

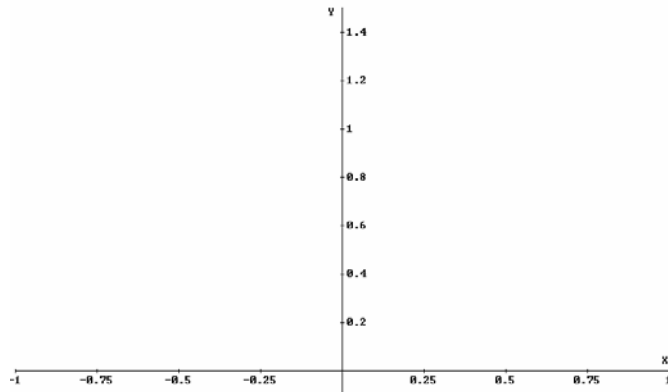
Assignment 5: Limits, Part I (1.1)
Please provide a handwritten response.

Name: _____

1a. Many ordinary limits can be found using *Derive*. For example, **Author** $\frac{3x+9}{x^2-9}$ and then click \lim . Enter a limit point of -3, select x as the variable, and click the dot next to “both” to have *Derive* calculate $\lim_{x \rightarrow -3} \frac{3x+9}{x^2-9}$. Click **OK** then use $=$ to simplify the expression and record the result below; is the answer what we would expect?

1b. Our text suggests $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$. **Author** the function and use *Derive* to find the limit. Does the answer support the conjecture made in the text?

2a. Examine $\lim_{x \rightarrow 0} \frac{\tan(x)}{\sin(x)}$ graphically and numerically. First, enter $f(x) = \frac{\tan(x)}{\sin(x)}$ by **Authoring** $f(x) := \tan(x)/\sin(x)$ then plot the function. Sketch the result on the axes to the right. What value for $\lim_{x \rightarrow 0} \frac{\tan(x)}{\sin(x)}$ does the graph suggest?



2b. Next, **Author** and use \approx to simplify $f(0.1)$, $f(0.01)$, etc to complete the table at right. What value for $\lim_{x \rightarrow 0} \frac{\tan(x)}{\sin(x)}$ does the table suggest?

x	$f(x)$
0.1	
0.01	
0.001	
-0.001	
-0.01	
-0.1	

2c. Finally, highlight $f(x)$ and use \lim to find the limit. Did all three approaches lead to the same conclusion? What trigonometric function does $f(x)$ simplify to?

3a. Round-off error can cause very misleading computed results. We will now use *Derive* to examine this. Enter the function $f(x) = \frac{\cos(x) - 1}{x^2}$ by **Authoring**

$f(x) := (\cos(x) - 1)/x^2$. Use *Derive* to complete the table at right. (Be sure to count the zeros!) Use the table to estimate $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x^2}$. Record the estimate below.

x	$f(x)$
0.1	
0.0001	
0.0000001	
0.00000001	
0.000000001	

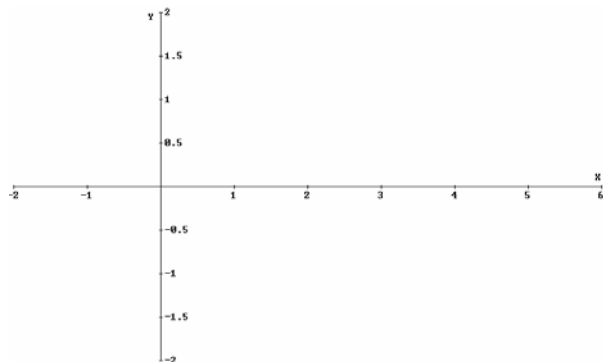
3b. Now use **lim** to find $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x^2}$ and record the result below.

3c. Do the results of 3a and 3b agree? If not, then which one do you think is correct? What does a plot of $f(x)$ show the limit to be? Repeatedly “zoom” in on the graph at $x = 0$. What do you see?

4a. To find one-sided limits, we simply select either right or left when using **lim**. For example, suppose we wanted to

examine $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$. First, we **Author**

$f(x) := \text{abs}(x - 2)/(x - 2)$ and plot the function to estimate the limit graphically. Sketch the result on the axes at right.



4b. Now find $\lim_{x \rightarrow 2^+} \frac{|x - 2|}{x - 2}$ by highlighting

$f(x)$, selecting **lim**, entering 2 as the limit point, x as the variable, and approach from “right”. Record the result below.

4c. Now find the limit from the left by selecting “left” and record the result below.

4d. Finally ask *Derive* to find the limit approaching from both directions. Record the result below; would you have expected this result? Why?