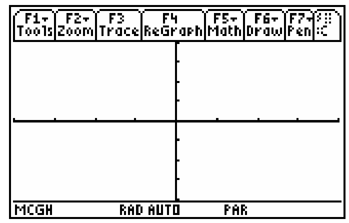
Assignment 26: Vector-Valued Functions, Part II (11.1-5)   Name_________________
Please provide a handwritten response.

1a. The Cornu spiral can be drawn on your calculator by defining $xt1(t) = \int(cos(\pi u \wedge 2/2), u, 0, t)$ and $yt1(t) = \int(sin(\pi u \wedge 2/2), u, 0, t)$. Set your window to $-\pi \le t \le \pi, -.8 \le x \le .8, -.8 \le y \le .8$ and graph. Sketch the result on the axes below. These integrals are known as "Fresnel integrals" and are used in applied mathematics.



$-.8 \le x \le .8, -.8 \le y \le .8$

1b. To find the arc length of the curve from $t = 0$ to $t = c$ you can use the formula $\int_0^c \sqrt{\left(\frac{dxt1}{dt}\right)^2 + \left(\frac{dyt1}{dt}\right)^2}\, dt$ by entering $\int\left(\sqrt{}\left(\left(d\left(xt1(t), t\right)\right) \wedge 2 + \left(d\left(yt1(t), t\right)\right) \wedge 2\right), t, 0, c\right)$. Record the result below. What does this say about the parameterization of this curve?

1c. To find the curvature at $t = c$ we can use the formula $\kappa = \frac{\|\vec{T}\,'(t)\|}{\|\vec{r}\,'(t)\|}$ where $\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|}$ is the unit tangent vector and define $\vec{v}(t) = d(\vec{r}(t), t)$. Then define the unit tangent vector as $\vec{u}(t) = \vec{v}(t) / norm(\vec{v}(t))$, its derivative as $\vec{s}(t) = d(\vec{u}(t), t)$ and define $k = norm(\vec{s}(t)) / norm(\vec{v}(t))$. Record the result below. What does this say about the curve?

2a. Given the vector valued function $\vec{r}(t) = \langle cos\, t, ln\, t, sin\, t \rangle$. Find the unit tangent vector $\vec{T}(t)$ at $t = \frac{\pi}{2}$ and record your result below.

2b. Now find the curvature $\boldsymbol{\kappa}$ for this function at $\boldsymbol{t = \frac{\pi}{2}}$ and record the result below. What does this say about the curve at this point?

3a. Find the unit tangent vector $\vec{\boldsymbol{T}}(\boldsymbol{t}) = \frac{\vec{\boldsymbol{r}}\,'(\boldsymbol{t})}{\|\vec{\boldsymbol{r}}\,'(\boldsymbol{t})\|}$ for $\vec{\boldsymbol{r}}(\boldsymbol{t}) = \langle \boldsymbol{t}, \boldsymbol{2t}, \boldsymbol{t^3} \rangle$ at $\boldsymbol{t = 0}$ and at $\boldsymbol{t = 1}$. Use the formulas from **1c.** Be sure to define $\vec{\boldsymbol{T}}(\boldsymbol{t})$ so you can use it for later calculations. Record your results below.

3b. Compute the curvature $\boldsymbol{\kappa}$ as in **2b** for this function at $\boldsymbol{t = 0}$ and at $\boldsymbol{t = 1}$ and record your results below.

3c. The principal unit normal vector, $\vec{\boldsymbol{N}}(\boldsymbol{t})$ can be found by computing $\vec{\boldsymbol{N}}(\boldsymbol{t}) = \frac{\vec{\boldsymbol{T}}\,'(\boldsymbol{t})}{\|\vec{\boldsymbol{T}}\,'(\boldsymbol{t})\|}$. Be sure to define $\vec{\boldsymbol{N}}(\boldsymbol{t})$ so you can use it for later calculations. Compute $\vec{\boldsymbol{N}}(\boldsymbol{t})$ at $\boldsymbol{t = 0}$ and at $\boldsymbol{t = 1}$ and record your results below.

3d. The binormal vector, $\vec{\boldsymbol{B}}(\boldsymbol{t})$, is defined to be $\vec{\boldsymbol{B}}(\boldsymbol{t}) = \vec{\boldsymbol{T}}(\boldsymbol{t}) \times \vec{\boldsymbol{N}}(\boldsymbol{t})$ and is orthogonal to both $\vec{\boldsymbol{T}}(\boldsymbol{t})$ and $\vec{\boldsymbol{N}}(\boldsymbol{t})$. Calculate $\vec{\boldsymbol{B}}(\boldsymbol{t}) = \boldsymbol{crossP}(\vec{\boldsymbol{T}}(\boldsymbol{t}), \vec{\boldsymbol{N}}(\boldsymbol{t}))$ at $\boldsymbol{t = 0}$ and at $\boldsymbol{t = 1}$ and record the results below.