

Assignment 28: Partial Derivatives (12.3-7) Name _____
Please provide a handwritten response.

1a. The function $f(x, y, z) = e^{2xy} - \frac{z^2}{y} + xz \sin(y)$ can be defined on your calculator by entering **Define** $f(x, y, z) = e^{(2x * y) - z^2 / y + x * z * \sin(y)}$. Partial derivatives can be taken with respect to x , y , and z by executing $d(f(x, y, z), x)$, $d(f(x, y, z), y)$, and $d(f(x, y, z), z)$. Execute these commands and record the results below.

1b. The second-order mixed partial derivative $f_{yx}(x, y, z) = \frac{\partial}{\partial x \partial y}(f(x, y, z)) = \frac{\partial}{\partial x} \left(\frac{\partial}{\partial y}(f(x, y, z)) \right)$ can be found by executing $d(d(f(x, y, z), y), x)$. Record the result below.

1c. Now find $f_{yy}(x, y, z) = \frac{\partial^2}{\partial y^2}(f(x, y, z))$ by executing $d(d(f(x, y, z), y), y)$ and record your result below. Further, evaluate $f_{yy}(-0.2, 3, \sqrt{7})$ by executing $d(d(f(x, y, z), y), y) / x = -0.2 \text{ and } y = 3 \text{ and } z = \sqrt{7}$ and recording the result below.

2a. Graph $f(x, y) = x^3 + 3xy - y^3$ by setting the **MODE** to **3d**, setting the format screen from the **Y=** editor by pressing **F1 9** and setting **Axes** to **BOX** and **Style** to **WIRE AND CONTOUR**, entering $z1 = x^3 + 3x * y - y^3$ and setting the window to $-0.5 \leq x \leq 1.5$, $-1.5 \leq y \leq 0.5$, and $-1 \leq z \leq 5$, $eye\theta = 70$, $eye\phi = 60$, $eye\Psi = 0$. Is it clear what type of critical point(s) f has over this range? What type are they?

2b. Change the window to $eye\theta = 135$, $eye\phi = 90$, $eye\Psi = 0$ and graph the function. Record what you can see regarding the critical points. What was the effect of the extra option?

2c. Calculate the gradient $\nabla f(x, y)$ as $v(x, y) = [d(z1(x, y), x), d(z1(x, y), y)]$ and record the result below.

2d. The critical points of the $f(x, y)$ occur where $\nabla f(x, y) = \mathbf{0}$. Find these critical points as indicated in the table below.

	TI-89	Voyage 200
Locating the critical of $z = f(x, y)$ fr $\nabla f(x, y) = \mathbf{0}$	Use the solver by entering: $\text{solve}(d(z1(x, y), x) = \mathbf{0} \text{ and}$ $d(z1(x, y), y) = \mathbf{0}, \{x, y\})$	Solve $d(z1(x, y), x) = \mathbf{0}$ and $d(z1(x, y), y) = \mathbf{0}$ for y and graph th simultaneously after setting the mode function . Use the intersection option the F5 Math menu to find the x and y values for the points of intersection.

Do these points appear consistent with the graph in **2b**? To compute $f(3, 2)$ enter $z1(3, 2)$. Use this command to find the corresponding z -values for your critical points. Record them below.

2e. To apply the second derivative test you must compute the discriminant, $d(d(z1(x, y), x), x) * d(d(z1(x, y), y), y) - d(d(z1(x, y), x), y)$. Evaluate the discriminant at each critical point found in **2d**. (You can use the technique from **1c** to accomplish this.)

3a. Graph $f(x, y) = (x^2 - 3xy + 3y^2 + 4x)e^{-2x^2 - \frac{1}{2}y^2} + \sin\left(\frac{x+y}{100}\right)$ by entering

$z1 = (x^2 - 3x * y + 3y^2 + 4x) * e^{(-2x^2 - y^2 / 2)} + \sin((x + y) / 100)$ and setting the window to $-1.5 \leq x \leq 1.5, -3 \leq y \leq 3, -1 \leq z \leq 4$. Make sure your calculator is in **3D MODE**. How many critical points does f seem to have over this range?

3b. Change the format (from the graph screen) press **F1 9 format style** to **CONTOUR LEVELS** and regraph the function setting . Use your results so far to give the rough coordinates of each of the critical points and what type of critical point it is.

3c. Using the solve command does not find all the critical points for this function. Use this command (as in **2d**) to find the critical point near $(-0.5, -0.1)$. Record the result below. Change the **format style** to **WIRE AND CONTOUR** and the window settings to $eye\theta = -90, eye\phi = \mathbf{0}, eye\Psi = \mathbf{0}$. Plot the function and use the trace function to approximate the other critical values within this range. Can you approximate them any more closely? Record your results below.