Assignment 16: Separable Differential Equations (6.5)   Name____________________
Please provide a handwritten response.

1a. The separable differential equation $y' = \dfrac{x^2 + \sqrt{x}}{e^{2y} + y - \sin(y)}$ is written $\int \left(e^{2y} + y - \sin(y)\right)dy = \int \left(x^2 + \sqrt{x}\right)dx$ with variables separated. To solve the equation in *Derive* we first treat each side separately; **Author** g(y):=int(exp(2y)+y-sin(y),y) and simplify using =. Record the result below.

Similarly, **Author** and simplify h(x):=int(x^2+sqrt(x),x); record the result below.

1b. The general solution of the differential equation is $g(y) = h(x) + c$. **Author** g(y)=h(x)+c and use = to simplify. Record the result below.

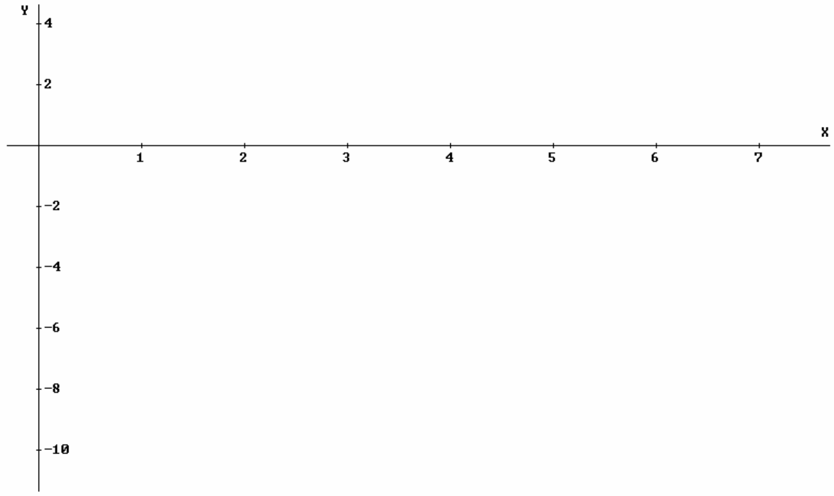
1c. We can form an Initial Value Problem (IVP) by adding the initial condition $y(1.5) = 1$. (This means *y* = 1 when *x* = 1.5.) To extract the value of *c* corresponding to this initial condition, first use SUB to substitute 1.5 for *x* into the solution we found in **1b**; secondly, substitute 1 for *y* into the solution we just found. Record the result below.

1d. Now, highlight this result and use  to algebraically find the exact value of *c*. Record the result below.

Before *Derive* can produce the particular solution, we must tell *Derive* that *c* is now a constant. To do this, highlight the last result. Using the mouse, right click in the **Author** expression box and choose **insert expression**. Insert a : immediately before the =. (Do not leave a space between the : and the =.) Press ENTER on the keyboard. Now *Derive* will "remember" the value of *c*!

To find the particular solution, substitute this value of *c* into the general solution. To do this, highlight the general solution obtained in **1b** and click = . Record the result below.

1e. We now want to graph the particular solution to the differential equation. Since it is impossible to solve for *y* we must create an implicit plot, which *Derive* does easily. Highlight the particular solution in **1d**, click and then again. Select an appropriate view and sketch the result on the axes at right. Use a large dot to mark the point on the curve corresponding to the initial condition.



1f. If there were no initial condition attached to our differential equation, we could create a family of particular solutions by letting *c* range, say, from -5 to 5; all these solutions could then be graphed on the same axes, showing how the solutions vary with *c*. To create these solutions, first highlight the general solution we found in **1b**. Click and type “vector(” then right click your mouse beside “(” and select **insert expression**. After the insertion, finish the vector by typing “,c,-5,5)” and press ENTER. *Derive* will create a solution with $c = -5$, then one with $c = -4$, then $c = -3$, and so on up to $c = 5$. (More than one attempt at this step may be needed. Keep trying!!)

Next, simplify our vector using =. Scroll across the screen and look at the different equations we just created. We have 11 different solutions to the original differential equation; each is different because of the value of *c*.

Click to return to the 2D-Plot window, click to delete the last plot, then click to plot our 11 different solutions. These solutions form our family of solutions. Sketch the result on the axes at right.