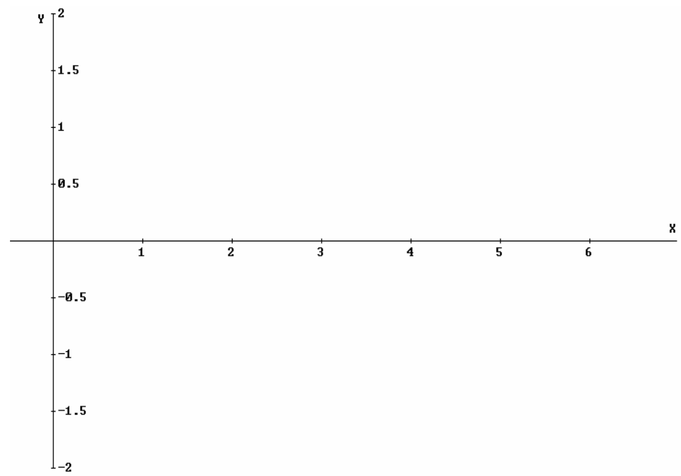
Assignment 14: Solids of Revolution (5.1-4) **Name____________________**
Please provide a handwritten response.

1. We know the integration used in calculating volume and surface area of solids of revolution can sometimes be difficult. Once we have an integral set up, *Derive* can make this task much easier by performing the actual integration. If the integral is not set up correctly, however, *Derive* will still produce an answer (usually). The result, of course, may not have anything to do with volume or surface area. So, although *Derive* can simplify the actual integration for us, we must still do most of the work by making sure our integral is correct.

For example, suppose the region bounded by $f(x) = \sin(x)$, the x-axis, $x = 0$, and $x = 2\pi$ is revolved around the x-axis. **Author** and plot f(x):=sin(x). Sketch the result on the axes at right. Shade the region being revolved and roughly sketch the 3-dimensional solid of revolution.

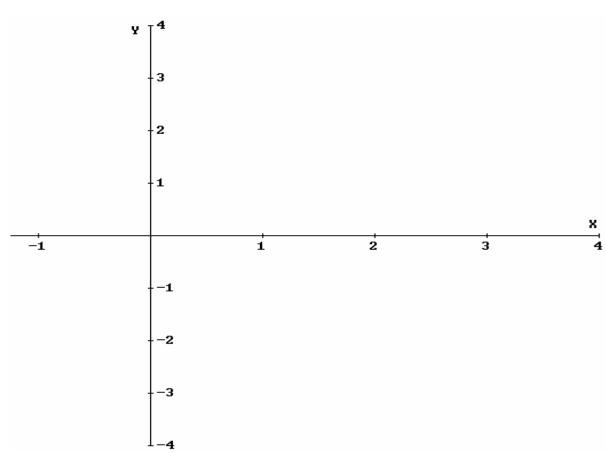


Since we can use either the disk method or the shell method to calculate volumes like this, we'll pick the most convenient method. For this example, the disk method will be easiest. (Why?) Also, note the symmetry of the graph. The solid of revolution is symmetric about $x = \pi$; we can calculate the volume between $x = 0$ and $x = \pi$ and double it. Therefore,

volume $= 2\int_0^{\pi} \pi[R(x)]^2\,dx = 2\int_0^{\pi} \pi[\sin(x)]^2\,dx$. Use $\int$ to set-up the definite integral; use $\approx$ to simplify the result numerically and record the result below. Is it correct?

2a. Use *Derive* to find the volume of the solid generated by revolving the region in the first quadrant bounded by $f(x) = x^2 \sin(x) - x$ and $y = 0$ (the x-axis) around the x-axis.

Author f(x):=x^2sin(x)-x and obtain a graph of the function. Sketch the results on the axes at right and shade the region being revolved. Also, roughly sketch the solid of revolution we are finding.



2b. We must first find the boundaries of the solid. This is difficult to do algebraically, so we'll use numerical solutions. Clearly one boundary is between $x = 1$ and $x = 2$; another one is between $x = 2$ and $x = 3$. Highlight $f(x)$ and click **Solve → Expression** and use the interval [1,2] to numerically solve for the first boundary. Simplify and record the result below. Now solve for the second boundary in a similar manner. Record its value below. From the graph, do these results appear correct?

2c. When revolving around the *x*-axis, the disk method should again be easier to use than the shell method. That is, the volume of this solid can be calculated as $\int_a^b \pi[f(x)]^2\,dx$; a is the lower boundary from **2b** and b is the upper boundary from **2b**. Enter this definite integral and approximate its value. Record the volume below; is the result plausible?

2d. Now calculate the volume generated by revolving the region around the *y*-axis. When revolving around the *y*-axis, the shell method should be easier to use than the disk method. (Why?) For this region, $p(x) = x$ and $h(x) = f(x)$. That is, the volume can be calculated as $\int_a^b 2\pi\, x\, f(x)\; dx$. Enter this definite integral and approximate its value. Record the result below. Does the result seem reasonable?

2e. Compare the two solids of revolution found in **2c** and **2d**. Using only the graph and our imagination, which one seems to be larger? Does this result agree with the approximations obtained from the integrals above?

3. Use *Derive* to calculate the surface area of the solid of revolution when the region in Question 3 is revolved around the *x*-axis. The formula given for surface area is $\int_a^b 2\pi\, f(x)\sqrt{1+[f'(x)]^2}\,dx$. (Recall that a and b were found in **2b**.) First **Author** 2pi*f(x)*sqrt(1+(f '(x))^2); then use ∫ to set-up the definite integral and ≈ to simplify; Record the result below.