R.8 Expressions, Equations and Graphing Calculators

Introduction

In the last 30 years, no other tool, technique or innovation could match the impact of graphing and calculating technology on the teaching and learning of mathematics. The ability to do complex calculations using a prescribed sequence of keystrokes has fueled a healthy debate over ...

① What knowledge students need to <u>*own*</u> (meaning connections, skills and calculations can be done mentally, or manually with little effort), and

② What knowledge students can gain from information that is <u>*accessed*</u> (meaning graphing or calculating technology can be used to construct a finished result from basic concepts, leaving subordinate skills and computations for the calculator).

The purpose of this section is to help you become better acquainted with graphing and calculating technology, leaving questions regarding the extent and timing of its use to you and your instructor. To facilitate this endeavor, all keystrokes and illustrations are given using a TI-84 Plus graphing calculator. The keystrokes can easily be adapted for other models and keystrokes for other popular models can be found at www.mhhe.com/coburn. We finish this introduction with one final note: To help prevent errors when using any form of technology, always attempt to *estimate the answer* or *forecast the magnitude of the result <u>first</u>*, then be sure the computed result is in line with this estimate.

Learning Objectives

In Section R.8 you will use a graphing calculator and learn how to:

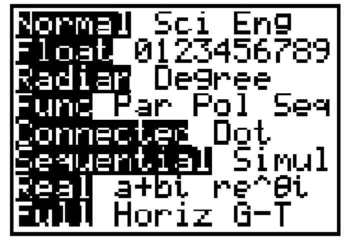
A) Perform basic operations
B) Calculate with exponents and roots
C) Use storage and recall features
D) Use the TABLE feature
E) Edit expressions on the home screen

Historical Reference/Points of Interest

- As a tool for doing basic computations, the abacus is fast, efficient and still widely used, particularly in the far east.
- The calculating device known as a **slide rule** was invented in 1632 and remained in common use until the mid to late 20th century.
- The first hand-held calculators made their appearance in the early 1940's.

A) Basic Operations

Figure 1



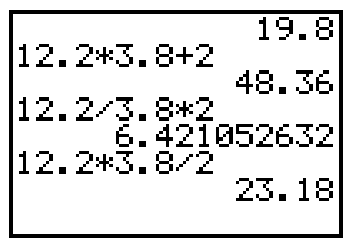
Before beginning, be sure your calculator is in the default mode -- after pressing the MODE key, all of the options on the left should be highlighted (see Figure 1). If one or two are not, use the arrow keys to navigate the cursor to overlay the "culprit" and press ENTER.

Basic operations on a graphing calculator mimic those of a standard calculator, with one notable exception -- the entire exercise can be reviewed, even after the calculation is complete. As a matter of good practice or as the need arises, the home screen can be cleared of all old calculations by pressing the CLEAR key located beneath the arrow keys. As you will soon notice, graphing calculators use the "∗" symbol for multiplication and a back-slash "/" for division.

• Order of Operations

Graphing calculators are programmed to use what is commonly known as the **order of operations**. Consider these exercises: ① 12.2 + 3.8 × 2; ② 12.2 × 3.8 + 2; ③ 12.2 ÷ 3.8 × 2; and ④ 12.2 × 3.8 ÷ 2. For ①, we expect the result to be near 20, since multiplications are completed before additions. The result of ② will be near 50 for the same reason. In ③ and ④, the operations have equal rank and are completed in order from left to right, giving estimated results of 6 and 24 respectively. See Figure 2.

Figure 2



Example 1: Evaluate the following expressions on a graphing calculator. Estimate each result first and compare the calculated answer with your estimate:

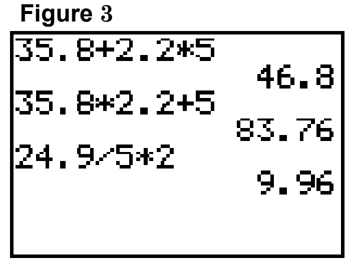
a. 35.8 + 2.2 × 5 b. 35.8 × 2.2 + 5 c. 24.9 ÷ 5 × 2

solution: a. Estimate: 36 + 2 × 5 = 46 multiply first

b. Estimate: 36 × 2 + 5 = 77 multiply first

c. Estimate: 25 ÷ 5 × 2 = 10 divide first

Figure 3



Now try Exercises 7 through 12

• Working with Negative Numbers

On a graphing calculator, a negative number is entered in the same way it is written -- by entering the negative sign using the [(–)] key, *then* entering the number. To compute the sum -43 + (-21), the sequence of keystrokes would be: [(–)] 43 [+] [(–)] 21 [ENTER] and the display reads -64. The keystrokes for (-6.05)(3.1) would be: [(–)] 6.05 [×] 3.01 [ENTER], yielding a result of -18.2105 (see Figure 4).

Figure 4

```
-43+-21
                -64
-6.05*3.01
           -18.2105
```

Example 2: Given x = -20.1, y = -7.9 and z = 5, evaluate each expression on a graphing calculator. Estimate first, then compare the calculated result (see Figure 5) with your estimate:

a. xy b. x + y c. yz ÷ x

solution: a. Estimate: (-20)(-8) = 160.

b. Estimate: -20 + (-8) = -28.

c. Estimate: (-8)(5) ÷ (-20) = 2 multiply first

Figure 5

```
-20.1*-7.9
             158.79
-20.1+-7.9
                -28
-7.9*5/-20.1
        1.965174129
```

Now try Exercises 13 through 20

B) Calculating with exponents and roots

• Exponents

The [^] key (located directly beneath the [CLEAR] key) is used to evaluate exponential terms. For instance $3^2 \rightarrow 3$ [^] 2 returns a value of 9, and $2^{-1} \rightarrow 2$ [^] [(–)] 1 gives a value of 0.5. Note that 2^{-1} could also be evaluated using the reciprocal key: 2 [x^{-1}], and that 0.5 can be converted to fraction form using the keystrokes 0.5 [MATH] **1:▶Frac**, with result $\frac{1}{2}$. Since the squaring operation is very common, many calculators provide a separate key for squaring a quantity. On the TI-84 Plus, [x^2] is located in the middle of the left-most column.

Example 3: Given x = 1.9 and z = -4.9, evaluate each expression on a graphing calculator. Estimate first, then compare the calculated result (see Figure 6) to your estimate:

a. z^2 b. $-z^2$ c. xz^3 d. $(x + z)^3$

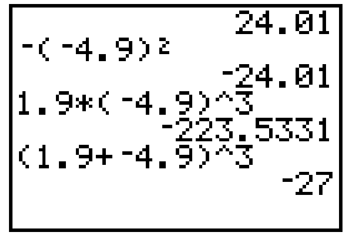
solution: a. Estimate: $(-5)^2 = 25$.

b. Estimate: $-(-5)^2 = -25$.

c. Estimate: $(2)(-5)^3 = -250$ exponent first

d. Estimate: $[2 + (-5)]^3 = -27$ simplify first

Figure 6



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Now try Exercises 21 through 26

• Rational Exponents

Many common rational exponents have a decimal equivalent that terminates after one, two or three decimal places. For instance, $\frac{1}{2} = 0.5$, $\frac{3}{4} = 0.75$ and $\frac{5}{8} = 0.625$. To evaluate expressions of this type, the decimal form is most convenient. The expressions $29^{\frac{1}{2}}$, $81^{\frac{3}{4}}$ and $92^{\frac{5}{8}}$ are evaluated on the home screen shown in Figure 7. If you are unsure of the decimal equivalent, or if the decimal equivalent has a non-terminating form, the rational exponent should be expressed as a division and ***grouped within parentheses***. The expressions $53^{\frac{1}{6}}$, $8^{\frac{-2}{3}}$ and $71^{\frac{4}{9}}$ are evaluated on the screen shown in Figure 8. The related keystrokes for $53^{\frac{1}{6}}$ are: 53 [^] [(] 1 [÷] 6 [)]. We know the answer should be between 1 and 2 since $\sqrt[6]{64} = 2$. Remember that you must press [ENTER] to evaluate the expression after it is entered.

Figure 7

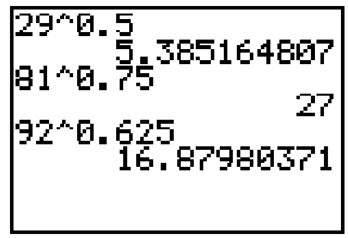
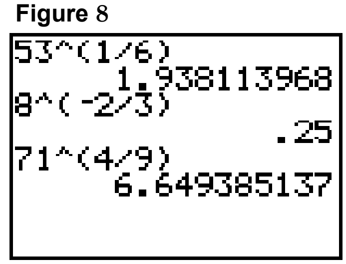


Figure 8

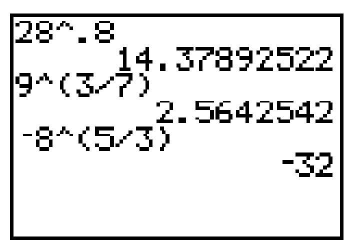


Example 4: Evaluate each expression on a graphing calculator. If the result is an integer, show why using properties of exponents. a. $28^{\frac{4}{5}}$ b. $9^{\frac{3}{7}}$ c. $-8^{\frac{5}{3}}$

solution: Only $-8^{\frac{5}{3}}$ yields a rational number, since

$$-8^{\frac{5}{3}} = -\left(8^{\frac{1}{3}}\right)^5 = -2^5 = -32.$$

Figure 9



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Now try Exercises 27 through 42

• Scientific Notation

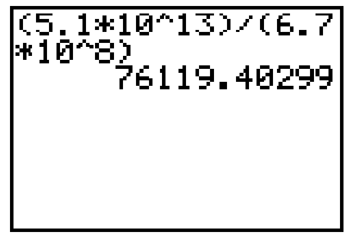
In many scientific applications we encounter numbers that are extremely large or very, very small. For example, light travels at approximately 186,000 miles per second, or close to

670,000,000 miles per hour. Even at this great speed, it still takes <u>years</u> for a beam of light from the earth to reach the nearest star. Consider the star known as Sirius A, which is about 51,000,000,000,000 miles away. We <u>could</u> compute the time it takes a beam of light to reach this star by hand, but with numbers this large, it's more convenient to use scientific notation. Using $T = \frac{D}{R}$ we have $\frac{51,000,000,000,000}{670,000,000} = \frac{5.1\times10^{13}}{6.7\times10^{8}}$. Although most graphing calculators have a "power of 10" key (often the [2nd] function for the [LOG] key), it may be easier to enter the division problem directly using the [^] key for exponents, and parentheses to group the numerator and denominator separately. The required keystrokes are:

[( ] 5.1 [×] 10 [^] 13 [ )] [÷] [( ] 6.7 [×] 10 [^] 8 [ )].

After pressing [ENTER] the number 76119.40299 shows on the display.

Figure 10

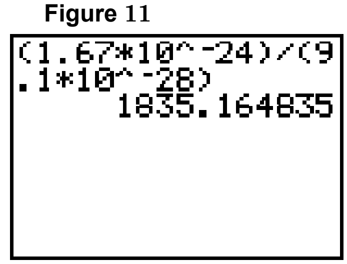


It would take approximately 76,119 hours for a beam of light to reach Sirius A, well over 8.5 years!

Example 5: The mass of an electron is approximately 0.000 000 000 000 000 000 000 000 000 91 grams. The mass of a proton is about 0.000 000 000 000 000 000 000 001 67 grams. How many times heavier is a proton than an electron?

solution: In scientific notation, a proton weighs 1.67 x 10^{-24} grams. An electron weighs 9.1 x 10^{-28} grams. To find how many times heavier a proton is, we have: $\frac{1.67\times10^{-24}}{9.1\times10^{-28}}$. A proton is over 1,835 times heavier.

Figure 11



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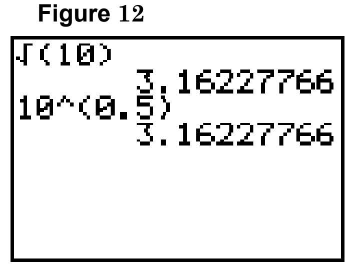
Now try Exercises 43 through 50

• The square root of a number

On the TI-84 Plus, the square root operation is the second function for the squaring operation. The positive square root of 10 is found using the keystrokes: [2nd] [x^2] 10 [ )] [ENTER]. Note the calculator supplies the left parenthesis automatically -- you need only

Figure 12



enter the number and close the group. Since $\sqrt{9} = 3$, we know $\sqrt{10}$ must be slightly more than 3. As an alternative, note that a rational exponent could be used (see Figure 12).

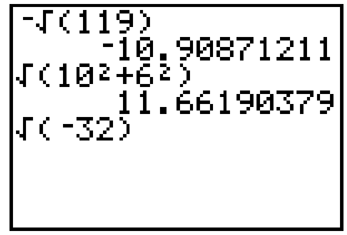
Example 6: Estimate the value of each expression, then use your calculator to obtain a decimal approximation. a. $-\sqrt{119}$ b. $\sqrt{10^2 + 6^2}$ c. $\sqrt{-32}$

solution: a. Estimate: $-\sqrt{121} = -11$.

b. Estimate: $\sqrt{144} = 12$ simplify first

c. $\sqrt{-32}$ is not a real number. After pressing [ENTER],

ERR: NONREAL ANS appears on a new screen.



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Now try Exercises 51 through 60

• The cube root of a number

For cube roots and other indices, we often have to select the operation from a sub-menu. On the TI-84 Plus these are accessed by pressing the [MATH] key, located below the green [ALPHA] key. The keystrokes for $\sqrt[3]{35}$ are: [MATH] **4:**$\sqrt[3]{\ }$(35 [ )] [ENTER]. Since $\sqrt[3]{27} = 3$, $\sqrt[3]{35}$ will be slightly larger (see Figures 13 and 14). The "4" in the sequence shown indicates your choice of "option 4" (press 4) from the sub-menu. You could also move the cursor to option 4 using the arrow keys.

Figure 13

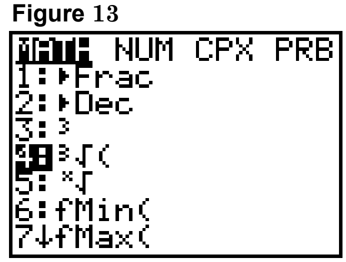
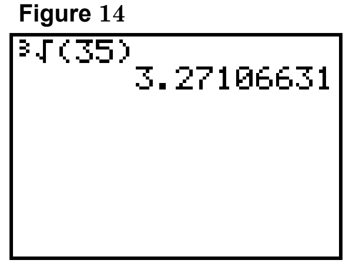


Figure 14



Example 7: Estimate the value of the given expression, then use your calculator to obtain a decimal approximation.

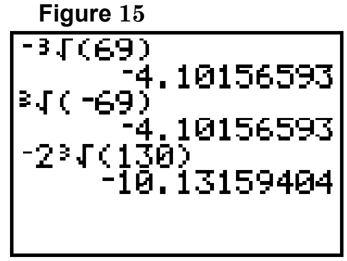
a. $-\sqrt[3]{69}$ b. $\sqrt[3]{-69}$ c. $-2\sqrt[3]{130}$

solution: a. Estimate: $-\sqrt[3]{64} = -4$.

b. Estimate: $\sqrt[3]{-69} = -4$.

c. Estimate: $-2(\sqrt[3]{125}) = -10$ simplify radical first

Figure 15



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Now try Exercises 61 through 64

The “nth root” operation is **option 5** from the same menu (using the [MATH] key). To use this operation, we first “give the calculator” the value of n (the index number), then enter the radicand. The TI-84 Plus does not supply the left parenthesis when using this function, so if the radicand contains more than one term, both the left and right parentheses must be entered by the user. As always, you should obtain an estimate first. The keystrokes for $\sqrt[5]{40}$ are: 5 [MATH] **5:**$\sqrt[x]{\ }$ [( ] 40 [ )] [ENTER]. Since $\sqrt[5]{32}$ = 2, $\sqrt[5]{40}$ is slightly larger than 2.

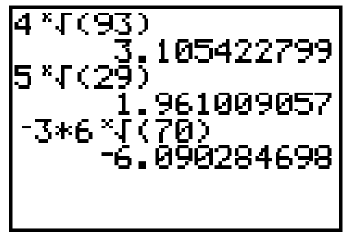
Example 8: Estimate the value of the given expression, then use your calculator to obtain a decimal approximation. a. $\sqrt[4]{93}$ b. $\sqrt[5]{29}$ c. $-3\sqrt[6]{70}$

solution: a. Estimate: $\sqrt[4]{81}$ = 3.

b. Estimate: $\sqrt[5]{32}$ = 2.

c. Estimate: $-3 \cdot \sqrt[6]{64}$ = -6. See Figure 16.

Figure 16



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Now try Exercises 65 and 66

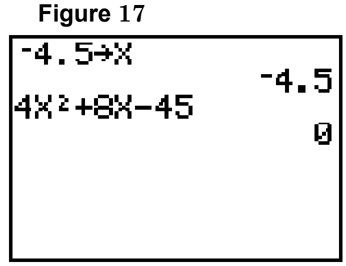
C) Using the storage and recall features

Two of the more common uses of a calculator’s storage and recall features are (1) checking the solution to an equation, and (2) situations where the same value might be used repeatedly.

• Checking Solutions to an Equation

To determine if x = -4.5 is a solution to $4x^2 + 8x - 45 = 0$, we place -4.5 in memory using these keystrokes: [(–)] 4.5 [STO►] [X,T,θ,n] [ENTER]. This temporarily stores -4.5 as the variable “X,” and allows us check the solution by constructing the expression directly on the home screen and pressing [ENTER]. Since the result is zero, we know x = -4.5 is indeed a solution (see Figure 17). The [X,T,θ,n] storage location is called “temporary” because it is often overwritten or replaced as a result of other calculations.

Figure 17



Example 9: Use the process illustrated above to determine if $x = -2\sqrt{5}$ is a solution to the equation $x^4 - 19x^2 - 20 = 0$.

solution: Store $x = -2\sqrt{5}$ in temporary memory: [(–)] 2 [2nd] [x^2] 5 [ )] [STO►] [X,T,θ,n] [ENTER]. On the next line enter $x^4 - 19x^2 - 20$. Pressing [ENTER] returns an output of zero. Yes, $x = -2\sqrt{5}$ is a solution to $x^4 - 19x^2 - 20 = 0$. ◆

Now try Exercises 67 and 68

When a given value is to be used repeatedly, we usually store it in an "ALPHA" location using the [ALPHA] key. For instance, suppose we need to repeatedly convert measures given in gallons to liters. The conversion factor is 1 gallon = 3.785306 liters. Instead of storing this value as "X" using the [X,T,θ,*n*] key, we store it in ALPHA location A where it is much less likely to be overwritten. The keystrokes are: 3.785306 [STO▶] [ALPHA] [MATH], since the "A" is the ALPHA location corresponding to the [MATH] key. This factor can now be recalled for future use as many times as necessary using the following keystrokes: [ALPHA] [MATH] [ENTER]. All computations can now be performed on the home screen by simply using "A," since the calculator knows A = 3.785306.

Example 10: A local pet shop has three sizes of aquariums: (a) 12 gallons, (b) 18 gallons, and (c) 24 gallons. How many liters of water are needed to fill each tank?

solution: Using 3.785306 ≈ 4, our estimates will be slightly high (see Figure 18).

a) Estimate: 12 x 4 = 48 liters.

b) Estimate: 18 x 4 = 72 liters.

c) Estimate: 24 x 4 = 96 liters.

Figure 18

```
12*A
          45.423672
18*A
          68.135508
24*A
          90.847344
```

◆

Now try Exercises 69 through 74

D) Using the TABLE feature

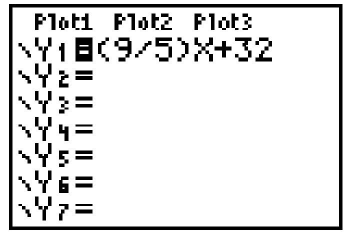
• Tables and formula use

The **TABLE** feature of a graphing calculator is a very efficient way of evaluating a formula for a large number of different inputs. To use this feature, we must: ① Input the formula we want to evaluate, ② Tell the calculator how we want the table to be set up (**TBLSET**), and ③ Access and use the resulting table.

① <u>Input the desired formula</u>

We'll illustrate the **TABLE** feature using the formula for temperature conversions, $F = \frac{9}{5}C + 32$. The input value will be "C" (Celsius temperature) and "F" (Fahrenheit temperature) is the output. However, the TI-84 Plus uses X for input values and Y_1 for outputs, meaning our formula will actually be entered as Y_1 = (9/5)X + 32. Press the Y= key, use CLEAR to delete existing expressions (if any), then enter the equation using the keystrokes: (9 ÷ 5) X,T,θ,*n* + 32 (see Figure 19).

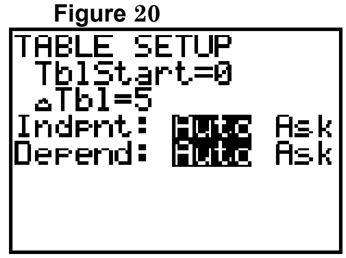
Figure 19



② <u>Set up the **TABLE**</u>

Access the **TBLSET** screen by pressing 2nd WINDOW, and the screen shown in Figure 20 appears. The first entry, "**TblStart =**" tells the calculator where we want the input values in our table to begin. For now, we'll start the table at **TblStart =** 0, although we could use any real number (decimals, negative numbers, and so on). Use the arrow keys ▼ and ▲ to move the cursor into position, then press "0" and ENTER. The next line "Δ**Tbl =**" (read "delta table equals") tells the calculator what increments (or steps) we want the calculator to use as it generates the inputs. In other words, what do we want the calculator to "count by." If Δ**Tbl =** 1, the input values will be 0, 1, 2, 3, 4, 5 ... . If Δ**Tbl =** 5, they would be 0, 5, 10, 15, 20 .... The calculator can also count by decimal values, so if Δ**Tbl =** 0.5, the inputs would be 0, 0.5, 1.0, 1.5, 2, 2.5 and so on. In this exercise, we'll have the calculator count by 5's. Navigate the cursor into position, enter a "5," then press ENTER. The next line "**Indpnt: Auto Ask**" is where we tell the calculator to either **Auto**matically generate the input values (the input values are sometimes called the "independent" values), or to **Ask** us what values we want to input manually. For this exercise, we will let the calculator **Auto**matically generate the input values. Navigate the cursor into position over the word "Auto" and highlight it by pressing ENTER: **Indpnt: Auto Ask**. For our purposes, the final line "**Depend: Auto Ask**," should always be in the default mode: **Depend: Auto Ask**.

Figure 20



③ <u>Access and use the TABLE created</u>

To access the table, press 2nd GRAPH. The TI-84 Plus displays a three column table, so the third column will be blank (unless another equation is entered in Y_2). One advantage of the **Indpnt: Auto Ask** mode is that you are able to scroll through the numbers using arrow keys.

Example 11: Answer the following questions using table feature and the formula $F = \frac{9}{5}C + 32$.

a. What Fahrenheit temperature corresponds to 20ºC?

b. If it is 32ºF outside, what is the temperature in degrees Celsius?

c. What Fahrenheit temperature corresponds to 100ºC?

d. The average temperature on Mars is -76ºF. Covert this to degrees Celsius.

solution: a. After completing steps ① and ②, pressing [2nd] [GRAPH] gives the table shown in Figure 21, where we note that 20ºC corresponds to 68°F.

Figure 21

X	Y1
0	32
5	41
10	50
15	59
20	68
25	77
30	86

X=0

b. Checking in the Fahrenheit column (Y1), we see that 32ºF corresponds to 0°C.

c. Scrolling down the table using the down arrow key [▼] reveals that 100ºC is equal to 212ºF.

d. Scrolling up the table using the up arrow key [▲] reveals that -76ºF is equivalent to -60ºC. The average temperature on Mars is -60ºC. ◆

Now try Exercises 75 and 76

Example 12: The total value V of goods and services (in billions of dollars) exported by the United States between 1999 and 2003 can be approximated by the polynomial $V = 17.17x^3 - 109.64x^2 + 170.90x + 697.51$, where x = 0 corresponds to 1999. According to this model, in which of these years were U.S. exports at a minimum? What was this minimum?

Source 2005 Statistical Abstract of the United States, Table 1293, page 811.

solution: Enter $17.17x^3 - 109.64x^2 + 170.90x + 697.51$ as Y_1 on the [Y=] screen, then go to [2nd] [WINDOW] **(TBLSET)** to set **TblStart =** 0, **∆Tbl =** 1, and **Indpnt:** **Auto** **Ask**. Access the table using [2nd] [GRAPH] and the table in Figure 22 appears, showing that U.S. exports reached a minimum of approximately 687 billion dollars in year 3 (2002).◆

Figure 22

X	Y1
0	697.51
1	775.94
2	738.11
3	687.04
4	725.75
5	957.26
6	1484.6

X=3

Now try Exercises 77 and 78

E) Editing expressions on the home screen

One of the primary advantages of a graphing calculator is the ability to review what's been entered, and editing an expression if an error has been made. Consider the expression $\frac{\sqrt{83}-4}{5}$. Since $\sqrt{83} \approx 9$, the expression has an approximate value of 1. But suppose we incorrectly enter the expression by forgetting to enclose the numerator in parentheses, multiplying instead of dividing, or adding instead of subtracting (see Figure 23). Instead entering the expression all over again, we can *recall the previous entry* by pressing [2nd] [ENTER]. You can actually recall a large number of previous entries, pressing the [2nd] [ENTER] keys each time. In this case we have recalled the entry "($\sqrt{83}+4$)/5," which has a sum in the numerator instead of the required difference. Use the arrow keys to navigate the cursor until it over-lays the "+" symbol, and enter the correct symbol "–" (see Figure 24). Pressing [ENTER] now gives a result of 1.022086716, which is more what we expected. You do not have to navigate the cursor to the end of the expression before pressing [ENTER] -- the cursor can be anywhere in the expression.

Figure 23

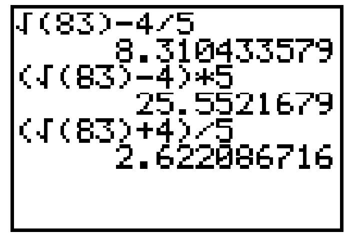
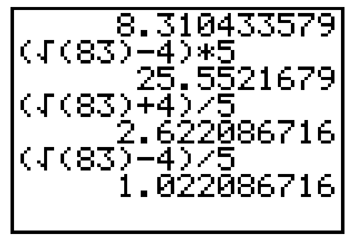


Figure 24



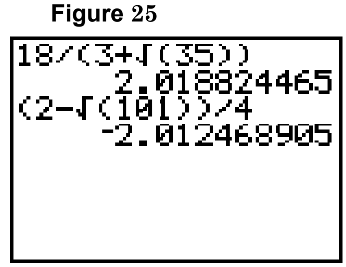
Example 13: Estimate the value of the expressions given, then use your calculator to obtain a decimal approximation rounded to the 100ths place. If it turns out you made an error while entering the expression, use the ideas discussed above to edit and re-evaluate. a. $\frac{18}{3+\sqrt{35}}$ b. $\frac{2-\sqrt{101}}{4}$

solution: a. Estimate: $\frac{18}{3+\sqrt{36}} = 2$.

b. Estimate: $\frac{2-\sqrt{100}}{4} = -2$.

See Figure 25.

Figure 25



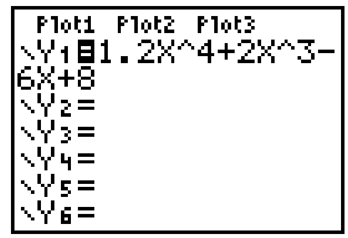
◆

• The insert and delete features of the TI-84 Plus

When using technology, even the most careful people sometimes make mistakes. The TI-84 Plus actually provides three ways that an error can be corrected: ① the incorrect expression can be cleared out entirely using the CLEAR key and replaced by the correct expression; ② the incorrect portion of the expression can be over-written; or ③ the incorrect portion of the expression can be deleted using the DEL key, and the corrections inserted by accessing the insert feature using the keystrokes 2nd DEL.

Suppose you wanted to evaluate the expression $1.2x^4 + 3x^2 - 6x + 8$, but after entering it on the Y = screen, you realize that you incorrectly entered $1.2x^4 + \mathbf{2x^3} - 6x + 8$ (see Figure 26).

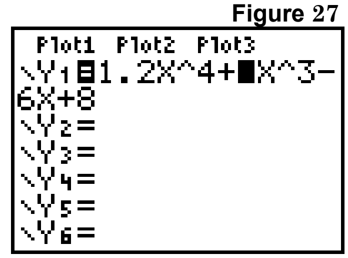
Figure 26



① To clear the old expression, press the CLEAR key and the entire expression is wiped out. The correct expression can now be entered. **Note**: The CLEAR feature will not operate if the cursor is over the equal sign (as though you were trying to activate or deactivate the function). The cursor must be somewhere *within* the expression you are clearing.

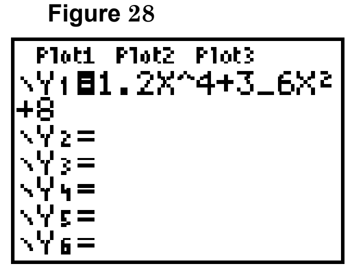
② To over-write the incorrect term, walk the cursor (using the arrow keys) into a position over-laying the first correction needed (over the “2” in $2x^3$). Be sure the cursor is in the “blinking box” mode (over-write) and not “blinking line” mode (insert). See Figure 27. Simply enter the correct term, noting the old entry is over-written (the 2 is replaced by a 3).

Figure 27



③ To make more expansive corrections, you can walk the cursor (using the arrow keys) into position over-laying the first error. After positioning the cursor over the number 2, press the DEL key (three times) to delete the incorrect entries. Follow this by 2nd DEL to access the *insert* feature, noting the cursor changes from a blinking box to a blinking under-line (see Figure 28). New information will be inserted until the ENTER key is pressed or some other feature is used.

Figure 28



F) Checking equivalent expressions

• Test values and equivalent expressions

One way to check whether two expressions are equivalent is to use test values. Consider the sum $-\sqrt{12x} + 5\sqrt{75x}$. After simplifying each radical, the sum becomes $-2\sqrt{3x} + 25\sqrt{3x}$ or $23\sqrt{3x}$. To check this result, enter the original sum as Y_1 on the Y = screen and the simplified result as Y_2 (see Figure 29). Next, access the **TBLSET** screen using 2nd WINDOW and set up the table so that you can enter your own choice of inputs (see Figure 30): **Indpnt: Auto Ask**. You are now ready to check the sum using a variety of test values and the TABLE feature. As a general rule you should check results using positive values, negative values, zero, and various fractional or decimal inputs (anticipate what might happen for this sum when you enter a negative input). If the table gives identical outputs for Y_1 and Y_2, you have likely computed the sum correctly. Pressing 2nd GRAPH and entering the inputs 2, -3, 0, $\frac{2}{3}$ produces the table shown in Figure 31.

Figure 29

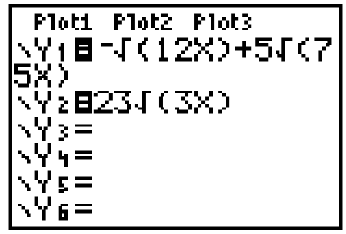
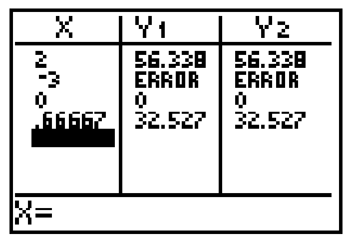


Figure 30

Figure 31



Example 14: Combine the radical expressions given and check the result using the ideas discussed here. a. $2\sqrt{18x} - 7\sqrt{50x}$ b. $12\sqrt[3]{54a} - 5\sqrt[3]{128a}$

solution: a. $2\sqrt{18x} - 7\sqrt{50x} = 6\sqrt{2x} - 35\sqrt{2x}$ or $-29\sqrt{2x}$. See Figures 32 and 33.

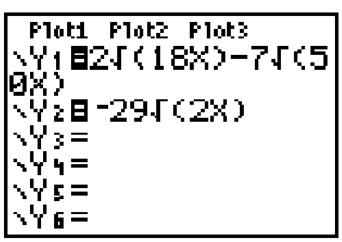


Figure 32

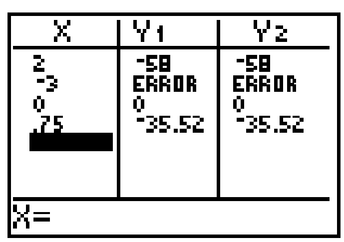


Figure 33

Since all outputs are identical, we assume the work has been completed correctly. Note that neither expression represents a real number when $x < 0$.

b. $12\sqrt[3]{54a} - 5\sqrt[3]{128a} = 12\sqrt[3]{27 \cdot 2a} - 5\sqrt[3]{64 \cdot 2a}$

$= 36\sqrt[3]{2a} - 20\sqrt[3]{2a}$

$= 16\sqrt[3]{2a}$.

See Figures 34 and 35.

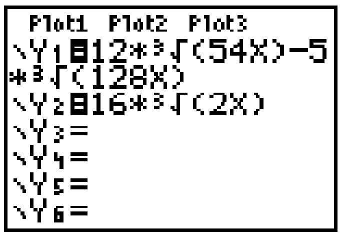


Figure 34

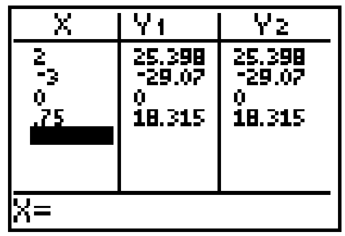


Figure 35

Since all outputs are identical, we assume the work has been completed correctly. Recall that cube root expressions are defined for all real numbers. ◆

Now try Exercises 79 through 90

SECTION R.8 EXERCISES

CONCEPTS AND VOCABULARY

Fill in each blank with the appropriate word or phrase. Carefully re-read the section, if necessary.

1. To help prevent errors when using any form of technology, always try to ________ the answer or *forecast the magnitude of the result* ***first***.

2. Many graphing calculators use the ____ symbol for multiplication and a __________ for division.

3. To use the TABLE feature of a graphing calculator the three basic steps are:

   a. ____________________________

   b. ____________________________

   c. ____________________________

4. The _________ storage location of a TI-84 Plus is [X,T,θ,n]. For the more permanent storage of a quantity, the ________ keys are used. The keystrokes 3.7141592 [STO▶] [ALPHA] [PRGM] stores 3.7141592 in memory location ____.

5. Discuss/Explain how graphing calculators distinguish between the symbol for subtraction and the symbol for a negative number.

6. What is the cube root of 343? Discuss/Explain two ways we can use a graphing calculator to find the cube root of this and other numbers.

DEVELOPING YOUR SKILLS

Complete all exercises using a graphing calculator. Obtain an estimate first, then compare your estimate to the calculated result. Reconcile any differences.

• Basic Operations/Order/Negative Numbers

7. $6 - 3^2 \cdot 21 + 7$

8. $-63.04 \div (\sqrt{121} - 3)^2$

9. $\frac{1}{4} + \frac{30}{7} \times \frac{14}{15} - \left(\frac{1}{3}\right)^2$

10. $\frac{4}{3}\pi(15)^2$

11. $2\pi(4.5)^2 + 2\pi(4.5)(9)$

12. $\dfrac{611.2}{6 + 11 \cdot (-4)^2 - (-9)}$

Evaluate the following expressions using the method of your choice (home screen or stored values). Use $x = 0.207$, $y = -12$ and $z = -\frac{1}{3}$ Round to hundredths as needed.

13. xyz

14. $x + yz$

15. $yz \div x$

16. $\frac{y}{z}$

17. $xz + y$

18. $yx \div z$

19. $xy + xz - yz$

20. $yz - xz - y$

Describe what is wrong with the expressions as entered (each gives an incorrect result).

21. $2^{0.5 \cdot 8}$

```
2^0.5*8
```

Exercise 21

22. $\frac{18}{2 \times 3}$

```
18/2*3
```

Exercise 22

23. $\sqrt{29 - 4}$

```
√(29)-4
```

Exercise 23

24. Evaluate $x^2 + 4x$ if $x = -5$:

```
⁻5²+4*⁻5
```

Exercise 24

25. $\frac{3 - 10}{1 - 15}$

```
3-10/1-16
```

Exercise 25

26. $\frac{-3}{4} \div \frac{1}{2}$

```
⁻3/4/1/2
```

Exercise 26

• Integer and Rational Exponents

Evaluate each expression. If the result is a rational number, show why using properties of exponents.

27. $32^{\frac{4}{5}}$
28. $9^{\frac{3}{2}}$
29. $-8^{\frac{2}{3}}$
30. $16^{\frac{-3}{2}}$
31. $20^{\frac{3}{5}}$
32. $-48^{\frac{5}{3}}$
33. $215^{\frac{2}{3}}$
34. $12^{\frac{5}{6}}$

Evaluate each expression if x = 0.5 and z = -4.

35. z^2
36. $-z^2$
37. xz^2
38. $(x-z)^2$
39. 2^z
40. -2^z
41. $z \cdot 25^x$
42. $(x+2)^{xz}$

• Scientific Notation

43. Light travels at approximately 670,000,000 miles per hour. Determine how long it takes a beam of light to reach the star Arcturus, one of the three brightest stars in the sky. It is approximately 220,000,000,000,000 miles from the earth.
44. From a study of physics, one kilomole of monatomic hydrogen consists of 6.02×10^{26} atoms. The mass of a single hydrogen atom is 1.67×10^{-27} kg. How much does a kilomole of hydrogen weigh?

Perform the indicated operations. Write the result in both scientific and ordinary notation.

45. $\frac{27,000,000,000,000,000}{1,200,000,000,000,000,000}$
46. (0.0000000000015)(0.000001)
47. $(5.02 \times 10^{17})(6.5 \times 10^{-12})$
48. $(6.9 \times 10^{-19})(4.1 \times 10^{15})$
49. $\frac{9 \times 10^{-32}}{7.2 \times 10^{-29}}$
50. $\frac{9.25 \times 10^{29}}{5.0 \times 10^{25}}$

• Roots and Radicals

Estimate the value of each expression, then use your calculator to obtain a decimal approximation. Round to hundredths as needed. If you make an error while entering the expression, edit and re-evaluate.

51. $-\sqrt{119}$
52. $\sqrt{2509}$
53. $\sqrt{15^2 - 8.9^2}$
54. $\sqrt{13^2 - 11.99^2}$
55. $\frac{\sqrt{34} - 12}{7 - \sqrt{10}}$
56. $\frac{46 - \sqrt{34}}{\sqrt{15} - (-6)}$
57. $-\sqrt{\frac{25}{35.2 - 10.3}}$
58. $\sqrt{\frac{144}{8.3 - (-8.1)}}$
59. $\frac{80}{\sqrt{23} - \sqrt{10}}$
60. $\frac{-50}{\sqrt{5} - \sqrt{17}}$
61. $-\sqrt[3]{120}$
62. $\sqrt[3]{-30}$
63. $(-9)^2 - \sqrt[3]{-79.507}$
64. $\sqrt[3]{3.375} - (-5)^2$
65. $\sqrt[5]{83}$
66. $\sqrt[4]{202}$

• Storage and Recall

Use the storage capabilities and them home screen to show $x = 2 + \sqrt{5}$ is a solution to:

67. $0 = x^3 - 2x^2 - 9x - 2$

68. $x^4 - 8x^3 + 14x^2 + 8x = -1$

To convert pounds to kilograms, we use 1 pound = 0.453592 kg. Store the factor 0.453592, then convert the following measures to the nearest tenth of a kilogram using the storage/recall feature of your calculator.

69. The elephant weighed 4,842 lbs.

70. The wheelbarrow full of dirt weighed 189 lbs.

To convert square miles to square kilometers, we use 1 mi^2 = 2.589 km^2. Store the factor 2.589, then convert the following measures to km^2 using the storage feature of your calculator.

71. The area of the North American continent is 9,400,000 mi^2.

72. The area of the Gulf of Mexico is 582,100 mi^2.

73. Which has the greater land area?
Spain: 505,798 **km^2** or California: 163,707 **mi^2**

74. Which has the greatest mass?
Titan: 1.4×10^{23} **kg** or Ganymede: 3.26×10^{23} **lbs**

• The TABLE feature and formulas

Use the formulas shown and the TABLE feature of your calculator.

75. **Volume of a Sphere: $V = \frac{4}{3}\pi r^3$**

a. What is the volume of a ball with radius r = 3.5? b. If you wanted to manufacture a large play-ball with a volume of approximately 6350 in^3, what must the radius be (to the nearest half-inch)?

76. **Compound Interest: $A = P(1 + \frac{r}{n})^{nt}$**

The amount of money **A** in an account paying **r** percent interest (written as a decimal) and compounded **n** times per year, depends on the initial deposit **P** and the amount of time **t** it is left on deposit. If a $5000 deposit is compounded quarterly at an annual rate of 8%: a. How much is in the account after 8 years? b. Approximately how many years (to the nearest quarter) will it take the deposit to grow to $10,000?

• The TABLE feature and evaluating expressions

77. From previous experience, a clothing company has found that its profit P (in thousands of dollars) can be modeled by the polynomial $P = -8x^3 + 110x^2 - 300x - 10$, where x represents the amount of money (in ten thousands of dollars) spent on advertising. a. How much is spent on advertising before the company starts making a profit? b. How much should be spent on advertising to maximize profits?

78. Due to the location of Mountain Pass Bed-and-Breakfast, their business is very seasonal. Suppose their profit function is modeled by $P = -2m^2 + 28m - 90$, where P represents the profit in thousands of dollars and m represents a month of the year (Jan → 1, Feb → 2, and so on). a. What month of the year should they open if they want to be profitable? b. What month should they close? c. In what month do they make the most profit? d. What is this maximum profit?

• Checking Equivalent Expressions

Use a table to determine if these statements are true or false. If false, state why and write the correct result.

79. $(x + 2)^3 = x^3 + 8$

80. $(x - 5)^2 = x^2 - 25$

81. $\sqrt{x^2 - 9} = x - 3$

82. $\sqrt{25 + x^2} = 5 + x$

83. $x^2 - 3x - 10 = (x + 5)(x - 2)$

84. $x^2 + 36 = (x + 6)(x + 6)$

85. $\frac{1}{2}x^2 - 4x = \frac{1}{2}(x^2 - 2x)$

86. $\frac{1}{3}x^2 - 15x = \frac{1}{3}(x^2 - 5x)$

87. $x^2 + 12x + 17 = (x + 6)^2 - 19$

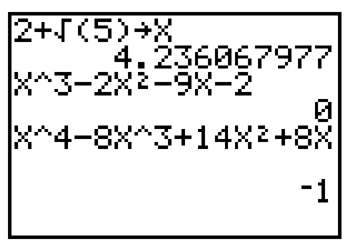
88. $x^2 - 10x + 1 = (x - 5)^2 - 24$

89. $2x^2 - 12x + 11 = 2(x - 3)^2 + 2$

90. $-3x^2 + 18x - 1 = -3(x - 3)^2 - 10$

Section R.8 Student Solutions

1. estimate
3. Enter an expression in **Y1**, set up the **TABLE**, access the **TABLE**
5. Answers will vary.
7. -176
9. $4.13\overline{8}$
11. $121.5\pi \approx 381.70$
13. 0.828
15. ≈ 19.324
17. -12.069
19. -6.553
21. "0.5 x 8" should be grouped as $2^{(0.5 \times 8)}$
23. 4 should be included within parentheses
25. the numerator and denominator should be grouped seperately
27. $16 = \left(32^{\frac{1}{5}}\right)^4$
29. $-4 = -\left(8^{\frac{1}{3}}\right)^2$
31. ≈ 6.034
33. ≈ 35.889
35. 16
37. 8
39. 0.0625
41. -20
43. $\approx$ 328,358 hr; about 37.5 yr
45. 2.25×10^{-2}; 0.0225
47. 3.263×10^{6}; 326,300
49. 1.25×10^{-3}; 0.0125
51. ≈ -10.91
53. ≈ 12.07
55. ≈ -1.61
57. ≈ -1.00
59. ≈ 48.97
61. ≈ -4.93
63. 85.3
65. ≈ -2.42
67.



69. 2196.3 kg
71. 24,336,600 km^2
73. Spain
75. a) ≈ 179.59 in^3
    b) 11.5 in
77. a) $\approx$ \$40,000
    b) $\approx$ \$75,000
79. False; $(x + 2)^3 = x^3 + 3x^2 + 3x + 8$
81. False; $\sqrt{x^2 - 9}$ cannot be simplified further
83. False; $x^2 - 3x - 10 = (x - 5)(x + 2)$
85. False; $\frac{1}{2}x^2 - 4x = \frac{1}{2}(x^2 - 8x)$
87. True
89. False; $2x^2 - 12x + 11 = 2(x - 3)^2 - 7$
90. False; $-3x^2 + 18x - 1 = -3(x - 3)^2 + 26$