R.7 Geometry Review with Unit Conversions

Introduction

Developing the ability to use mathematics as a descriptive tool is a major goal of this text. Without a solid understanding of basic geometry, this goal would be difficult to achieve – as many of the tasks we perform daily are based on decisions regarding size, measurement, configuration and the like.

Learning Objectives

In Section R.7 you will learn how to:

A) Find the perimeter and area of common figures

B) Compute the volume of common figures

C) Identify and use the Pythagorean Theorem

D) Use unit conversion factors

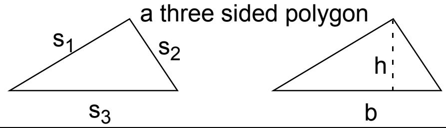
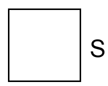
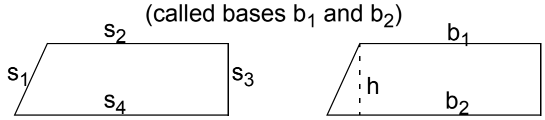
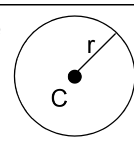
Point of Interest

In the Middle Ages, units of measure varied a great deal from country to country, with most having ancient origins which based units of length on parts of the human body (the cubit, span, palm, etc). This caused a great deal of inefficiency and confusion, because the human body comes in different sizes -- hence was not a good "standard."

A) Perimeter and Area Formulas

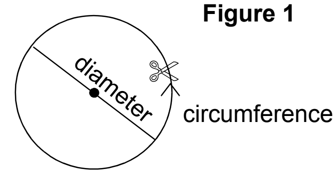
Basic geometry plays an important role in the application of mathematics. For your convenience, many of the more common formulas have been collected in Table 1, and a focused effort should be made to commit them to memory. Note that some of the formulas use **subscripted variables**, or a variable with a small case number to the lower right (s_1, s_2 and so on). To help understand the table, we quickly rehearse some fundamental terms and their meaning. A **plane** is the infinite extension of length and width along a flat surface. **Perimeter** is the distance around a two dimensional figure, or a closed figure that lies in a plane. Many times these figures are **polygons**, or closed figures composed of line segments. The general name for a four-sided polygon is a **quadrilateral**. A **right angle** is an angle measuring 90º. A quadrilateral with four right angles is called a **rectangle**. **Area** is a measure of the amount of surface covered by a plane figure, with the measurement given in **square units**.

Table 1

	definition and diagram	perimeter formula (linear units or *units*)	area formula (square units or *units*^{2})
triangle	a three sided polygon 	$P = s_1 + s_2 + s_3$	$A = \frac{bh}{2}$
rectangle	a quadrilateral with four right angles and opposite sides parallel 	$P = 2L + 2W$	$A = LW$
square	a rectangle with four equal sides 	$P = 4S$	$A = S^2$
trapezoid	a quadrilateral with one pair of parallel sides (called bases b_1 and b_2) 	sum of all sides $P = s_1 + s_2 + s_3 + s_4$	$A = \frac{h(b_1 + b_2)}{2}$
circle	the set of all points lying in a plane that are an equal distance (called the radius r) from a given point (called the center C). 	$C = 2\pi r$ or $C = \pi d$	$A = \pi r^2$

Worthy of Note: The formulas $C = \pi d$ and $C = 2\pi r$ both use the symbol "π," which represents the ratio of a circle's circumference to its diameter. We will use a two decimal approximation in calculations done by hand: $\pi \approx 3.14$. On the TI-84 Plus, π is the $\boxed{\text{2nd}}$ function to the $\boxed{\wedge}$ key and produces a much better approximation (see Figure 1). When using a calculator, we most often use all displayed digits and round only the answer to the desired level of accuracy.

Figure 1

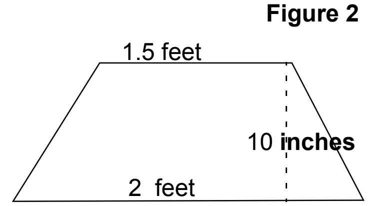


$$\frac{\text{circumference}}{\text{diameter}} \approx 3.141592653\ldots$$

If a problem or application uses a formula, begin by stating the formula rather than by immediately making any substitutions. This will help to prevent many careless errors.

Example 1: A basement window is shaped like an isosceles trapezoid (base angles equal, opposite sides equal in length), with a height of 10 in. and bases of 1.5 ft. and 2 ft. What is the area of the glass in the window?

Figure 2



solution: Before applying the area formula, all measures must use the same unit. In inches we have 1.5 ft. = 18 in. and 2 ft. = 24 in.

$$A = \frac{h(b_1 + b_2)}{2} \qquad \text{given formula}$$

$A = \frac{10\text{in}(18\text{in} + 24\text{in})}{2}$ substitute 10 for h, 18 for b_1 and 24 for b_2

$A = \frac{10\text{in}(42\text{in})}{2}$ simplify

$A = 210\ \text{in}^2$ result

The area of the glass in the window is 210 in^2. ◆

Now try Exercises 7 through 16

Worthy of Note In actual practice, most calculations are done without using the units of measure, with the correct units supplied in the final answer. When like units occur in an exercise, they are treated just as the numeric factors. If they are part of a product, we write the units with an appropriate exponent as in Example 1. If the like units occur in the numerator and denominator, they "cancel" as in Example 6.

• Composite Figures

The largest part of geometric applications, whether in art, construction or architecture, involve **composite figures**, or figures that combine basic shapes. In many cases we are able to **partition** or break the figure into more common shapes using an **auxiliary line**, or a dashed line drawn to highlight certain features of the diagram. When computing a perimeter, we use only the exposed, outer edges, much as a soldier would guard the base camp by marching along the outer edge -- the perimeter. For composite figures, it's helpful to verbally describe the situation given, creating an English language model which can easily be translated into an equation model.

Example 2: Find the perimeter and area of the composite figure shown. Use $\pi \approx 3.14$.

solution: To compute a perimeter, we use only the exposed, outer edges.

Figure 3

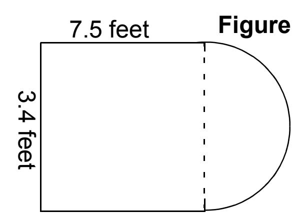
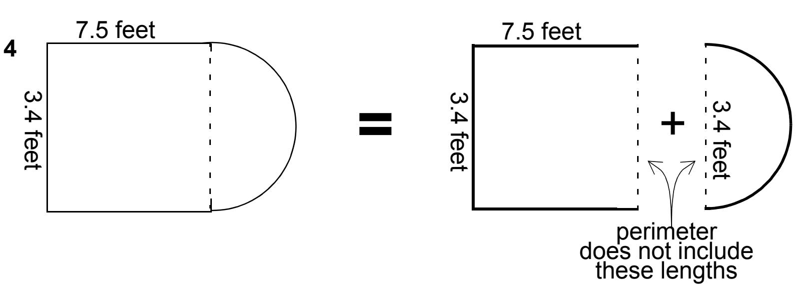


Figure 4



Perimeter = three sides of rectangle + one-half circle   verbal model

$$P = 2L + W + \frac{\pi d}{2} \qquad \text{formula model}$$

$$\approx 2(7.5) + 3.4 + \frac{(3.14)(3.4)}{2} \qquad \text{substitute 7.5 for L, 3.4 for W, and 3.14 for } \pi$$

$$\approx 18.4 + 5.338 \qquad \text{simplify}$$

$$\approx 23.738 \qquad \text{result}$$

The perimeter of the figure is about 23.7 ft.

Total Area = area of rectangle + one-half area of circle   verbal model

$$A = LW + \frac{\pi r^2}{2} \qquad \text{formula model}$$

$$\approx (7.5)(3.4) + \frac{(3.14)(1.7)^2}{2} \qquad \text{substitute 1.7 for r (one-half the diameter)}$$

$$\approx 25.5 + 4.5373 \qquad \text{simplify}$$

$$\approx 30.0373 \qquad \text{result}$$

The area of the figure is about 30.0 ft^2. ◆

Now try Exercises 17 through 32

B) Volume

Volume is a measure of the amount of space occupied by a three dimensional object and is measured in **cubic units**. Some of the more common formulas are given in Table 2.

Table 2

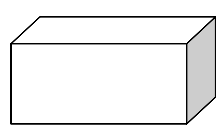
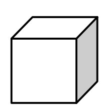
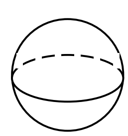
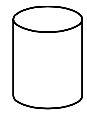
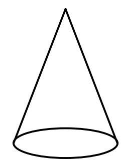
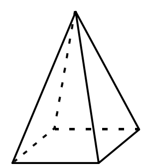
	Illustration		volume formula (cubic units or *units*^{3})
rectangular solid	a six sided, solid figure with opposite faces congruent and adjacent faces meeting at right angles		$V = LWH$
cube	a rectangular solid with six congruent, square faces		$V = S^3$
sphere	the set of all points an equal distance (called the radius) from a given point (called the center)		$V = \frac{4}{3}\pi r^3$

	Illustration		volume formula (cubic units or *units*^{3})
right circular cylinder	union of all line segments connecting two congruent circles in parallel planes, meeting each at a right angle		$V = \pi r^2 h$
right circular cone	union of all line segments connecting a given point (vertex) to a given circle (base) and whose altitude meets the center of the base at a right angle		$V = \frac{1}{3}\pi r^2 h$
right pyramid	union of all line segments connecting a given point (vertex) to a given polygon (base) and whose altitude meets the center of the base at a right angle		$V = \frac{1}{3} s^2 h$

Example 3: Sand at a cement factory is being dumped from a conveyor belt into a pile which is shaped like a right circular cone atop a right circular cylinder (see Figure 5). How many cubic feet of sand are there at the moment the cone is 6 ft high with a diameter of 10 ft?

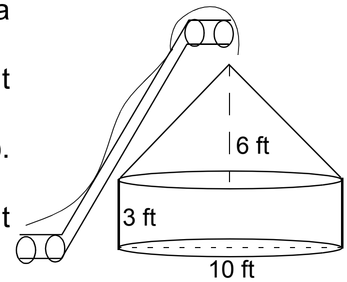


Figure 5

solution: Total Volume = volume of cylinder + volume of cone verbal model

$$V = \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 \quad \text{formula model}$$

$$= \pi(5)^2(3) + \frac{1}{3}\pi(5)^2(6) \quad \text{substitute 5 for r, 3 for } h_1\text{, and 6 for } h_2$$

$$= 75\pi + 50\pi \quad \text{simplify}$$

$$= 125\pi \quad \text{result (exact form)}$$

There are about 392.7 ft^3 of sand in the pile. ◆

Now try Exercises 33 and 34

Worthy of Note: It is again worth noting that units of measure are treated as though they were numeric factors. For the cylinder in Example 3: $\pi \cdot (5\text{ ft})^2 \cdot (3\text{ ft}) = \pi \cdot 25\text{ ft}^2 \cdot 3\text{ ft} = 75\pi\text{ ft}^3$. This concept is an important part of the unit conversions often used in the application of mathematics.

C) Right Triangles and the Pythagorean Theorem

A right triangle is one having a 90º angle (a "perfect corner"). The longest side (opposite the right angle) is called the **hypotenuse** (see Figure 6). Over two thousand years ago, a unique relationship was discovered between the sides of these triangles, namely, if you added the square of each leg, the result was always equal to the square of the hypotenuse: $\text{leg}^2 + \text{leg}^2 = \text{hyp}^2$. A geometric interpretation of the theorem is shown in Figure 7 (verifying $3^2 + 4^2 = 5^2$), with additional illustrations in Figure 9. The result is called the **Pythagorean Theorem** and is often stated as $\mathbf{a^2 + b^2 = c^2}$, where c is the hypotenuse.

Figure 6

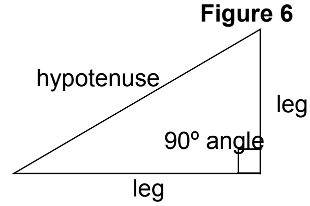


Figure 7

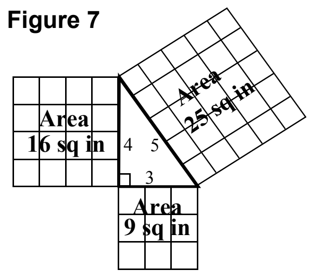
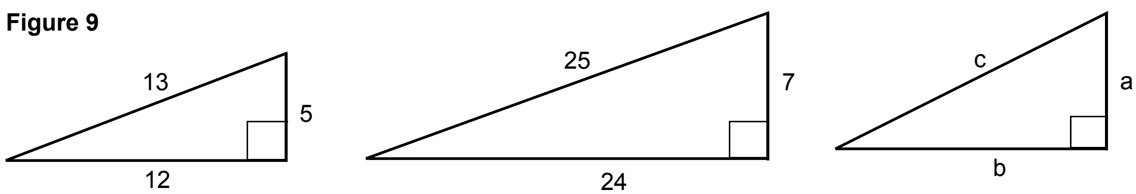


Figure 9



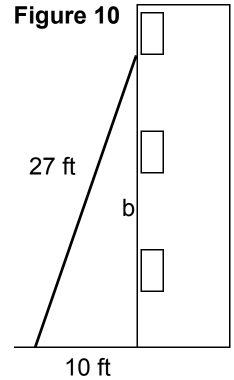
$5^2 + 12^2 = 13^2$	$7^2 + 24^2 = 25^2$	$a^2 + b^2 = c^2$
25 + 144 = 169 ✓	49 + 576 = 625	(general case)

Example 4: A 27 foot extension ladder is placed 10 feet from the base of a building in an effort to reach a third story window (see Figure 10). Is the ladder long enough to reach a window sill that is 25 feet high?

solution: Let b represent the height the ladder will reach.

$a^2 + \mathbf{b}^2$	=	c^2	focus on specified variable
$10^2 + \mathbf{b}^2$	=	27^2	substitute 10 for a and 27 for c
$100 + \mathbf{b}^2$	=	729	simplify
$\mathbf{b}^2$	=	629	subtract 100
$\sqrt{\mathbf{b}^2}$	=	$\sqrt{629}$	solve for b
$\mathbf{b}$	≈	25.1 feet	rounded to 10ths

Figure 10



The ladder just reaches the window sill. ◆

Now try Exercises 35 and 36

D) Unit Conversion Factors

Unit conversion factors are used to convert from one unit of measure to a related unit, like ounces to pounds. The procedure used is based on the fact that multiplying any quantity by the equivalent of 1 does not change its value, and any fraction of the form $\frac{\text{like amount or value}}{\text{like amount or value}} = 1$. Several examples of unit conversion factors are shown here.

(A) $\frac{12 \text{ inches}}{1 \text{ foot}} = 1$ OR $\frac{1 \text{ foot}}{12 \text{ inches}} = 1$

(B) $\frac{2000 \text{ lbs}}{1 \text{ ton}} = 1$ OR $\frac{1 \text{ ton}}{2000 \text{ lbs}} = 1$

(C) $\frac{4 \text{ qts}}{1 \text{ gallon}} = 1$ OR $\frac{1 \text{ gallon}}{4 \text{ qts}} = 1$

The unit factor is set up so that the units you are converting *from* cancel out, leaving only the unit *to which you are converting*. When doing these conversions, it is helpful to set up the units first and then write the numeric values with their related unit.

Example 5: Mt. Everest is 29,035 feet tall and is the highest mountain in the world when measured from the surface of the earth. Convert 29,035 feet to miles.

solution: a) Set up the units of the conversion factor so that "feet" will cancel.

$$\frac{29{,}035 \text{ feet}}{1}\left(\frac{\text{mile}}{\text{feet}}\right)$$

b) Recall that 5,280 feet = 1 mile, and write the numeric values with their related unit: "5,280" with feet and "1" with mile. Then simplify the result and state your answer.

$$\frac{29{,}035 \cancel{\text{feet}}}{1}\left(\frac{1 \text{ mile}}{5280 \cancel{\text{feet}}}\right) = \frac{29{,}035}{5280} \text{ miles}$$

$$= 5.49905303$$

Mount Everest is about 5.5 miles high. ◆

Now try Exercises 37 and 38

• Volume and Capacity

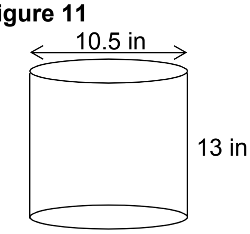
Strictly speaking, there is a distinction between the **volume** of a three dimensional object and its' **capacity**. Volume is the amount of space occupied by the object, capacity is considered to be a measure of the object's potential for holding or storing something. Some of the common relationships between units of volume and capacity are given in the following table.

VOLUME AND CAPACITY		
1 gal = 231 in^3	1 L = 1000 cm^3 = 1 dm^3	1 mL = 1 cm^3

Example 6: While taking inventory of the warehouse, workers find a large cylindrical paint can with no label. The cylinder measures 13 inches high and has a diameter of 10.5 inches (see Figure 11). How many <u>gallons</u> of paint does the container hold?

solution: a) Compute the volume in cubic inches.

Figure 11



$V = \pi r^2 h$ volume of a cylinder

$V = (3.14)(5.25)^2(13)$ substitute $\frac{10.5}{2} = 5.25$ for r

$V \approx 1125 \text{ in}^3$ simplify

b) Convert cubic inches to gallons using a unit conversion factor.

$$1125\,\text{in}^3 = \frac{1125\ \cancel{\text{in}^3}}{1} \cdot \frac{1\text{ gal}}{231\ \cancel{\text{in}^3}} \quad \text{set up units}$$

$$\approx 4.87 \text{ gallons} \quad \text{result}$$

There are just under 4.9 gallons of paint in the container. ◆

Now try Exercises 39 through 42

• US Units and Metric Units

Since U.S. Customary units and metric units were developed independently of each other, there is no “direct link” between the two and we usually look up equivalent values in a reference book or appendix. Since two equal quantities have a ratio of 1, any needed conversion factor can be formed. The length/distance conversions shown in Table 3 are correct to three decimal places.

Table 3: Length/Distance Conversion Factors

U.S. to Metric	Metric to U.S.
1 inch ≈ 2.504 centimeters	1 centimeter ≈ 0.394 inch
1 foot ≈ 0.305 meter	1 meter ≈ 3.281 feet
1 yard ≈ 0.914 meter	1 meter ≈ 1.094 yards
1 mile ≈ 1.609 kilometers	1 kilometer ≈ 0.621 mile

Using the left-half of the table when doing U.S. to metric conversions, and the right-half when doing metric to U.S. conversions, helps to simplify the calculation.

Example 7: The flight distance between New York City and London is 5,536 kilometers. Approximately how many <u>miles</u> is the related flight?

solution: Set up the conversion factor, noting from the table that 1 km $\approx$ 0.621 miles.

$$\frac{5536\,\cancel{\text{km}}}{1}\left(\frac{0.621\ \text{miles}}{1\ \cancel{\text{km}}}\right)$$ set up units

$$\approx 3438 \text{ miles}$$ result

The flight distance is approximately 3438 miles. ◆

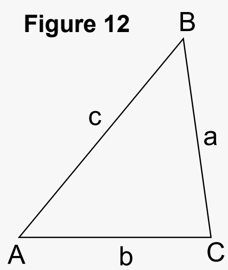
Now try Exercises 43 and 44

Conversion factors for weight, volume and capacity can likewise be found and used.

• **Similar Triangles**

Another important geometric relationship is that of **similar triangles**. Two triangles are similar if corresponding angles are equal. The angles are usually named with capital letters and the side opposite each angle is named used the related lower case letter (see Figures 12 and 13). Similar triangles have the following useful property:

Figure 12



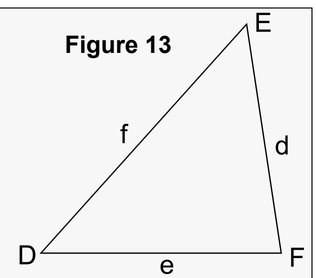
Similar Triangles

Given Δ ABC and Δ DEF as shown.
If $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$,

then Δ ABC and Δ DEF are ***similar triangles*** and corresponding sides are in proportion.

$$\frac{a}{d} = \frac{b}{e} \qquad \frac{b}{e} = \frac{c}{f} \qquad \frac{c}{f} = \frac{a}{d}$$

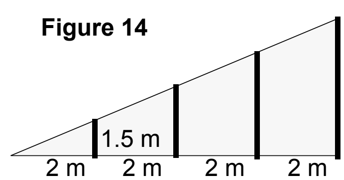
Figure 13



The phrase *corresponding sides* means sides that are in the same relative position in each triangle. This property allows us to find the length of a missing side by setting up and solving a proportion. One important application of similar triangles involves ramps or other inclined planes.

Example 8: The ramp leading up to a loading dock has a base of 8m with vertical braces placed every 2m. If the first vertical support is 1.5 m high, how high is the dock?

Figure 14



solution: Since the two triangles in question are similar, we set up and solve a proportion. Let h represent the height of the dock (see Figure 14).

$$\frac{\text{support height}}{\text{horizontal length}} = \frac{\text{dock height}}{\text{dock length}} \qquad \text{corresponding sides are in proportion}$$

$$\frac{1.5}{2} = \frac{h}{8} \qquad \text{substitute known values}$$

$$6 = h \qquad \text{solve for h}$$

The dock is 6 feet high. ◆

Now try Exercises 47 and 48

• **Additional Applications**

Cubes and rectangular boxes are part of a larger family of solids called **prisms**. A prism is any solid figure with bases of the same size and shape (the top and bottom are both called **bases**). A **right prism** has sides which are perpendicular to the bases, with all cross-sections congruent (see Figures 15 and 16). The volume of a right prism is the area of its base times its height. This is easily seen in the case of a rectangular box: V = LWH = (LW)H = (area of base) · H.

triangular prism

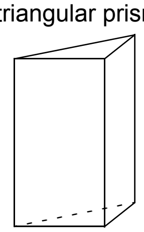


Figure 15

VOLUME OF A RIGHT PRISM
V = Bh,
where B represents the area of the base and h represents the height.

trapezoidal prism

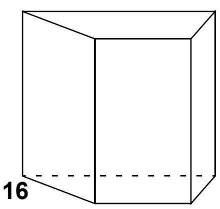
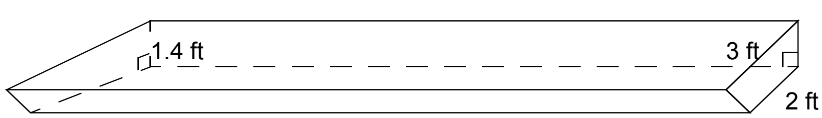


Figure 16

Example 9: The feeding trough at a large cattle ranch is a right prism with trapezoidal bases (see Figure 17). The trough is 1.4 feet high and 38 feet long. If the bases of the trapezoid are 2 feet and 3 feet, how many loads of feed are needed to fill the trough if each load is 54 cubic feet?

Figure 17



solution: Notice the trough is a right trapezoidal prism.

a) Find the area of the trapezoidal bases:

$$A = \frac{h(b_1 + b_2)}{2} \qquad \text{area of a trapezoid}$$

$$A = \frac{1.4(3+2)}{2} \quad \text{substitute 1.4 for } h\text{, 3 for } b_1\text{, and 2 for } b_2$$

$$A = 3.5 \text{ ft}^2 \quad \text{result}$$

The area of each base is 3.5 ft^2.

b) Compute the volume of the trough (the right prism).

$$V = Bh$$

$$V = (3.5)(38)$$

$$V = 133 \text{ ft}^3$$

The volume of the trough is 133 ft^3.

c) Find how many 54 ft^3 loads are needed for 133 ft^3 (divide).

$$\frac{133 \text{ ft}^3}{54 \text{ ft}^3} = \frac{133}{54} \approx 2.463$$

Approximately 2.5 loads of feed are needed to fill the trough. ◆

Now try Exercises 49 through 66

SECTION R.7 EXERCISES

CONCEPTS AND VOCABULARY

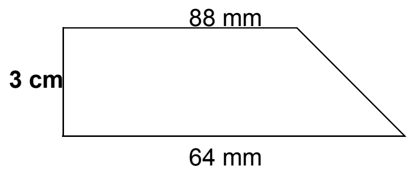
Fill in each blank with the appropriate word or phrase. Carefully re-read the section, if necessary.

1. A geometric figure composed of more than one basic shape is a ____________ figure.
2. A trapezoid is any _________________ that has two _______________ sides.
3. Volume is a measure of the amount of __________ occupied by an object.
4. Capacity is a measure of an object's potential to ________ or __________ something.
5. Discuss/Explain the methods available to find the perimeter, area and volume of composite figures.
6. Discuss/Explain the formula for computing the volume of a prism. Does the concept apply to the formula for the volume of a cylinder? Explain.

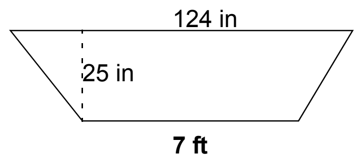
DEVELOPING YOUR SKILLS

Compute the area of each trapezoid using the dimensions given.

7.

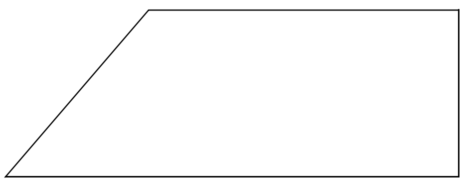


8.



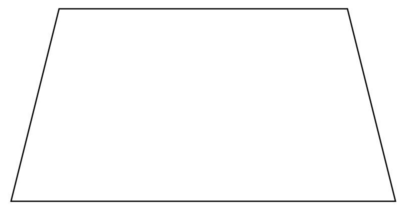
Use a ruler to estimate the area of each trapezoid using the actual measurements.

9. .... in square millimeters



Exercise 9

10. .... in square inches

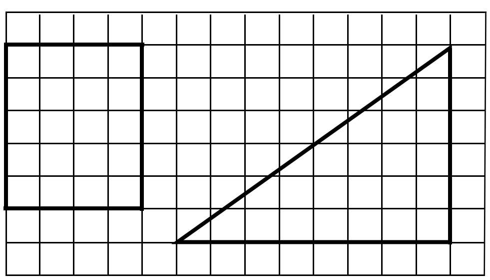


Exercise 10

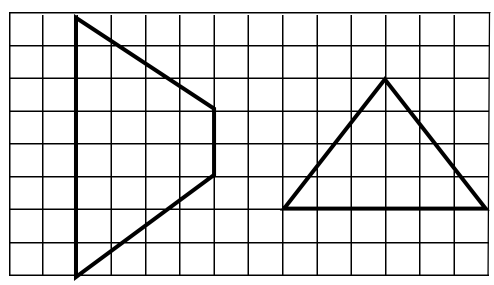
Graph paper can be an excellent aid in understanding why perimeter must be measured in linear units, while area is measured in square units. Suppose the graph paper shown has squares which are 1 cm by 1 cm. Use the grid to answer each question by counting and by computation.

11. What is the perimeter and area of the rectangle shown?

12. What is the perimeter and area of the trapezoid shown?



Exercises 11 and 13



Exercises 12 and 14

13. What is the perimeter and area of the triangle shown?

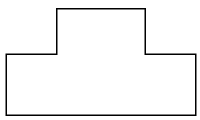
14. What is the perimeter and area of the triangle shown?

15. What is the outer circumference of a circular gear with a radius (center to teeth) of 2.5 cm? Round to tenths.

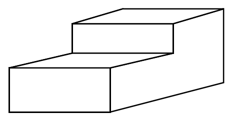
16. If the inner radius of the gear from Example 15 (center to base of teeth) is 2.2 cm, what is the working depth of the gear (height of gear teeth)?

For each figure: a. draw auxiliary lines which partition the figure into basic shapes and label the sides as needed, b. give a verbal model, and a formula model that could be used to complete the exercise.

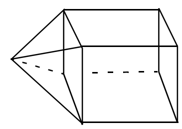
17. Area



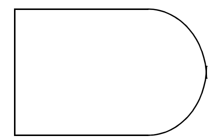
18. Volume



19. Volume

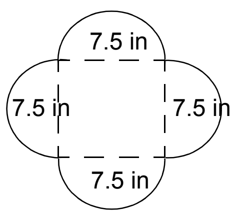


20. Perimeter

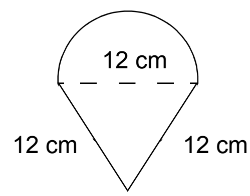


• A composite figure is shown. Compute the perimeter, area, or volume as indicated.

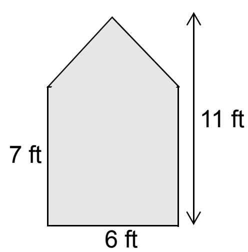
21. perimeter



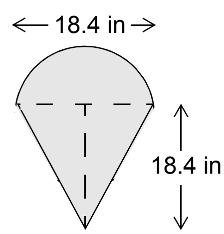
22. perimeter



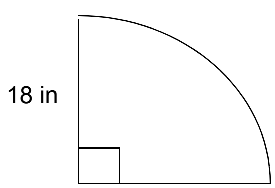
23. area



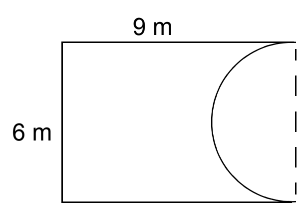
24. area



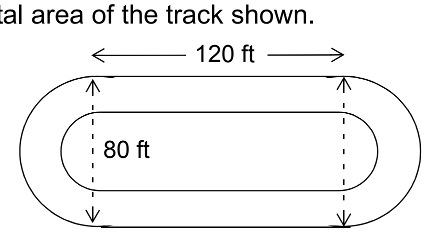
25. area



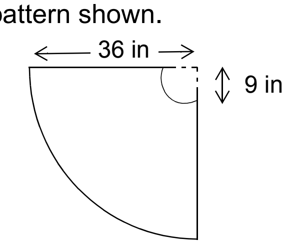
26. area



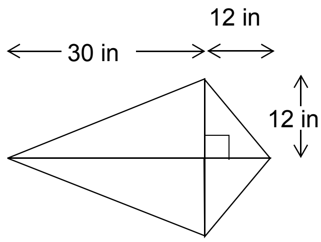
27. Determine the outer perimeter and total area of the track shown.



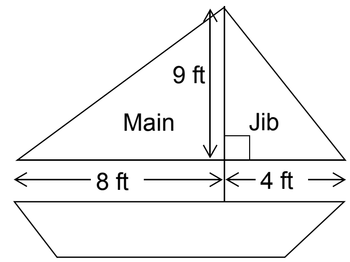
28. Find the perimeter and area of the skirt pattern shown.



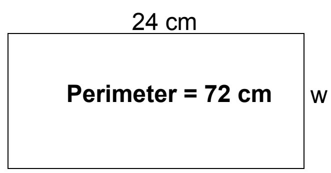
29. How many square inches of paper are needed to cover the kite?



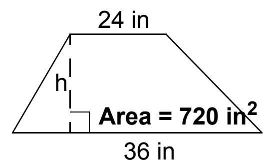
30. Find the area of the main sail and the area of the jib-sail of the sailboat.



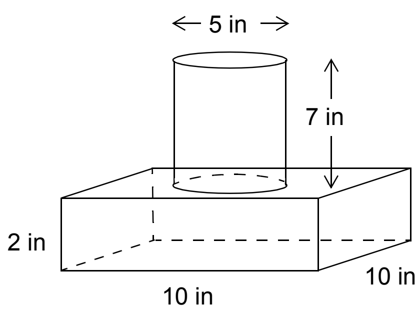
31. Find the length of the missing side.



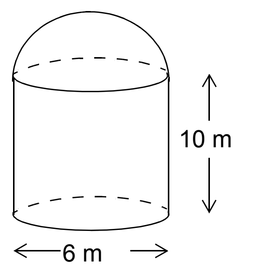
32. Find the height of the trapezoid.



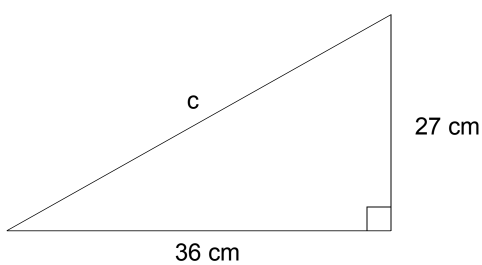
33. volume



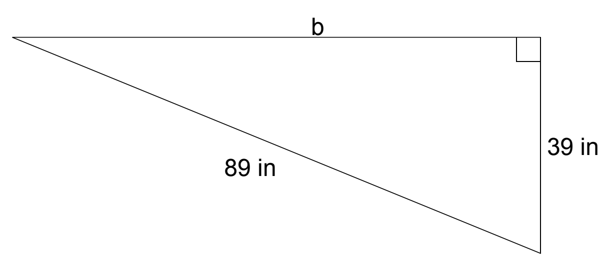
34. volume



35. Find the missing length.



36. Find the missing length.



37. Convert 625 quarts to gallons.

38. Convert 52,600 pounds to tons.

39. Convert 2887.5 in^3 to gallons.

40. Convert 6500 cm^3 to liters.

41. How many gallons in a container with a volume of 623.7 in^3?

42. A container has a volume of 18,200 cm^3. How many liters of liquid will it hold?

43. Convert 300 pounds to kilograms.

44. Convert 95 kilometers per hour to miles per hour

WORKING WITH FORMULAS

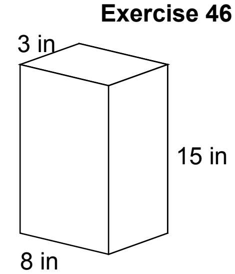
45. Matching: Place the correct letter in the corresponding blank.

A. perimeter of a square	B. perimeter of a rectangle	C. perimeter of a triangle
D. area of a triangle	E. circumference of a circle	F. volume of a cube
G. area of a square	H. perimeter of a trapezoid	I. volume of a rectangular box
J. area of a circle	K. area of a rectangle	L. area of a trapezoid

____ LW	____ $s_1 + s_2 + s_3$	____ πr^2
____ $4S$	____ LWH	____ $\frac{1}{2}BH$
____ $s_1 + s_2 + s_3 + s_4$	____ $2\pi r$	____ S^2
____ $2L + 2W$	____ S^3	____ $\frac{h(b_1 + b_2)}{2}$

Surface Area of a Rectangular Box: SA = 2(LW + LH + WH)

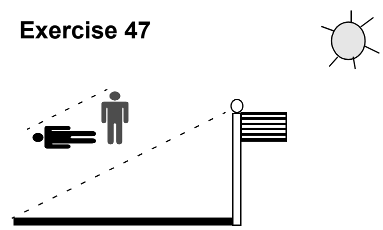
Exercise 46



46. The surface area of a rectangular box is found by summing the area of all six sides. Find the surface area of a box 15 inches long, 8 inches wide and 3 inches high.

APPLICATIONS (round to tenths as needed).

Exercise 47

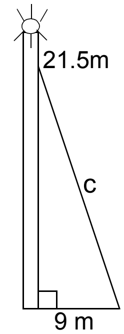


47. **Similar Triangles**: To estimate the height of a flagpole, Mitchell reasons his shadow must be proportional to the shadow of the flagpole due to similar triangles. Mitchell is 4 feet tall and his shadow measures 7.5 feet. The shadow of the flagpole measures 60 feet. How high is the pole?

48. **Similar Triangles**: A triangular image measuring 8 in x 9 in x 10 in is projected on a screen using an overhead projector. If the smallest side of the projected image is 2 feet, what are the other dimensions of the projected image.

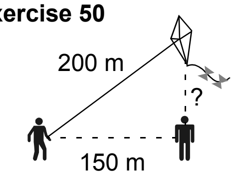
49. **Length of a Cable**: A radio tower is secured by cables which are clamped 21.5 m up the tower and anchored in the ground 9 m from its base. If 30 cm lengths are needed to secure the cable at each end, how long are the cables? Round to hundredths of a meter.

Exercise 49



50. **Height of a Kite:** Benjamin Franklin is flying his kite in a storm once again ... and has let out 200 meters of string. John Adams has walked to a position directly under the kite and is 150 meters from Ben. How high is the kite to the nearest meter?

Exercise 50



51. **Unit Conversions**: A baseball pitcher at the major-league level can throw the ball around 145 feet per second. What is the speed of the ball in miles per hour?

Exercise 51

52. **Unit Conversions**: A U.S. citizen living in Brazil wants a Brazilian carpenter to build a wooden chest 3 feet high, 2.5 ft wide, and 4 ft long. If the carpenter knows only the Metric System, what dimensions should the carpenter be given?

53. **Most Economical Purchase:** Missy's Famous Pizza Emporium is running a special -- one large for \$8.99 or two mediums for \$13.49. If a large pizza has a 14 inch diameter and a medium pizza has a 12 inch diameter, which is the better buy (least expensive per square inch)?

54. **Most Economical Purchase:** A large can of peaches costs 79¢ and is 15 cm high with a radius of 6 cm. A small can of peaches costs 39¢ and is 15 cm high with a radius of 4 cm. Which is the better buy (least expensive per cubic cm)?

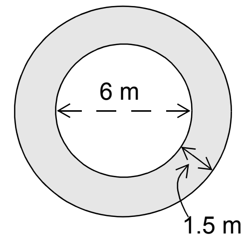
55. **Volume of a Jacuzzi**: A certain Jacuzzi tub is 84 inches long 60 inches wide and 30 inches deep. How many gallons of water will it take to fill this tub?

56. **Volume of a Bird Bath**: I am trying to fill an outdoor bird bath, and all I can find is a plastic dish pan 7 in. high, with a 12 in. by 15 in. base. If the bird bath holds 15 gal, how many trips will have to be made?

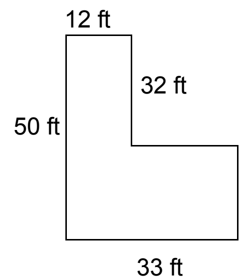
Exercise 56

57. **Paving a Walkway**: Current plans call for building a circular fountain 6 meters in diameter with a circular walkway around it that is 1.5 meters wide. a. What is the approximate area of the walkway? b. If the concrete for the walkway is to be 6 cm deep, what volume of cement must be used? c. If the cement costs \$125 m^3 and the sales tax is 7%, what is the cost of paving the walkway?

Exercise 57



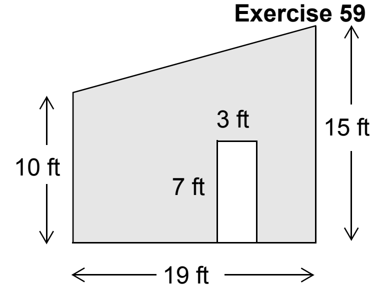
Exercise 58



58. **Paving a Driveway**: A driveway and turn-about has the dimensions shown in the figure. a. What is the area of the driveway? b. If the concrete was poured to a depth of four inches, what volume of cement was used to construct the driveway? c. If the cement cost \$3.50 per cubic foot and a 5.65% tax was paid, what was the total cost of the driveway?

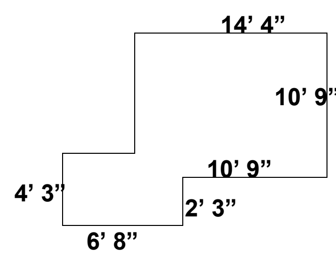
59. **Cost of Drywall**: After the studs are up, the wall shown in the figure must be covered in dry wall. a. How many square feet of dry wall are needed? b. If drywall is sold only in 4 ft by 8 ft sheets, approximately how many sheets are required for this job? c. If drywall costs \$8.25 per sheet and a 5.75% tax must be paid, what is the total cost?

Exercise 59



60. **Cost of Baseboards**: The dimensions for the living room/dining room of a home are shown. a. How many feet and inches of molding are needed for the baseboards around the perimeter of the room? b. If the molding is only sold in 8 foot lengths, how many are needed? c. If the molding costs \$1.74 per foot and sales tax is 5.75%, what is the total cost of the baseboards?

Exercise 60

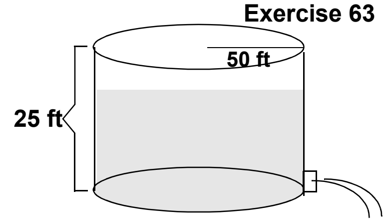


61. **Dimensions of an Index Card**: A popular sized index card has a length that is one inch less than twice its width. If the card has an area of fifteen square inches, find the length and width.

62. **Dimensions of a Ruler**: The plastic ruler that Albert uses for graphing lines has a length which is one centimeter more than seven times its width. If the ruler has an area of thirty square centimeters, find its length and width.

63. **Tracking an Oil Leak**: At an oil storage facility, one of the tanks has a slow leak. If the tank shown was full to begin with, how many gallons have been lost at the moment the height of the oil in the tank is 24 feet?

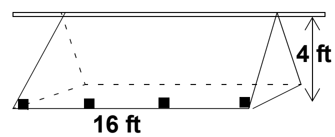
Exercise 63



64. **Tracking a Water Leak**: A cylindrical water tank has developed a slow leak. If the tank is standing vertically and is 4 feet tall with a radius of 9 inches, how many gallons have leaked out at the moment the height of the water in the tank is 3 feet?

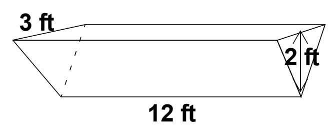
65. **Volume of a Prism**: Using a sheet of canvas that is 10 feet by 16 feet, a simple tent is made using a long pole through the middle and pegging the sides into the ground as shown. If the tent is 4 feet high at the apex, what is the volume of space contained in the tent?

Exercise 65



66. **Volume of a Prism**: The feeding trough at a pig farm is a right prism with bases that are isosceles triangles (two equal sides). The trough is 3 ft wide, 2 ft deep and 12 ft long. If each load is 7 ft^3, how many loads of slop are needed to fill the trough?

Exercise 66



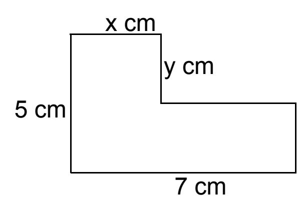
EXTENDING THE CONCEPT

67. The area (in cm^2) of the region shown in the figure is:

    a. 35 – xy   b. 35 + xy   c. 35 + 7y – 5x

    d. 35 + xy – 7y   e. 35 – xy + 5x   f. none of these

Exercise 67



68. A 2-quart saucepan is 75% full of leftover soup. Which plastic container will best store the leftover soup (i.e. with minimum wasted volume)?

    a. a hemispheric bowl with radius 3.5 in.   b. a cylinder with radius 3 in. and height 5 in.

    c. a rectangular container 4 in. x 6 in. x 3 in.   d. None of these will hold the soup.

Section R.7 Student Solutions

1. composite

3. space

5. Answers will vary.

7. $A = 2280\ mm^2$ or $22.8\ cm^2$

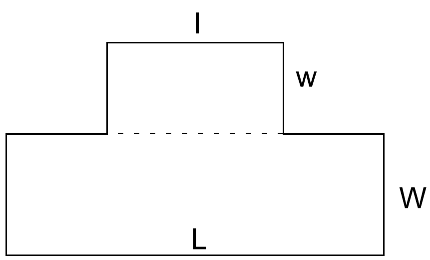
9. $A \approx 1111\ mm^2$

11. $P = 18$ cm
$A = 20\ cm^2$

13. $P = 24$ cm
$A = 24\ cm^2$

15. $C \approx 15.7$ cm

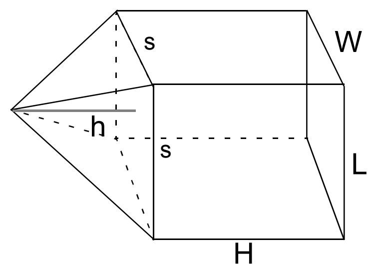
17. a.



b. Total Area = area of larger rectangle plus area of smaller rectangle

c. $A = LW + lw$

19. a.



b. Total volume = volume of rectangular solid plus volume of pyramid

c. $V = LWH + LWH$
or
$V = LWH + s^2h$

21. $15\pi \approx 47.1$

23. $A = \frac{bh}{2} + LW$
$A = 54\ ft^2$

25. $A = \frac{\pi r^2}{4}$
$A = 81\pi$
$A \approx 254.5\ in^2$

27. $P = 2L + \pi d$
$P = 240 + 80\pi$
$P \approx 491.3$ ft;
$A = LW + \pi r^2$
$A = 9600 + 1600\pi$
$A \approx 14{,}626.5\ ft^2$

29. $504\ in^2$

31. $W = 12$ cm

33. $V = LWH + \pi r^2 h$
$V = 200 + 43.75\pi$
$V \approx 337.4\ in^3$

35. $c = 45$ cm

37. 156.25 gal

39. 12.5 gal

41. 2.7 gal

43. 661.5 kg

45. B: LW   C: $s_1 + s_2 + s_3$ J: pr^2

A: 4S   I: LWH   D: $\frac{1}{2}BH$

H: $s_1 + s_2 + s_3 + s_4$ E: 2pr   G: S^2

B: 2L + 2W   F: S^3 L: $\frac{h(b_1 + b_2)}{2}$

47. 32 ft

49. 23.91 m

51. about 98.9 mph

53. one large at about 5.8¢/in^2

55. about 654.5 gal

57. (a) 63.6 m^2
(b) 3.8 m^3
(c) about $508.25

59. (a) 216.5 ft^2
(b) about 7
(c) about $61.07

61. 5 in x 3 in

63. $2500\pi \approx 7854$ ft^3
about 58,752 gal

65. 192 ft^3

67. d. 35 + xy – 7y