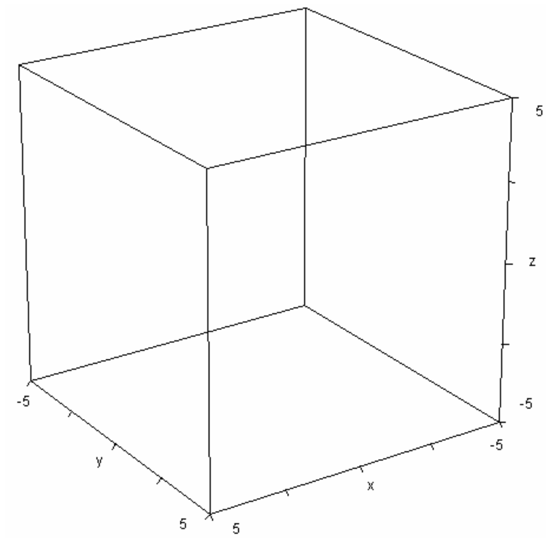
Assignment 27: Functions of Two Variables (12.1-2) **Name____________________**
Please provide a handwritten response.

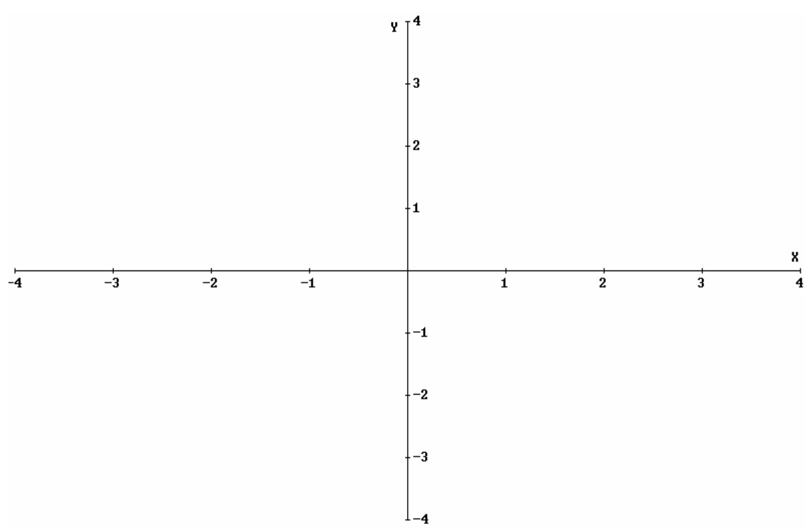
1a. To graph the function $f(x,y)=\sin\left(y-x^2\right)$, **Author** f(x,y):=sin(y-x^2) and click to open a 3D-Plot Window. This window operates much like the 2D-Plot Window. If the function we want to plot is highlighted, simply click from the shortcut menu. To rotate the graph, use the arrow keys on your keyboard or the rotation buttons ← → ↑ ↓. Click ; what happens?

Sketch the result in the box at right; rather than trying to draw every line generated by *Derive*, just use general outlines and shading to give the overall shape.



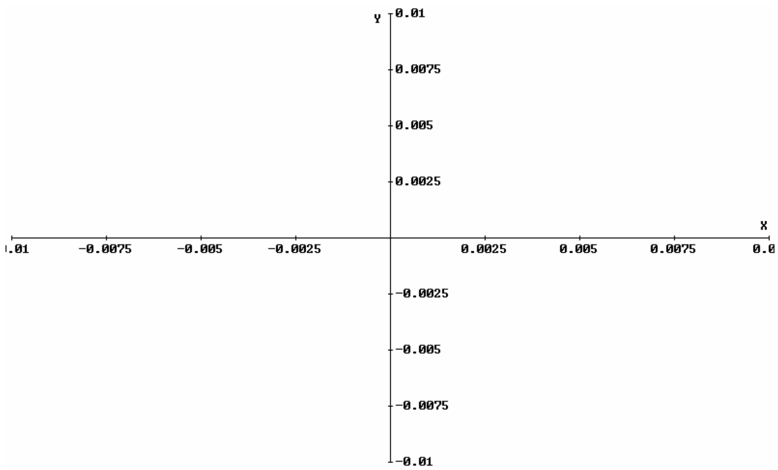
1b. We can use the zoom buttons, and , to maneuver around the axis and the buttons and to magnify and shrink the plot. Graph *f* over a wider range by changing the length of the *x*- and *y*-axis with **Set→Plot Range**. Describe the general appearance of the resulting surface.

1c. To draw a contour plot of *f*, we must create the level curves using the vector command. **Author** vector(f(x,y)=c,c,0,2,0.5) and simplify with = to generate a series of level curves. Open a 2D-Plot window with and plot the curves. Sketch the level curves on the axes at right; do the results agree with the 3D-Plot obtained above?



1d. Notice that we only used positive values of z (or c) above. Change the 0 to -2 and execute the commands again. How does this change the contour plot?

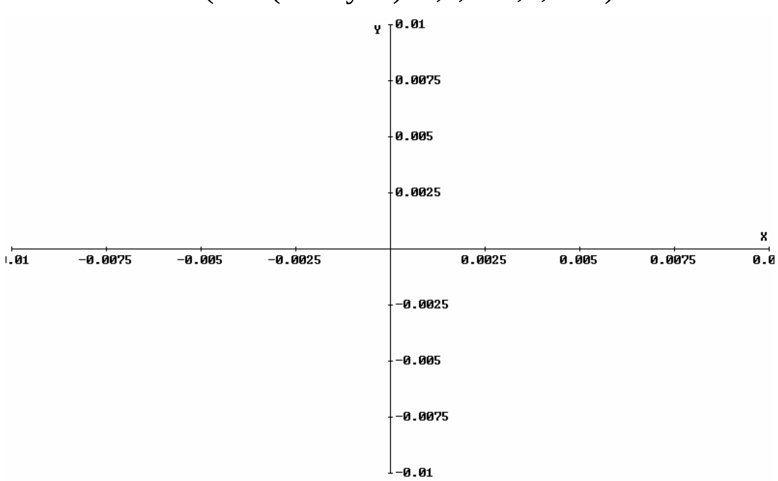
2a. The facts that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^2+y^2} = 0$ and that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2}{x^2+y^2}$ does not exist can be detected using contour plots. To examine the first limit, we **Author** vector(x^2y/(x^2+y^2)=c,c,-0.01,0.01,0.0001) and simplify using =; then plot the level curves with a range of -0.01 to 0.01 on both axes. (Use **Set→Plot Range**.) Sketch the result on the axes at right.



How did we come up with these values for c? We know the z-value should be close to 0, so we have used c-values on either side of zero to try to detect a pattern. Try changing the step-size (from 0.0001) and/or the min/max values. How do these changes affect the graph?

2b. How do these graphs support the conclusion that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^2+y^2} = 0$?

2c. Now examine the second limit. **Author** vector(x^2/(x^2+y^2)=c,c,-0.1,1,0.05) and simplify using =; then plot the level curves with a range of -0.01 to 0.01 on both axes. Sketch the result on the axes at right.



2d. How do these graphs support the conclusion that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2}{x^2+y^2}$ does not exist?

2e. Based on contour plots, does it appear that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x\sin(y)}{x^2+y^2}$ exists? Explain.