y, y^2 \right\rangle$. Draw the vector field by **Author**ing, simplifying, and plotting direction_field(y^2/(x^2-y),x,-2,2,20,y,-2,2,20) as in **1a**. Sketch the result on the axes at right. Now **Author** r(t):=[cos(t),sin(t)] to parameterize the curve *C* by $\mathbf{r}(t) = \langle \cos(t), \sin(t) \rangle$, $0 \le t \le 2\pi$. Highlight this vector and plot its graph with the vector field; add the result to the axes at right.



3b. The commands r(t)sub1 and r(t)sub2 give the first and second components of **r**(*t*). (Try them.) Thus, **Author** f (r(t)sub1, r(t)sub2).r'(t) to calculate $\mathbf{F}(\mathbf{r}(t)) \bullet \mathbf{r}'(t)$. Now apply ∫ over $0 \le t \le 2\pi$ to find $\oint_C \mathbf{F} \bullet d\mathbf{r}$. Record this result below and tell whether Green's Theorem gives the same result.

3c. Suppose the integration in **3b** were taken over $\frac{3\pi}{4} \le t \le \pi$ instead; would the graph lead you to expect a positive or negative result? Why? What result does *Derive* give? Repeat for $\frac{3\pi}{2} \le t \le 2\pi$.