Special Constants

$\pi \approx 3.1416$ $\quad \frac{\pi}{2} \approx 3.1416$ $\quad \frac{\pi}{3} \approx 1.0472$ $\quad \frac{\pi}{4} \approx 0.7854$ $\quad \frac{\pi}{6} \approx 0.5236$ $\quad \frac{\pi}{12} \approx 0.2618$

$e \approx 2.7183$ $\quad \sqrt{2} \approx 1.4142$ $\quad \sqrt{3} \approx 1.7321$ $\quad \frac{\sqrt{2}}{2} \approx 0.7071$ $\quad \frac{\sqrt{3}}{2} \approx 0.8660$ $\quad \frac{\sqrt{3}}{3} \approx 0.5774$

Special Products

$$(x + a)(x + b) = x^2 + (a + b)x + ab \qquad (a + b)(a - b) = a^2 - b^2$$
$$(a + b)^2 = a^2 + 2ab + b^2 \qquad (a - b)^2 = a^2 - 2ab + b^2$$
$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \qquad (a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Special Factorizations

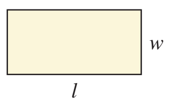
$$x^2 + (a + b)x + ab = (x + a)(x + b) \qquad a^2 - b^2 = (a + b)(a - b)$$
$$a^2 + 2ab + b^2 = (a + b)^2 \qquad a^2 - 2ab + b^2 = (a - b)^2$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2) \qquad a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Formulas from Plane Geometry: $P \longrightarrow$ perimeter, $C \longrightarrow$ circumference, $A \longrightarrow$ area

Rectangle

$P = 2l + 2w$

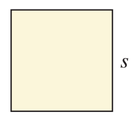
$A = lw$



Square

$P = 4s$

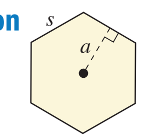
$A = s^2$



Regular Polygon

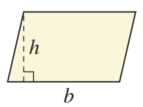
$P = ns$

$A = \frac{a}{2}P$



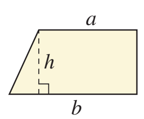
Parallelogram

$A = bh$



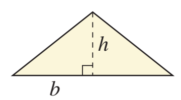
Trapezoid

$A = \frac{h}{2}(a + b)$



Triangle

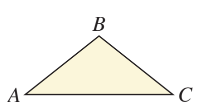
$A = \frac{1}{2}bh$



Triangle

Sum of angles

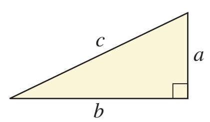
$A + B + C = 180°$



Right Triangle

Pythagorean Theorem

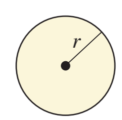
$a^2 + b^2 = c^2$



Circle

$A = \pi r^2$

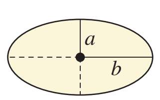
$C = 2\pi r = \pi d$



Ellipse

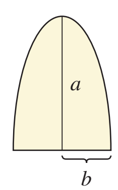
$A = \pi ab$

$C \approx \pi\sqrt{2(a^2 + b^2)}$



Right Parabolic Segment

$A = \frac{4}{3}ab$



Formulas from Solid Geometry: $S \longrightarrow$ surface area, $V \longrightarrow$ volume

Rectangular Solid

$V = lwh$

$S = lw + lh + wh$

Cube

$V = s^3$

$S = 6s^2$

Right Circular Cylinder

$V = \pi r^2 h$

$S = 2\pi r(r + h)$

Right Circular Cone

$V = \frac{1}{3}\pi r^2 h$

$S = \pi r(r + s)$

Right Square Pyramid

$V = \frac{1}{3}b^2 h$

$S = b^2 + b\sqrt{b^2 + 4h^2}$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Formulas from Analytical Geometry: $P_1 \longrightarrow (x_1, y_1)$, $P_2 \longrightarrow (x_2, y_2)$

Distance between P_1 and P_2

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Slope of Line Containing P_1 and P_2

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of Line Containing P_1 and P_2

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Equation of Line Containing P_1 and P_2

Slope-Intercept Form (slope m, y-intercept b)

$$y = mx + b, \text{ where } b = y_1 - mx_1$$

Parallel Lines

Slopes Are Equal: $m_1 = m_2$

Perpendicular Lines

Slopes Have a Product of -1: $m_1 m_2 = -1$

Intersecting Lines

Slopes Are Unequal: $m_1 \neq m_2$

Dependent (Coincident) Lines

Slopes and y-Intercepts Are Equal: $m_1 = m_2$, $b_1 = b_2$

Logarithms and Logarithmic Properties

$y = \log_b x \Leftrightarrow b^y = x$ $\qquad \log_b b = 1$ $\qquad \log_b 1 = 0$

$\log_b b^x = x$ $\qquad b^{\log_b x} = x$ $\qquad \log_c x = \frac{\log_b x}{\log_b c}$

$\log_b MN = \log_b M + \log_b N$ $\qquad \log_b \frac{M}{N} = \log_b M - \log_b N$ $\qquad \log_b M^P = P \cdot \log_b M$

Applications of Exponentials and Logarithms

$A \rightarrow$ amount accumulated $\qquad P \rightarrow$ initial deposit, $p \rightarrow$ periodic payment $\qquad n \rightarrow$ compounding periods/year

$r \rightarrow$ interest rate per year $\qquad R \rightarrow$ interest rate per time period$\left(\frac{r}{n}\right)$ $\qquad t \rightarrow$ time in years

Interest Compounded n Times per Year

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Interest Compounded Continuously

$$A = Pe^{rt}$$

Accumulated Value of an Annuity

$$A = \frac{p}{R}[(1 + R)^{nt} - 1]$$

Payments Required to Accumulate Amount A

$$p = \frac{AR}{(1 + R)^{nt} - 1}$$

Sequences and Series:

$a_1 \rightarrow$ 1st term, $a_n \rightarrow$ nth term, $S_n \rightarrow$ sum of n terms, $d \rightarrow$ common difference, $r \rightarrow$ common ratio

Arithmetic Sequences

$$a_1, a_2 = a_1 + d, a_3 = a_1 + 2d, \ldots, a_n = a_1 + (n - 1)d$$
$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

Geometric Sequences

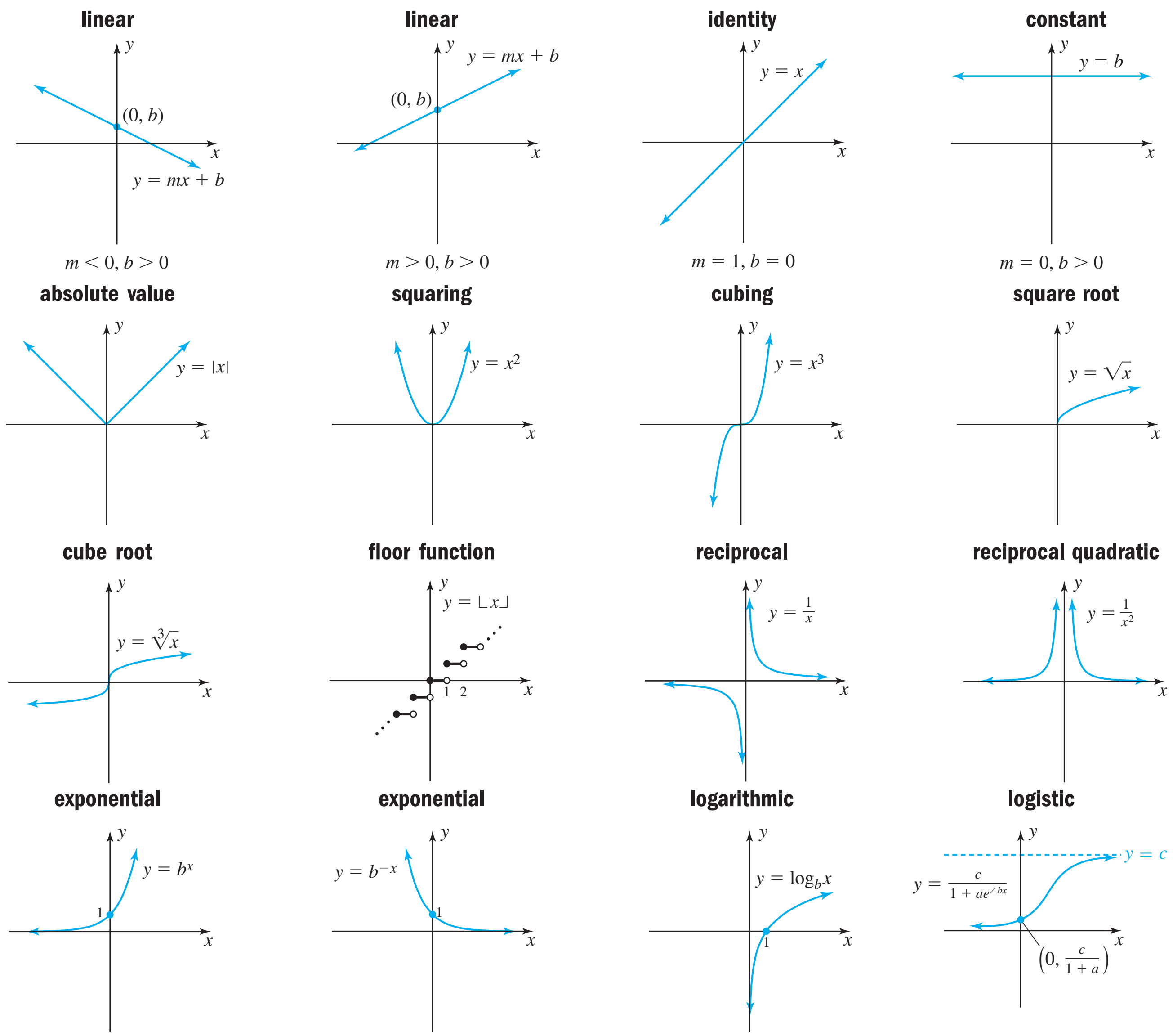
$$a_1, a_2 = a_1 r, a_3 = a_1 r^2, \ldots, a_n = a_1 r^{n-1}$$
$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$
$$S_\infty = \frac{a_1}{1 - r}; \ |r| < 1$$

Binomial Theorem

$$(a + b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n$$

$$n! = n(n - 1)(n - 2)\cdots(3)(2)(1) \qquad \binom{n}{k} = \frac{n!}{k!(n - k)!}; \qquad 0! = 1$$

The Toolbox and Other Functions



Transformations of Basic Graphs

Given Function

$y = f(x)$

Transformation of Given Function

$y = af(x \pm h) \pm k$

- a: vertical reflections, vertical stretches/compressions
- h: horizontal shift h units, opposite direction of sign
- k: vertical shift k units, same direction as sign

Average Rate of Change of $f(x)$

For linear function models, the average rate of change on the interval $[x_1, x_2]$ is constant, and given by the slope formula: $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$. The average rate of change for other function models is non-constant. By writing the slope formula in function form using $y_1 = f(x_1)$ and $y_2 = f(x_2)$, we can compute the average rate of change of other functions on this interval:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Quick Counting and Probability

Fundamental Counting Principle: Given an experiment with two tasks completed in sequence, if the first can be completed in m ways and the second in n ways, the experiment can be completed in $\boldsymbol{m \times n}$ ways.

Permutations—Order Is a Consideration: (Al, Bo, Ray) and (Ray, Bo, Al) finish the race in a different order.

The permutations of r objects selected from a set of n (unique) objects is given by $\boldsymbol{{}_nP_r = \frac{n!}{(n-r)!}}$.

Combinations—Order Is Not a Consideration: (Al, Bo, Ray) and (Ray, Bo, Al) form the same committee.

The combinations of r objects selected from a set of n (unique) objects is given by $\boldsymbol{{}_nC_r = \frac{n!}{r!(n-r)!}}$.

Basic Probability: Given S is a sample space of equally likely events and E is an event defined relative to S.

The probability of E is $\boldsymbol{P(E) = \frac{n(E)}{n(S)}}$, where $n(E)$ and $n(S)$ represent the number of elements in each.

For any event E_1: $0 \le P(E_1) \le 1$ and $P(E_1) + P(\sim E_1) = 1$.

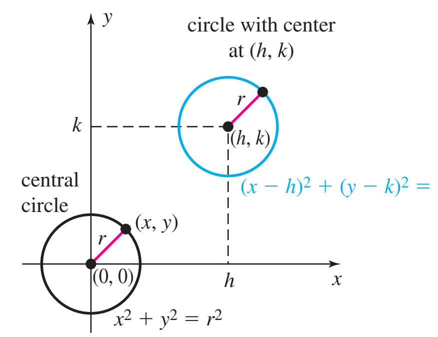
Probability of E_1 and E_2

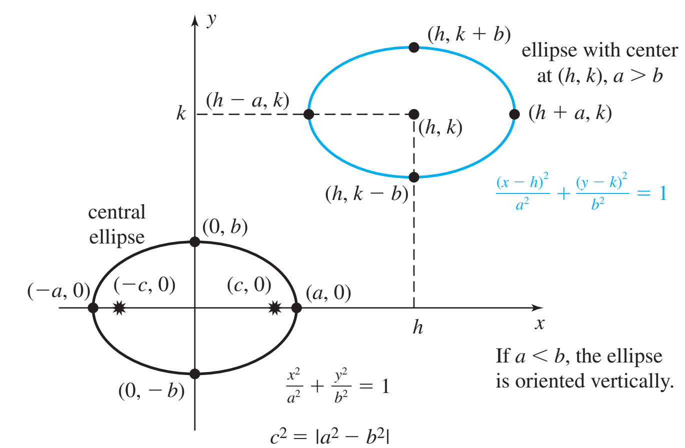
$$P(E_1 \cap E_2) = P(E_1)P(E_2)$$

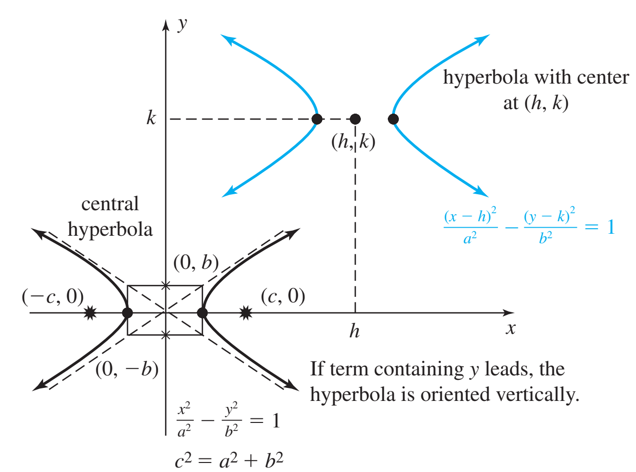
Probability of E_1 or E_2

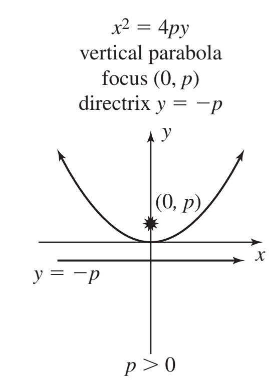
$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

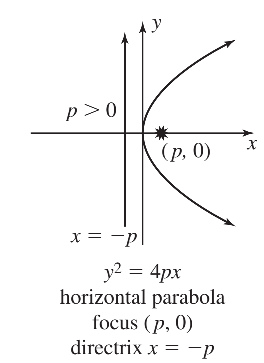
Conic Sections











Commonly used, small case Greek letters

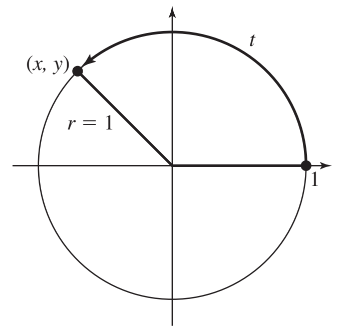
α	alpha	β	beta	γ	gamma	δ	delta	ϵ	epsilon
ζ	zeta	θ	theta	λ	lamda	μ	mu	π	pi
ρ	rho	σ	sigma	ϕ	phi	ψ	psi	ω	omega

Trigonometric Functions of a Real Number

For any real number t and point $P(x, y)$ on the unit circle associated with t:

$\cos t = x$ $\quad \sin t = y$ $\quad \tan t = \frac{y}{x};\ x \neq 0$

$\sec t = \frac{1}{x};\ x \neq 0$ $\quad \csc t = \frac{1}{y};\ y \neq 0$ $\quad \cot t = \frac{x}{y};\ y \neq 0$

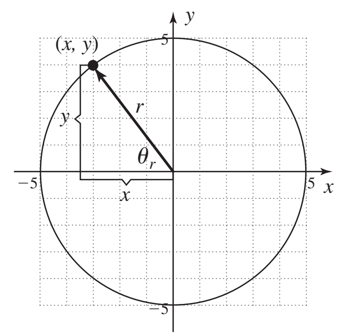


Trigonometry and the Coordinate Plane

For $P(x, y)$ a point on the terminal side of an angle θ in standard position:

$\cos\theta = \frac{x}{r}$ $\quad \sin\theta = \frac{y}{r}$ $\quad \tan\theta = \frac{y}{x},\ x \neq 0$

$\sec\theta = \frac{r}{x},\ x \neq 0$ $\quad \csc\theta = \frac{r}{y},\ y \neq 0$ $\quad \cot\theta = \frac{x}{y},\ y \neq 0$

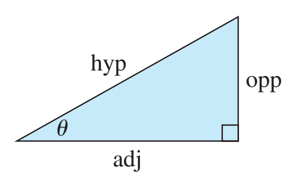


Right Triangle Trigonometry

For right ΔABC with indicated sides **adj**acent and **opp**osite to acute angle θ:

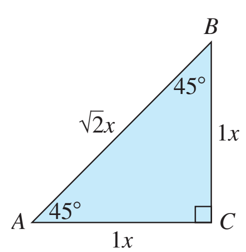
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\quad \sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\quad \tan\theta = \frac{\text{opp}}{\text{adj}}$

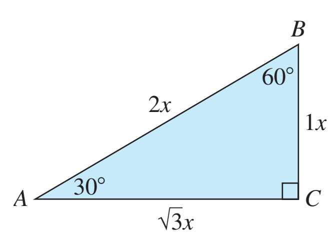
$\sec\theta = \frac{\text{hyp}}{\text{adj}}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}}$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}}$



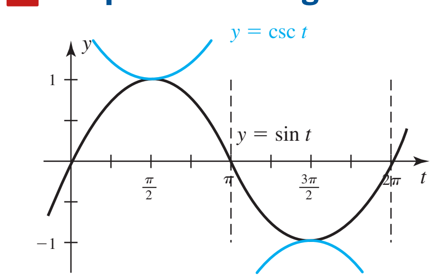
Special Triangles and Special Angles

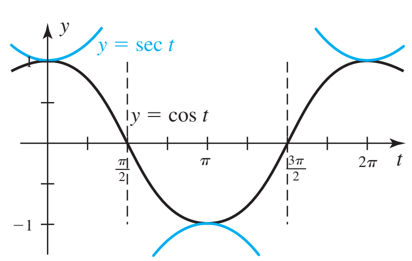
θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0°	0	1	0	—	1	—
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90°	1	0	—	1	—	0

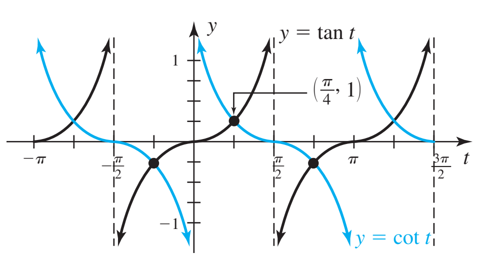




Graphs of the Trigonometric Functions







Fundamental Identities

Reciprocal Identities

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Ratio Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Identities due to Symmetry

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Sum and Difference Identities

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Double-Angle Identities

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

Half-Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1 - \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 + \cos\theta}$$

Power Reduction Identities

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Product-to-Sum Identities

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

Sum-to-Product Identities

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

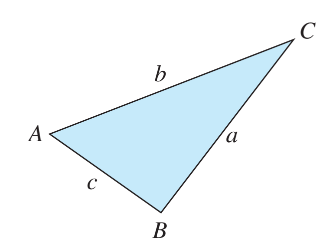
$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Area of a Triangle

$$A = \frac{1}{2}bc\sin A$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$